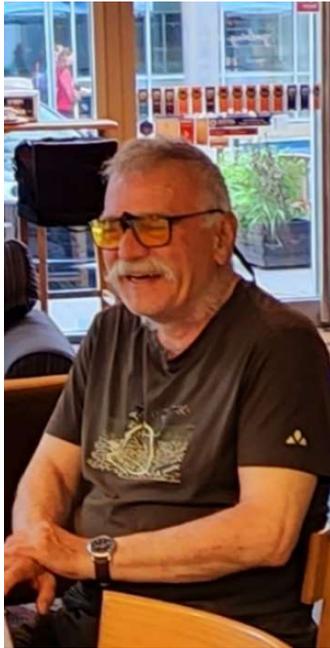


Editorial from Bronisław Czarnocho, the Chief Editor of MTRJ



We would like to introduce this issue of MTRJ through two papers on learning and teaching trigonometry, the subject of mathematics whose pedagogy has not yet been deeply investigated. It could be due to the trigonometry's position between algebra and geometry. Such a position of a mathematical subject between two different areas of mathematics, while possibly difficult to address yet it hides within itself very creative possibilities, whose one of the primary examples is algebraic geometry. One can therefore surmise that pedagogy of trigonometry might hide some new and interesting ideas.

The first paper by colleagues from Indonesia addresses the connection between epistemological obstacles manifested by students and their misconceptions. Their work focuses on two such obstacles: the relationship between degrees and radians as measures of angles and related to it the meaning of π : $22/7$ or 180 .

The second paper by the colleague from Malaysia addresses more limited issue of the misconception related to finding maxima and minima of trigonometric functions. At the same time, the author provides a clearly designed Japanese Lesson Study in relevant

trigonometry as an example of the helpful professional development of teachers; the author finds out that the teachers' main obstacle is the successful analysis of the function $y = a \sin x \pm b \cos x$ and $y = a \sin kx \pm b \cos kx + c$. Interestingly enough teachers could deal with such functions if a , b , k and c are given numerically. Consequently, the problem may lie in the generalization process (quite involved as included examples show).

The third paper in this series by Edo and Tasik relates to student misconceptions in algebra, which occur in the context of solving Pisa-like problems of high difficulty. These misconceptions undermine student competence in algebraic modelling, and at the same time they show the serious absence of conceptual understanding as it relates to the role of algebraic symbols.

The two following papers have also an internal relationship. While one team of colleagues from Indonesia (Wijaya et al) investigates the development and role of analytic questions asked by students during the process of learning, another colleague, Aloisius Son from the same country investigates the synthetical abilities of students in terms of their ability to connect different concepts in mathematics as well as in the relations of mathematics with real World and other domains of science. Vijaya et al find that analytical questions, within a collaborative framework are generated by weaker students towards stronger students in a group aimed mostly at cognitive understanding. Their work is analyzed with the help of the revised Bloom's taxonomy. They have

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chosen five stages of the learning taxonomy: observing, asking, experimenting, associating, and communicating. They found out that the first stage of observing is the dominant stage when analytical questions are formulated and the last stage of communicating is where the least number of such questions appear.

On the other hand, Son has found out that the principles characterized by the CORE RME model, that is connecting, organizing, reflecting, and extending used to solve real world problems result in the highest degree of the development of student connecting abilities, as compared with other models.

Comparing work conducted in both papers we see that the lower stages of the Bloom taxonomy encourage analytical questions while the higher stages of the taxonomy promote synthetic abilities of students.

The next two papers come from Eastern Europe which is at present engaged in the tragic in my opinion war. We welcome both papers as the expression of commitment to reason and peace. Both papers are concerned with the efficiency in our work; the first one with the efficiency of teachers' assessment in courses with large (circa 1000) students using computer generated test questions, the second one with the efficiency of understanding mathematics by medical students pointing to the special role of visual representation. Both papers refer to courses in advanced mathematics. Colleagues from Russia bring forward a very nice metaphor for the composition of functions: the nested matryoshka dolls.

Following our regular by now Problem Corner, we present an unusual paper submitted by the 19-year-old student of the Indian Institute of Science Education and Research in Bhopal, Samir Sharma. Samir was inspired by the challenge posted by Professor James Tanton's Youtube channel (<https://www.youtube.com/watch?v=ImEcUQFw6DU>) titled Math Mystery for Young Mathematicians and proved an extension of properties shown there. for the relationship between loops, intersections and enclosed spaces in a two dimensional space. It might of interest for the reader to look up that video and then to look what a nice theory of loops, intersections and spaces has been created by Sameer Sharma.

The teaching-research report by colleagues from Germany, Caspari – Sadeghi et al, addresses an interesting theme of the role of student self-assessment with the help of a relatively new instrument called Certainty-Based Marking (CBM). The goal of CBM instrument is to cause student reflection upon their own answers and thus to increase student involvement in STEM learning. Student self-assessment is a subtle approach, it can play both positive and negative role in student motivation.

The next paper by colleagues from Indonesia by Ary Woro Kurniasih et al, addresses equally interesting and subtle work, facilitation of creative thinking by teachers in mathematics. The authors use the traditional by now approach grounded in Guilford/Torrance formulation of fluency, flexibility and originality as the critical markers.

The presentation by colleagues from Spain, Gavillán Isquierdo et al introduce here at MTRJ a new approach to learning formulated in the last decade and called called commognition, a purely socio-

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cultural approach based upon the analysis of discourse in mathematics classes. The authors focus their attention on the commognitive conflicts, which in that approach are the sources of learning, and in particular of learning through creativity. The authors have found object level and metalevel conflicts, each type related to learning at different cognitive levels.

We complete the issue with the review of a mathematics book introducing Gödel incompleteness theorem by Powell and Trimmer from US through a nicely narrated story by Hiroshi Yuko who specializes in writing books about mathematics from the student point of view. Nice and helpful reading about one of the deepest results in mathematics.

One cannot help but to admire the intensity of Indonesian work in Mathematics Education: 50% of the presented papers in this issue come from Indonesia.

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