Quantitative Reasoning Across Campus

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Abstract: *Quantitative reasoning takes on different meanings based on experience and background. Across the curriculum students have various needs for their quantitative reasoning experience. As a result, at our university students have a range of choices that more closely fit their future needs. Seven different departments across campus offer quantitative reasoning courses engaging students through a variety of avenues.*

Keywords: quantitative reasoning, quantitative literacy, interdisciplinary, alternative pathways

INTRODUCTION

The need for students to attain quantitative reasoning (QR) or quantitative literacy (QL) is not a new idea. The nation needs employees who are quantitatively literate across the spectrum of jobs (Rocconi, et al., 2013). Steen declares that there are "estimates that 60% of all new jobs in the early 21st century will require skills possessed by only 20% of the current workforce" (Steen, 2004). This should be a cause for concern.

The faculty at our small, liberal arts, private university in the Midwest identified quantitative reasoning as a valued skill for all students. As a result, all students are required to complete a quantitative reasoning course as part of their graduation requirements.

Although people often think of college algebra when it comes to a quantitative reasoning course in higher education, several authors have pointed out that there are other opportunities to teach quantitative reasoning skills across the college curriculum (Elrod, 2014; Mathematical Association of America Committee on the Undergraduate Program in Mathematics, 2014; Steen, 2004; Sweet & Strand, 2006). A quantitative reasoning course is context driven, in other words, the topics covered are relevant to the enrolled students' courses and lives (Briggs, 2018). Quantitative reasoning is more than math and includes problem-solving skills and applying reasoning and data interpretation skills in everyday decision making as well as critical thinking (Botts, Carter, & Crockett, 2018; Briggs, 2018). It is a habit of mind (Steen, 2004). At our university, we've taken this notion to heart.

QUANTITATIVE REASONING

At Millikin University, quantitative reasoning is defined broadly as students in different areas have various needs. A sociologist needs to understand and interpret a statistical analysis while an exercise science major may need to interpret a change in a client's performance. A philosophy or political science student may take the information learned in Introduction to Logic and use it to make a sound argument while an English major may take the information learned in a financial mathematics unit and successfully negotiate a manageable interest rate on a car loan or mortgage. The needs are vast and extensive.

Institutions of higher education across the country have been shifting away from traditional mathematics courses (such as College Algebra) to QR courses as students need QR/QL skills to be successful in their professional lives (Mathematical Association of America Committee on the Undergraduate Program in Mathematics, 2014; Steen, 2004). Institutions have created alternative courses to the traditional college algebra. The Carnegie Foundation introduced Quantway/Statway that is used by several schools (Howington, Hartfield, & Hillyard, 2015). Across the country, colleges, universities, and community colleges of all sizes are implementing QR programs (Gillman, 2006; Tunstall, et al., 2016). This is evident in the number of presentations that are happening at national meetings relating the implementation and changes to quantitative reasoning requirements and programs.

The concept of a QR requirement is not novel. However, giving students more non-traditional options outside of the mathematics department is still a growing area. Students can take a class in their major that satisfies the requirement and allows them to critically think and explore topics that are relevant to their coursework and life.

For consistency across campus, our University has identified two learning goals for quantitative reasoning courses regardless of what QR course a student may take. Students who complete a QR course will demonstrate the ability to:

> "(1) use deductive reasoning in a formal, symbolic, axiomatic system, and
>
> (2) apply the theorems of the system to solve appropriate problems."
>
> (Millikin University, 2018)

When the University's Quantitative Reasoning Task Force created the QR learning goals and rubric for assessment, the group was 50% mathematicians. As a result, the descriptions sound very mathematical. However, since the group was 50% non-mathematicians, the descriptors are very broad to allow for wide interpretations by various areas. The University's QR rubric (see Table 1) varies greatly from the Association of American Colleges and Universities' (AAC&U) Quantitative Literacy VALUE rubric as our University considers two areas (with three ratings) and AAC&U considers six areas on a 4-point scale (AAC&U, 2009). It is important to note that Millikin's QR work pre-dates the AAC&U's release of their rubric by more than two years.

In addition, the learning goals are merely a part of what the University hopes to accomplish through the QR courses. The University wants to offer basic quantitative skills necessary for the students to have professional success regardless of their chosen career path. This work includes the understanding of "basics of numerical, statistical, or logical analysis" (Millikin University Quantitative Reasoning Task Force, 2007). This is fundamental to success in this world and there is no area that is exempt from this type of knowing. In addition, we want "To prepare students to be competent citizens by developing the quantitative skills necessary to understand fundamental reasoning that involves numbers, statistics, or logical reasoning". In today's world, it is necessary to understand "graphs, detect faulty statistical analysis, or spot basic flaws in reasoning". The QR courses help develop these skills (Millikin University Quantitative Reasoning Task Force, 2007).

	Good	Average	Poor
Goal 1: Deductive Reasoning	• Student uses proper symbolic notation in the context of the stated problem • Student manipulates these symbols according to the rules of the axiomatic system • Student achieves desired directive of the problem with no (or a few minor) errors	• Student work achieves exactly two of the following: 1. Student uses proper symbolic notation in the context of the stated problem 2. Student manipulates these symbols according to the rules of the axiomatic system 3. Student achieves desired directive of the problem with no (or a few minor) errors	• Student work achieves no more than one of the following: 1. Student uses proper symbolic notation in the context of the stated problem 2. Student manipulates these symbols according to the rules of the axiomatic system 3. Student achieves desired directive of the problem with no (or a few minor) errors
Goal 2: Theorem Application	• Student correctly selects theorem(s) necessary to solve stated problem · Student performs all necessary calculations needed to apply theorem with no (or a few minor) errors • Student uses selected theorem(s) to form a	• Student correctly selects theorem(s) necessary to solve stated problem ·Student's work falls into one of the following two categories: 1. Student made major computational errors, but made a correct conclusion based on the computations	• Student work falls into one of the following two categories: 1. Student did not correctly select theorem(s) necessary to solve stated problem 2. Student did correctly select theorem(s) necessary to solve, but made major computational errors and made an incorrect

	correct conclusion on the basis of the computations	2. Student made no (or a few minor) errors in computations, but made an incorrect conclusion made on the computations	conclusion based on the computations

Table 1: Rubric for assessing student achievement of quantitative reasoning learning outcome goals (Millikin University Quantitative Reasoning Task Force, 2007)

To establish that students meet a baseline of mathematical skills, all students must meet a mathematics competency requirement in addition to the QR requirement. The competency requirement may be met by an ACT mathematics score of 22 (or equivalent) or by completing the University's Intermediate Algebra course.

QUANTITATIVE REASONING COURSES

Currently, there are eight courses offered that can satisfy the QR requirement. (See Table 2.) The number of offerings has increased from the initial few as the Department of Mathematics and Computational Sciences has encouraged colleagues across campus to offer courses that are more suitable to their majors or have looked for ways to make connections in their current course offerings. For example, the two semesters of music theory that satisfy the QR requirement have long been a requirement for music majors. Through discussions between mathematics and music faculty it was determined that what the students were already doing in the courses satisfied the QR goals. It was a matter of documenting the experience for assessment purposes.

Course Satisfying Quantitative Reasoning	Department that Offers the Course
College Algebra	Mathematics and Computational Sciences
Finite Mathematics	Mathematics and Computational Sciences
Introduction to Logic	Philosophy
Analysis of Biological Data	Biology
Statistical Methods in the Behavioral Sciences	Psychology/Sociology
Music Theory (2 semesters)	Music
Technical Direction	Theatre
Honors Seminar (Infinity, Ethnomathematics, Big Data, etc.)	Interdepartmental (various faculty from across campus)

Table 2: Course offerings that satisfy the quantitative reasoning goals

Course Examples

Since there are currently eight courses across seven departments that satisfy the QR learning goals, there is a wide range of ways in which the goals are met. The following is a sampling of the way the different courses meet the QR learning goals. We purposely are leaving out mathematics courses in the examples since they are traditionally considered QR courses, and there are existing examples in other publications (Hastings, 2006; Tunstall & Bossé, 2015).

Analysis of Biological Data

The Analysis of Biological Data course is a sophomore-level class. It is a course intended for biology majors who are interested in understanding the mathematical reasoning of statistical tests that biologists regularly use. The focus is on selecting the appropriate statistical test, designing an experiment that is most appropriate for statistical analysis, and interpreting and presenting the results of the tests.

The second learning goal of theorem application may be met through a case study. Students are given a real data set that includes descriptions of the variables. (Multiple data sets are distributed to students so there isn't unwanted collaboration.) Students formulate a question from which they can test a statistical null hypothesis. Then, the student must select the appropriate statistical test given that the only information they have is the data set and the description of variables. See Figure 1 for a case study item from Analysis of Biological Data.

Music Theory

The two-semester sequence of Music Theory I and II is a foundational course for all music majors and minors. In the courses, students learn the principles of music such as key, scales, triads, chords, chord modulations, and critical listening.

The first learning goal of deductive reasoning may be met by completing a Roman numeral analysis of a piece of music. A Roman numeral analysis of a piece reveals items such as chord progressions, inversions, and augmentations. Roman numerals represent chords (triads). Typically, upper case represents major chords and lower case represents minor chords. Arabic numerals complement the Roman numerals by denoting the inversion of a chord. A superscript of $^{+}$ (plus sign) indicates an augmented triad and a ° (small circle) indicates a diminished triad.
The analysis allows for deeper musical understanding of a piece. See Figure 2 for a Roman numeral analysis item from Music Theory.

Data Set: Gonadotropin-releasing hormone challenge of Florida scrub-jays

Description of Variables:

Bird: Bird band combination

Territory: Territory in which bird was captured

CORT: corticosterone (stress hormone) level at time of capture (baseline)

BaselineLH: luteinizing hormone (LH) level at time of capture

@15minLH: 15 minute after capture LH levels

BaselineT: testosterone level at time of capture

@30minT: testosterone level 30 minutes after capture

Treatment: 1 = saline injection only (control), 2 = gonadotropin-releasing hormone injection

AgeCategory: 1 = 2-4 years, 2 = 5-7 years, 3 = 8-12 years

LHResponse: Difference between 15 minute and baseline LH levels

TResponse: Difference between 30 minute and baseline testosterone levels

For this assignment, you must:

1. Provide a statement of the question of interest.
2. Provide a one paragraph justification of choice of statistical test(s).
3. Email the full SPSS Output showing the tests you have run.
4. Include discussion of the assumptions of the test you choose and how you tested if those assumptions were met and if you violated them, justify that.
5. Type a full paragraph describing the study, with sample size, null and alternative hypotheses, relevant test statistic and degrees of freedom, p-values, post hoc tests/results (if applicable), effect

Figure 1: Analysis of Biological Data Case Study Task (Wilcoxen, 2017).

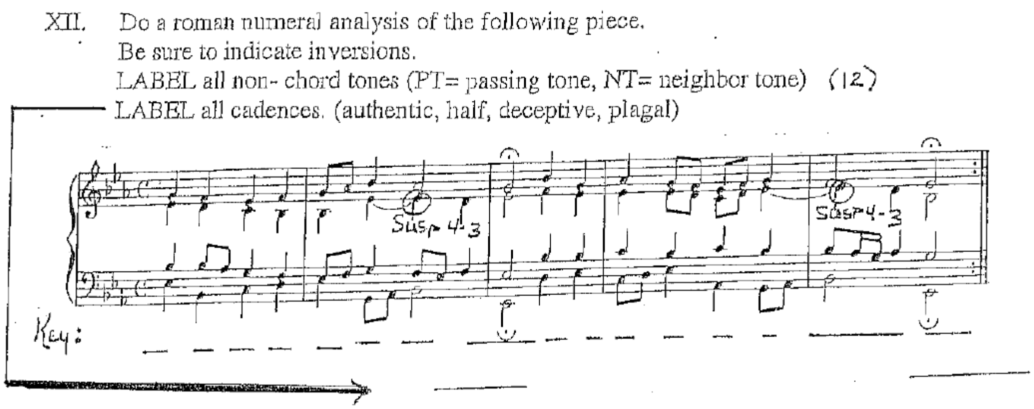

Figure 2: A Roman Numeral Analysis Task from Music Theory (Nicholson, 2014).

Introduction to Logic

The Introduction to Logic course is a freshman-level course offered by the philosophy department. It is a required course for philosophy and pre-law majors. The goal of the course is to promote the development of sound reasoning. The course covers concepts such as deductive and inductive reasoning, critical thinking through methods such as logic puzzles, Venn diagrams, and symbolic proofs.

The first learning goal of deductive reasoning may be met by determining if an argument is valid, conditionally valid, or invalid using a Venn diagram. The second learning goal of theorem application is met through the construction of a truth table to determine if an argument is valid or invalid, and if the argument is invalid determining what line(s) of the truth table fails.

Honors Seminars: Ethnomathematics and Infinity

Within the honors program, students take three seminars that are topic-specific courses that introduce students to the ways of knowing in the discipline. Seminars are offered in creative arts, humanities, natural science, social science, and mathematics. In the mathematics honors seminar, topics have included big data, data analytics, ethnomathematics, and infinity. In ethnomathematics, the second learning goal of theorem application may be met through a question requiring students to calculate the date of Easter for the year 2026 and show their work. In Infinity, the second learning goal of theorem application is met through an application of Cantor's theorem. See Figure 3 for an example.

Define a "laugh" to be a countably infinite sequence of the words "HA" and/or "HO". For example, HAHAHAHAHA.... and HAHOHAHOHAHOHAHO... and HAHAHOHAHOHOHAHOHA ... are all laughs. Prove that the set of laughs is uncountable.

Figure 3: Application of Cantor's Theorem Item from Infinity (Rauff, 2008).

Technical Direction

Technical Direction is a senior-level elective class that builds on the foundational skills learned in Scenic Construction, a freshman-level course required for design and production majors (theatre) and serves as an option to meet the technical theatre requirement for other theatre degree programs. Students learn skills in scenic construction, drafting, rigging, and problem solving through lecture and hands-on activities. Students are also introduced to mechanics, scenery automation, and budget planning for the cost of scenery in a production.

The second learning goal of theorem application is met by the item in Figure 4. In the rigging problem, students must apply their knowledge regarding force, stresses, and strains on beams in different situations. Specifically, the students much determine the torque based on the load and shaft length.

What quantity of torque is necessary on the shaft to keep this sheave stationary?

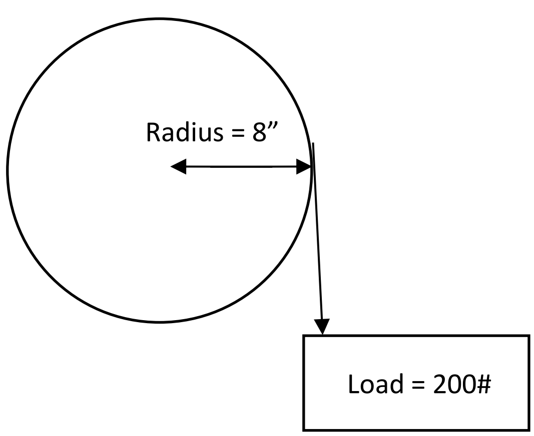

Figure 4: Rigging Item from Technical Theatre (Black, 2021).

Statistical Methods in Behavioral Sciences

Statistical Methods in Behavioral Sciences is a freshman-level class that serves as a required course for sociology, psychology, nursing, human services, and criminal justice majors. Several other students opt to take the course to meet their QR requirement. The focus is on research design and applying elementary statistical methods to psychological and sociological data using descriptive and inferential statistics.

The first learning goal of deductive reasoning is met by the exam problems in Figure 5. In the first item students are required to conduct hypothesis testing for a chi square and interpret the results in terms of the context of the problem. In the second item students must calculate the least squares regression line, conduct hypothesis testing on the slope for the data, and then interpret the results in terms of the problem context.

16.) A medical researcher is interested in determining if there is a relationship between adults over 50 who walk regularly and low, moderate, and high blood pressure. A random sampled of 240 adults over 50 is selected and the results are given below. Conduct hypothesis testing for independence at Use $\alpha = .01$.

Blood pressure

	Low	Moderate	High
Walkers	71	32	23
Non-walkers	20	27	67

17.) Interpret the results of the hypothesis testing conducted in question #16 i**n the terms of the problem**. (You can't just say reject or fail to reject. That is part of your answer for 16.)

11.) Suppose you are studying skill development among 14-year-old pianists in a city music club and wonder if time/years of membership in the club (X) is related to the number of awards and trophies won (Y). Calculate the least squares lines for these data. Use the back of this page if needed.

Years in Music Club	Merit Awards and Trophies
7	8
4	6
2	1
4	3

Figure 5: Hypothesis Testing Items from Statistical Methods in Behavioral Sciences (Collinsworth, 2022).

DATA COLLECTION

Data is collected at the end of each semester by the instructors teaching the courses. The University's QR coordinator organizes the data collection. Instructors identify questions on the final exam (or equivalent culminating assessment) that demonstrate satisfaction of the QR learning goals. After completion of final exams/culminating assessments, instructors randomly select five samples of each of the questions that satisfy the goals and indicate at what level the learning goal was met or not. A student response is classified as good, average, or poor. The rating is assigned by the course instructor.

The rubric used for assessment in all QR courses is in Table 1. The information is submitted to the QR coordinator who organizes and synthesizes the data. An annual QR report is written and submitted to the administration.

RESULTS

The collection of data is an ongoing process so at any point in time results are merely a snapshot of what students are accomplishing. Recent results of the past 9 semesters of students enrolled in the QR courses can be found in Table 3. (As more courses now count for QR, earlier data with mostly mathematics courses creates an incomplete picture.) It can be observed that for both goals, approximately half the students or more are performing at what is considered the good level. As a university, we would like to see movement from average to good and poor to average. However, it is a challenge to cause movement collectively when the students are in different areas. Instructors must reflect on their individual courses to determine what the best path is for their course to achieve greater student success. Less than a quarter of the students are performing poorly for each goal. We are pleased with this low percentage.

	Good	Average	Poor
Goal 1	115 (49.57%)	60 (25.86%)	57 (24.57%)
Goal 2	132 (56.41%)	62 (26.50%)	40 (17.09%)

Table 3: Student performance on quantitative reasoning learning goals

CONCLUSIONS

The quantitative reasoning learning goals in their current format were put into place in 2007. At that time, the only courses that satisfied the QR requirement were mathematics courses, behavioral statistics, and Introduction to Logic from the philosophy department. Over the years, the Mathematics and Computational Sciences department (where the QR coordinator comes from) has encouraged other areas to submit courses in their area that they believe meet the QR learning goals. We believe that a course in students' discipline or related area has a greater impact on the students and their understanding of quantitative reasoning, and it enhances their skills as indicated by Briggs (2018). Over the last several years, with repeated reaching out to areas across campus and working together, the QR offerings have grown to the eight courses previously identified.

The Mathematics and Computational Sciences department continues to encourage areas to submit courses they believe satisfy the QR learning goals or work together to develop a course or enhance parts of a course to better serve the needs of the students in their given areas. More areas are reaching out with proposals for courses to count as QR. This begins a conversation between the mathematics department and the instructor, with the QR coordinator as the go between, as to how the class can sufficiently meet the University's QR goals. Faculty in other areas are interested in finding courses within their disciplines that are possible candidates for QR as it reduces the number of courses their students must take outside the discipline. If a major course also satisfies the QR requirement, then the course can be "double-dipped", i.e., it counts as a major course and satisfies the QR requirement. Additionally, our faculty understand that it is more beneficial to students for the QR experience to be context-driven (Briggs, 2018) in an area that students can relate to. Students are completing tasks relevant to their fields of study and gaining critical thinking skills necessary to succeed outside of the institution.

With more options for QR, the enrolment of the College Algebra course is now mostly STEM majors. As a result, the D-F-W rate is more reflective of students' success in the course as students with no interest (and some would argue no need) in College Algebra are taking a more appropriate course for QR. This has opened seat availability in College Algebra which was needed as our STEM enrolment has increased each of the last three years.

We hope to continue to find ways to work with areas on campus to find the best option for students in their respective areas to meet the QR requirement and be a meaningful learning experience. A context-driven QR course will result in a more meaningful experience for students. The more opportunities that we (and other institutions) can provide for students the better educational experience students will have. We encourage other institutions to explore alternative pathways to meeting QR goals including reaching out to colleagues across their campuses to work towards better and more appropriate options for students.

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