Editorial from Rully Charitas Indra Prahmana, Southeast Asia Editor of MTRJ

Freudenthal’s ideas on mathematics stated that mathematics is a human activity and must be connected to reality. This has influenced the learning of mathematics all over the world. Realistic Mathematics Education (RME) has been adopted in many countries, including Indonesia. The Indonesian version of RME is known as Pendidikan Matematika Realistik Indonesia (PMRI) or Pendidikan Matematika Realistik (PMR). RME continues to grow in Indonesia, as seen from the increasing number of RME research. There are multiple variations of RME in Indonesia, such as design research, qualitative research, development research, and mix method research approach. RME in Indonesia for two decades has been discussed by Zulkardi, Ratu Ilma Ali Haji, and Sabariah, Uswatun Hasanah (University of Nahdlatul Ulama NTB, Indonesia) develop intervention of learning to promote students’ understanding of percentage. Their findings show that their design supports the students in developing their understanding of several fundamental ideas of percentage. Furthermore, the second paper entitled “Learning Trajectory of Algebraic Expression: Supporting Students’ Mathematical Representation Ability” written by Cut Khairunnisa, Rahmah Johar, Yuhasriati, Cut Morina Zubainur, Suhartati, and Putri Sasalia from Research Center of Realistic Mathematics Education, Universitas Syiah Kuala, Indonesia. In this paper, they evaluate the RME-based learning trajectory oriented to enhance students’ mathematical representation ability on algebraic expression. Their findings indicated that the designed RME-based learning trajectory (LT) oriented to support students' mathematical representation ability in algebraic expression has been valid and could be implemented in the pilot experiment. On the other hand, the third paper design the learning trajectories in the topic of Quadrilateral applying the RME. This learning trajectory consist of four activities, i.e., origami shape, finding the properties, solid (stacking sticks), and origami puzzle. From these activities, students can understand the concept of a quadrilateral smoothly. Lastly, Farida Nursyahidah and Irkham Ulil Albab examines a mathematics learning design on the area and volume of cylinders using ethnomathematics context carried by traditional cake mold assisted by GeoGebra. Students show excellent reasoning on how increasing cylindrical radius gives a more significant effect than increasing its height. The student also construes the design of the cylinder that provides the most considerable volume by expanding its base or radius.

The next four paper present the RME research by using qualitative research approach. The fifth manuscript entitled “Students’ Ability to Solve Mathematical Problems in The Context of Environmental Issues” is presented by Jeinne Mumu, Vera Sabariah, Benidiktus Tanujaya, Roni Bawole, Hugo Warami, and Harina Orpa Lefina Monim from Universitas Papua, Manokwari, Indonesia, and Rully Charitas.
Indra Prahanma from Universitas Ahmad Dahlan, Yogyakarta, Indonesia. In their article, the environmental topics as a global issue used as a context in teaching mathematics. This paper describes students' mathematical abilities in solving problems related to environmental issues and determines students' knowledge and responses to the environment. Next, the paper entitled, “Realistic Mathematics Learning on Students’ Ways of Thinking”, is written by Resy Nirawati, Darhim, Siti Fatimah, and Dadang Juandi from Universitas Pendidikan Indonesia and STKIP Singkawang. This paper explores the students' thinking in solving geometry problems in schools that applied realistic mathematics learning was described in high-ability students, the mental acts displayed were interpreting, problem-solving and inferring. On the other hand, I Putu Ade Andre Payadnya, I Ketut Suwijia, and Kadek Adi Wibawa from Universitas Mahasaraswati Denpasar analyze the students’ abilities in solving realistic mathematics problems using "What-If"-Ethnomathematics Instruments with content focused on plane and space materials. Their research shows us the students’ abilities in solving realistic ethnomathematics problems using "What-If"-Ethnomathematics Instruments are still lacking which include: errors in understanding the problems, errors in representation, errors in reasoning, errors in answering "What-If" Questions. Lastly, the eighth paper entitled Pedagogical Content Knowledge of Teachers in Teaching Decimals through Realistic Mathematics Education written by Rahmah Johar, Fitriadi, Cut Morina Zubainur, M. Ikhsan, Tuti Zubaidah from Universitas Syiah Kuala and SMK 1 Al Mubarkeya. This qualitative study involved two teachers teaching fourth grade at the school partner of PRP-PMRI, Aceh Province, Indonesia. Their paper showed that teachers’ PCK in teaching focused more on the reality principle, activity principle, interactivity principle, and guidance principle of RME.

The next three paper discuss RME development research. Atiqah Meutia Hilda and Rizki Dwi Siswanto from Universitas Muhammadiyah Prof. DR. HAMKA develop an android application of probability theory of the same element permutation material based on RME as a learning medium, as well as to assess the quality of the apps created for utilize in learning mathematics. Furthermore, the tenth article entitled Developing Realistic Mathematics Problems based on Sidoarjo Local Wisdom is written by Eka Nurma Sari Agustina, Sofif Widadah, and Putri Afinanun Nisa from STKIP PGRI Sidoarjo. In this paper, they develop mathematical problems based on Sidoarjo’s local wisdom on valid and reliable flat-shaped materials. This study produced 15 mathematical questions based on Sidoarjo's local wisdom on flat-shaped material that had been declared valid and reliable. On the other hand, Munawarah, Siti Zuhaerah Thalhal, Andi Dian Angriani, Fitriani Nur, and Andi Kusumayanti from IAIN Bone and Universitas Islam Negeri Alauddin Makassar develop an instrument test for computational thinking (CT) skills in the mathematics based RME (Realistic Mathematics Education) class of the Grade VIII students of JHS/IIHS. This is a Research and Development research carried out using the Plomp model.

Lastly, this issue closes with RME research which uses the mix method approach. In this paper, Namirah Fatmanissa and Nur Qomaria from Sampoerna University and Universitas Trunojoyo investigate prospective teachers' beliefs toward the realism of mathematics word problems. The study employed both quantitative and qualitative analysis. Prospective teachers with realistic beliefs emphasized that any information presented in the word problem should simulate real life as accurately as possible. In contrast, those who have non-realistic beliefs stated that it was acceptable if it can be imagined. Neutral prospective teachers believe that word problems' realism is relative to cultural setting and students' background.

Rully Charitas Indra Prahanma
Southeast Asia Editor of MTRJ
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Developing Students’ Understanding of Percentage:  
The Role of Spatial Representation

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Abstract: The main goal of the current study is to develop intervention of learning to promote students’ understanding of percentage. The design of the interventions employed the spatial representation of percentage in the form of a bar model and was designed based on the pedagogical concepts of Realistic Mathematics Education (RME). The participants were taken from year-four primary students (around 10 to 11 years old). The data were collected from classroom observation during the implementation of the interventions. The findings show that the design of the learning interventions supports the students in developing their understanding of several fundamental ideas of percentage. The students could make sense of the proportional relationship underlying percentage in applying counting strategies involving proportional relationship, such as doubling, halving, multiplying, and dividing. They could add or subtract two percentages and treat a percentage as an operator. The spatial representation of percentage in the form of a bar plays a critical role during learning. First, the bar helps the students in visualizing the proportional relationship underlying the two magnitudes of percentage. Second, the bar aids the student in noticing the part-whole relationship underlying the percentage. Third, the bar model triggers students to perform flexible counting or computation strategies. Fourth, the bar helps the student to perform a mental computation. Fifth, the bar facilitates the students in keeping track of their computation process. Sixth, the bar model helps students to switch their thinking easily and mentally between the two magnitudes. Seventh, the bar triggers the students in estimating their counting.

INTRODUCTION

As percentage is often employed in many practical applications therefore it is a substantial topic in the core curriculum for science and social subjects at schools (Baroody, Baroody, & Coslick, 1998; Parker & Leinhardt, 1995; Schwartz & Riedesel, 1994; van Galen & van Eerde, 2013). It is a mathematical idea for communicating proportional relationships in a hundredth. However, many studies report that percentage is highly challenging to make sense for many students for its
complex mathematical relationships underlying the concepts (van Galen & van Eerde, 2013; Cole & Weissenfluh, 1974; Jannah & Prahmana, 2019; Parker & Leinhardt, 1995). For example, students are still facing challenges in defining the meaning of percentage and how a percentage relates to another percentage. For example, what does 10% mean? How 10% means can be related to 50%?

By employing the power of spatial representation (Putrawangsa, 2021; van Galen & van Eerde, 2013), this study aims to investigate the characteristics of instructional tasks that support early-grade students in constructing their conception of percentage. The instructional activity employs the use of the spatial representation of the percentage in the form of a bar model to trigger or facilitate students to do mathematical exploration. The main learning goal of the instructional activity is to support students in developing their understanding of the proportional relationship of percentage and percentage as an operator.

**Developing Understanding**

In this study, we view developing understanding as to the process of establishing a rich and meaningful connection among the mental representations of mathematical ideas. Mathematical understanding is constructed by making connections between the new knowledge and the existing knowledge (Barnby, et al. 2007; Piaget, 1976; Skemp, 1982). The new knowledge is assimilated into a proper existing knowledge constructing an ability to identify the new knowledge (Piaget, 1976; Slavin, 2019). If there is sufficient existing knowledge to assimilate the new knowledge, it will create associations between them that produce an understanding. Nickerson (1985) defined this relationship as “the more one knows about a subject, the better one understands it, and the richer the conceptual context in which one can embed a new fact, the more one can be said to understand the fact.” (p. 235-236). This indicates that developing an understanding of a subject involves forming as many as possible links between the subject and the existing knowledge, for example, by connecting between two different ideas which have not been related before.

Therefore, in this study, developing understanding is defined as the progress of making the connection between the new knowledge and the relevant existing knowledge such that the new knowledge can be employed as a way of thinking or reasoning.

**The Percentage**

The fundamental notion of percentage is the idea of proportionality where it explains the proportional relationship between two magnitudes or ratios, namely the percentage and its reference (Parker & Leinhardt, 1995; van Galen & van Eerde, 2013). The proportional relationships involve one-hundred part-whole relationships that give a non-absolute measure but a relative measure (Fosnot & Dolk, 2002). It implies that the percentage as part-whole relationship expresses the relative value of the part compared to the whole. For example, the value of 20% is not absolute. It is always relative depending on the whole that the 20% refers to. Here, students are not required to explain the relation in such a formal manner, but they have to exhibit an
awareness that percentages are always associated with something (the referent magnitude) and, therefore, they cannot be associated without taking into account what they refer to (Marja Van Den Heuvel-Panhuizen, 1994). As it represents part-whole relationships, percentage involves mathematical ideas of ratio (Parker & Leinhardt, 1995). In addition, percentages offer a uniform operator or fractional comparison of distinct amounts (Fosnot & Dolk, 2002). For example, in discounts and interest rates context, percentages act as an operator, e.g., 50% off whatever price. In the context of 50% of $120, for example, the percentage acts as an operator of looking at or finding half of the $120.

The recent study aims to develop students’ understanding of percentage as a proportional relationship (e.g., ratio and part-whole relationship) and the percentage as an operator since these notions are among the most critical ideas in learning percentage for early grades.

**Spatial Representation**

In general, spatial representation in mathematics can be regarded as the representation of mathematical ideas which trigger the use of spatial reasoning to think and reason about the ideas. For example, the spatial representation of numbers, such as number lines, is used to trigger students to see and explore the structure and the relation underlying the magnitude of numbers (Fosnot & Dolk, 2001b). The array representation of multiplication fosters students to think about the multiplicative structure of rectangular surface area (Putrawangsa, 2013). Many studies highlight that the use of spatial representations of mathematical ideas in mathematics learning, such as number line, bar, and array, influence the way students think and the reason that fosters mathematical understanding (Barmby, Harries, Higgins, & Suggate, 2009; Fosnot & Dolk, 2001a, 2001b; Putrawangsa, 2013; Putrawangsa & Hasanah, 2020a, 2020b; Hendroanto, et al. 2018; Marja Van Den Heuvel-Panhuizen, 2003; van Galen & van Eerde, 2013).

In the context of percentage, it is suggested that the spatial representation of percentage in the form of a bar model effectively facilitates students in making sense of mathematical ideas underpinning percentage (Marja Van Den Heuvel-Panhuizen, 2003; van Galen & van Eerde, 2013). The bar model allows students to imagine, reason, and communicate the proportional or the part-whole relationship represented in percentage (van Galen & van Eerde, 2013). Moreover, the representation supplies a stronghold for estimating the percentage and the relative value represented by the percentage, especially for the problems involving numbers that are not simply converted to a simple fraction or percentage (Marja Van Den Heuvel-Panhuizen, 2003). The bar model, furthermore, supports the students with more opportunities to make progress.

Since a percentage represents complex proportional relationships between two magnitudes, it is required an external representation can be used to show the relationships. Therefore, the bar model is advised as it indorses several benefits (Fosnot & Dolk, 2002; van Galen & van Eerde, 2013). First, the bar model has a surface area that makes it easier to speak in the terms of the part and the whole area. Next, the bar model can be used to record the track in making estimations involving percentages. Third, the bar model supports students to progress their understanding. Moreover, the
bar model is an effective visualization used for teaching percentage as it lets the student think and reason flexibly through the visualization. It helps students to easily look at the relationships between the magnitude of the percentage and the magnitude where the percentage belongs to. It assists students to flexibly switch over their thinking from one magnitude to another which prompts students to develop various computation strategies, such as doubling, halving, and preserving ratios.

The Instructional Design

The instructional activities in this study are designed based on the view of Realistic Mathematics Education (RME). RME suggests that students will learn mathematics effectively when they are allowed to investigate phenomena that are meaningful (realistic) for students. RME calls this heuristic as didactical phenomenology (Freudenthal, 1986; Gravemeijer, 1999; Larsen, 2018; M. Van den Heuvel-Panhuizen & Drijvers, 2020). The didactical phenomenology is the core idea of RME (Larsen, 2018) suggesting that learning mathematics should begin from phenomena that are meaningful for the student, that request to be organized, and that promote the progression of learning processes. According to Gravemeijer (1999), the goal of a phenomenological exploration is to encounter “situations for which specific approaches can be generalized and to find situations that can arouse paradigmatic solution procedures that can be taken as the basis for vertical matematization”. The didactical phenomenology principle suggests that the instructional designer should provide students with contextual problems in the form of phenomena that are meaningful for students. The phenomena should trigger students to experience reinventing mathematical ideas and the emergent model of mathematical thinking and reasoning under the teacher’s support, facilitation, and guidance (Gravemeijer, 1999).

Considering the didactical phenomenology principle, the recent study uses the power indicator of a computer or laptop as the learning context to actualize the didactical phenomenology principle of RME. The choice of such a context is based on several considerations: First, the computer power indicator is a familiar context among students due to the massive use of computer-like devices recently. Second, a computer power indicator is usually presented in form of a bar representing a percentage and of the remaining power represented by the percentage. This feature makes it possible to connect the context to the notion of percentage and the bar model to represent the percentage. Moreover, the bar representation of the power indicator can be an effective context to connect students not only to the bar model but also to the idea about double number line model where this model allows students to think and reason flexibly about percentage and the relative value represented by the percentage (Fosnot & Dolk, 2002).

RESEARCH METHOD

This study aims to not only understand how the students can be supported to learn percentage but also to understand how each characteristic or element of the intervention impact students’ thinking
and responses. Therefore, to acquire such a deep understanding, this study is conducted in a small-scale study involving two year-four primary students (around 10 to 11 years old). The selection of the participants was conducted randomly and administrated by the school.

This study was administrated according to the Design Research framework involving three main phases: design preparation, design experiments, and retrospective analysis (Gravemeijer & Cobb, 2013; Plomp & Nieveen, 2013; Putrawangsa, 2019). During the preparation, the literature review was conducted to clarify the essential mathematical ideas underlying percentage and research findings regarding the teaching and learning of percentage. The findings from the literature review were then used to inspire the formulating of a learning intervention to support students in learning percentage. The learning intervention was articulated in the form of a hypothetical learning trajectory (HLT) depicting the learning goals, the sequence and the form of the learning activities to achieve the learning goals, and the conjectures of students’ responses (thinking and reasoning) toward the learning activities (Simon, 1995). The overview of the intervention is elaborated in the result section.

In the next phase, the design experiments, the design of the learning intervention has experimented with the targeted students in a classroom setting. The purpose of the experiment is to critically observe and understand how the interventions work (or do not work) in shaping students’ thinking or reasoning to the conjectured responses. In the experiment, one of the researchers acted as the teacher while the other researchers observed the learning. As the source of the data, the whole classroom activities, including students’ actions and conversations, were video recorded, and students’ works both on paper and onboard were collected and documented. Researchers’ findings and impressions on the learning were discussed and documented right after the learning as additional supporting data to clarify the learning.

Finally, in the retrospective analysis, the whole set of the data generated by the classroom experiment were analyzed to identify how the interventions shape students’ understanding (thinking and reasoning) of percentage. The analysis was guided by three reflective questions formulated in what, how, and why questions, namely: What learning activity in HLT does work to promote the emergent of the conjectures of students’ responses elaborate in the HLT? How does it work (or not work)? Why does it work (or not work)? To answer the questions, the data from classroom experiments, including student works and other learning artifacts, were analyzed comprehensively and critically to acquire understanding and explanation on what, how, and why the interventions shape students’ cognitive development. To answer the first question (what), students’ actual responses during the learning were compared with the conjectures of students’ responses in the HLT episode by episode. Then, the data relating to the episode were investigated to answer the second question (how). Finally, the last question (why) was answered by explaining or discussing the findings from the previous two questions (what and how) through the eye of the relevant theories to gain a better understanding of the phenomena.
RESULTS

The presentation of the result is elaborated into two sections. The first section elaborates the overview of the learning intervention, and the second section focuses on presenting students’ responses regarding the intervention.

Overview of the Learning Intervention

Based on the literature review on percentage, the learning interventions were designed to develop students’ understanding of the part-whole relationship underlying percentage and the percentage as an operator.

The computer power indicator was used as the context of the learning (see Figure 1). There were two percentage problems proposed to the students through the context. The first problem was about finding the time if the percentage of the remaining time is given and the whole time is known, for example, “If the power indicator shows 100% or fully charged, the laptop lasts about 4 hours 40 minutes or 280 minutes, so how long the laptop last if the percentage of the power indicator is shown 60%?” This problem is questioning about the part if the whole and the percentage of the part are known. Through the problem, it is estimated that students will have the opportunity to explore the part-whole relationship and the relative value of percentage. The students could likely see the part-whole relation between the percentage and the value represented through the percentage. For example, they understand that halving the percentage will change the value of the percentage in the same ratio. For example, halving the percentage means halving the value represented by the percentage. As the students investigate the part-whole relationship of percentage, the students will be aware of the use of percentage as an operator. For example, defining 50% of 280 as multiplying 280 by ½ or dividing 280 by 2.

While the first problem asks for “What is P if P is X% of the whole W?”, the second percentage problem involves “What is the percentage of P from the whole W?”, for example, “If the power lasts for 280 minutes indicated by 100%, what is the percentage of the power lasts for 112 minutes?” The problem is about obtaining the percentage of the remaining time if the remaining time and the full time are identified.

In each problem, some follow-up problems were given to progress students’ understanding to a higher level. For example, the students were asked to determine the remaining time if the laptop is charged for 20%, 45%, or 80%. Meanwhile, the follow-up questions for the second problem questioned the percentage for 56 minutes, 84 minutes, or 112 minutes.
To foster the RME’s principle of progressive mathematization (Gravemeijer, 1999), the students were given non-contextual problems of percentage to check students’ ability in generalizing their understanding beyond the context. For example, the students were asked “What is 70% of 400?” It is expected that the students can generalize the use of the bar model to think about mathematics problems involving percentages.

**Students’ Responses and Reasoning**

To start the learning, the teacher introduced the learning context to the students by showing a laptop where the power indicator of the laptop shows 85% indicating that the laptop will hold up for 2 hours and 20 minutes. First of all, the students transformed the time into minutes. They found that 2 hours and 20 minutes is 140 minutes. As the teacher asked them about the 85%, the students knew that the number stands for the percentage of the remaining power of the laptop. The students also knew that 100% implies that the laptop is charged fully. It is noticed that the discussion on the context successfully motivated students to explore more about the percentage in the context of the power indicator.
As the students were getting motivated, the teacher then gave students the first percentage problem. The problem questioned the student to figure out the time if the laptop had been charged for 60% and if it was fully charging it lasted for 4 hours and 40 minutes (see Figure 2). The students knew that 4 hours and 40 minutes refer to 280 minutes. Students’ first response to the problem shows that the student fooled by the variable of the percentage and the time (i.e., adding 1 hour and 20 minutes with 10%) as it is shown in the following students’ talk: “Since 100% is 2 hours and 40 minutes. So half of it is 1 hour and 20 minutes which is 50%, but I need 10 (10%) more to get 60%, So, it is 1 hour and 20 minutes plus 10 which is 1 hour and 40 minutes”.

![Figure 3: Students’ first strategy to determine the minutes for 60%](image)

To prompt the students to reflect on their thinking, the teacher drew the replication of the power indicator in the form of a bar-like representation on the classroom board. The teacher then asked the student to write down what they knew from the given problem on the representation (see Figure 3). To express their initial solution, the student drew the line in the middle of the bar to indicate 50% and wrote 140 minutes representing the time for the 50%. They recognized that 50% is one-half of 100%, therefore, they need to halve 280 minutes as well resulting in 140 minutes intended for 50%. Since they intended to get 60%, they added 10% to both the percentage and the time magnitudes, therefore, they concluded that 150 minutes is for 60% (see Figure 3). Since the finding was considered incorrect, the teacher asked students to reflect on their findings by looking at the bar carefully. After a while, they realized that they made a mistake, but they were still in difficulties in finding the minutes for 60%.

To help the students, the teacher proposed finding the minutes for other familiar percentages, such as 25%. It was easy for the students to get the minutes for 25% by just looking at the bar. Students’ reasoning is shown by the following students (S) and teacher (T) discussion:

*Student:*  “It is 70 minutes”.

*Teacher:*  “How do you know that?”

*Student:*  “Since it is half of the half, you divide 140 by 2.”
Half of the half indicates here is the half of 140 (50%) which is 70 since 140 is the one-half of 280 (100%) (see Figure 4).

![Figure 4: Students’ solution to get the minutes for 25%](image)

The teacher then questioned whether 25% would aid them to get the minutes for 60%. They said that 20% would not help them. Instead, they thought that 20% or 10% might help them to get the minutes for 60% as they looked at the bar. Interestingly, the student could think that 25% would not help them to get 60%, instead, they realized that 10% or 20% could help.

Then, the students opted to work with 10%. Here is their strategy to get the minutes for 10%. they sketched a line on the left side of the bar in proportion and wrote 10% on the percentage magnitude (see Figure 5). The following record shows their thinking.

*Teacher:* “So, how many minutes for 10%? Probably, you can find the relation between 10% and 50%.”

Then, the students looked at the bar and said:

*Student:* “I divided this by 5 (pointing 140).”

*Teacher:* “How do you know that you have to divide it by 5?”

*Student:* “Because the distance between 10% and 50% is 5. So, if I divide this (pointing 140) by 5, I will get 10%.”

*Teacher:* “That will be interesting. Why don’t you find it out for 10%?”

After a while, they said:

*Student:* “It’s 28 minutes”

*Teacher:* “28 minutes. How do know you that?”

*Student:* “Cos I divide this by 5 (pointing 140).”
Then, the teacher incited the student to think about the connection between the minutes for 50% and 10% to get the minutes for 60%. However, the students still used their earlier strategy which adds the same amount on both magnitudes without realizing that each magnitude represents a different whole. Here, the students were still confused in understanding the difference between the two magnitudes (the percentage and the minutes) since each of them refer to a different unit and whole.

After a while, instead of looking at the relationships between 10% and 50%, the students invented a different strategy. They multiplied both magnitudes by 6. They claimed that 6×10% leads to 60%, and consequently, 6×28 minutes produces 168 minutes. Therefore, they claimed that 168 minutes are for 60%. Students’ reasoning can be seen in the following excerpt. The word ‘distance’ in the conversation refers to the difference between 10 and 60.

**Student:** “I do 28 times 6”

**Teacher:** “28 times 6? How do you know why you have to time it by 6?”

**Student:** “mmm… the distance between 10 and 60 is 6. So, this is the lowest we can go (pointing the minutes for 10% which is 28 minutes). So, I do 28 times 6 (he got 168 for 28 times 6)”.

**Teacher:** “That is interesting.”

Next, the teacher suggested the students see the relationship between 10%, 50%, and 60%. By looking at the bar, the students were encouraged to look at the minutes for 10% (28 minutes) and 50% (140 minutes) and how the information can be connected to the minutes for 60% (168 minutes). Finally, the students could see that if 10% is added to 50% it will be 60%. Therefore, they need to add the minutes of the 10% (28 minutes) to the minutes for the 50% (140) to get the minutes for 60% which is 28+140 = 168 minutes.

**Student:** “Since we have 10% and 10% is 28, we do 140 (50%) plus 28 (10%) which is 168 which is the minutes for 60%.”

Before resuming the activity, the teacher engaged the student to do a reflection of what they had accomplished to find the minutes for 60%. They said that they had two approaches to get the
minutes for 60%. The first is multiplying the percentage and the minutes by 6, and the second is combining (adding) two combine percentages and consequently their respective minutes.

To enhance student understanding, the teacher offered the student a follow-up problem which was figuring out the minutes for 45%. The students knew that 45% represents 126 minutes. The following excerpt shows students’ thinking toward the problem.

Student: “I subtracted 50% by 10% to get 40% that is 140 (50%) minus 28 (10%) which is 112. Then, I halved 28 (10%) to get 5% (the minutes for 5%) that is 14. Then, I added 14 (5%) to 112 (40%) to get 45% which is 126 minutes (see Figure 6).”

![Figure 6: Students’ computation to get the minutes for 45%](image)

Another follow-up problem was given to the students. They are asked to determine the minutes for 30%. The students looked at the bar for a while and they found it in 84 minutes. They drew the line for 30% on the bar (see Figure 7) to check their findings.

Student: “It is 84 (minutes).”

Teacher: “How do you know that?”

Student: “It is from 28 times 3. Here is 10 (10%). 10 times 3 is 30 (30%). 10% is 28. So, 28 times 3, which is 84 (minutes).”

![Figure 7: Students’ solution to get the minutes for 30%](image)
Afterward, the students were asked to select their percentages and were asked to determine the minutes of the selected percentages. The students selected 20%. By looking at the bar, they calculated the minutes for 20% mentally and said it is 56 minutes for 20%. They multiplied 10% by 2 to get 20%. Consequently, they the minutes for 10% (28 minutes) by 2, and they got 56 minutes (see Figure 8). It seems that the student could conserve the ratio on both proportions.

![Figure 8: Students’ solution to get the minutes for 20%](image)

The student then tested whether they could find the minutes for 1% by asking the students to find out the minutes for 21%. However, the students would not be able to solve the problem although they had been suggested to see the connection among the known percentages and to use his previous counting strategy. The teacher then transformed the problem into finding the minutes for 85%. To figure out the minutes for 85%, they multiplied 28 minutes for the 10% by 8 to get 80% resulting in 204 minutes. Then, they added 14 (5%) to 204 minutes to get the minutes for 85% which is 218 minutes (see Figure 9).

**Student:** “It is 28 times 8 because 28 is 10(%), so 28 times 8 which is 204. I need 5% more, which is half of 10(%). It will be 5%, and 5% will be 14 minutes. So, I added 204 plus 14, which is 218”.

![Figure 9: Students’ counting strategies to get the minutes for 85%](image)
As the students could make sense of the problems regarding ‘finding the minutes if the percentage and the whole minutes are known’, the teacher then pointed to the second percentage problem as follows: “If the power indicator shows that the laptop lasts for 280 minutes indicated by 100%, what is the percentage of the power lasts for 112 minutes?” It is identified that it was not difficult for the student to deal with the problem as they just looked at the bar for a while and said: “56 (20%) times 2 which are 112 minutes. So, it for 40%” Then, they said that if the laptop lasts for 112 minutes it has been charged for 40%. It seems their previous experience of working with the bar helps them in reasoning and visualizing their thinking about the problem. As the follow-up problems, the students were asked to determine the percentage for 42 minutes and 210 minutes. Figure 10 shows students’ solutions to the problems. Their solutions indicate that they could reason and calculate through the bar by seeing the change on the ratio on both magnitudes to produce the value of other percentages or to figure out the percentage of a given value.

“What is its percentage if the laptop lasts for 42 minutes?”
“I added 28 (10%) plus 14 (5%) which is equal to 42 minutes and 42 is for 15%.”

“What is its percentage if the laptop lasts for 210 minutes?”
“I did 28 times 8 which is 224. It (224) is also (for) 80%. Then, I halved this (pointing 28 minutes for 10%) I got 14 (5%). So, I did 224 minus 14 which is 210. So, that is (for) 75%.”

![Figure 10: Students’ solution in finding the percentage for 42 minutes and 210 minutes](image)

To facilitate the process of progressive mathematization (the process of abstracting mathematical ideas out of the context, such that the ideas become mental mathematics which finally can be utilized as a tool to think or reason mathematically), the students were asked the following problem: “What is 70% of 400?” Interestingly, they could answer the problem by developing counting strategies on the bar model. Firstly, they found 40 for 10% since 10×10% is 100% and 10×40 is 400. Then, they multiplied 10% by 7 to obtain 70% and also for 40 and they got 280 minutes for 70% (see Figure 11).

Here, they could maintain the ratio of the change on both magnitudes (the percentage magnitude and the value where the percentage refers to). Here, they could recognize the link among percentages, such as dividing 100% by 10 to obtain 10% and multiplying 10% by 7 to produce 70%.
DISCUSSION

Regarding the findings, it is claimed that the learning context together with the bar representation of the percentage plays a significant role in developing students’ understanding of the percentage. The students were able to recognize the part-whole relationship (e.g. 50% means one-half), recognize the transformation on the percentage magnitudes proportionally (e.g. multiplying or dividing both magnitudes by the same number), and consider a percentage as an operator (e.g. seeing 50% of 400 as halving 400). The next paragraphs will discuss in detail students’ cognitive development and the role of spatial representation in learning.

Student’s Development of Understanding

In the learning, the students showed their understanding of the proportional relationships underlying percentage and treated percentage as an operator. Such understanding can be observed through the use of proportional-based strategies in solving percentage problems, such as doubling and halving, multiplying or dividing percentages, adding or subtracting percentages, and working with ratios.

Doubling and Halving

The doubling and halving are mostly applied simultaneously as doubling is the inverted operation of halving. The idea of doubling and halving as a counting method has a long root in human-life history, especially among Egyptians and Russians (Fosnot & Dolk, 2002). To get 4 x 12, Egyptians will begin counting from 1 x 12 = 12, 2 x 12 = 24, 4 x 12 = 48, meanwhile Russians calculate as the following: 4 x 12 = 2 x 24 = 1 x 48 = 48.

The findings of the current study display that the students were able to utilize doubling and halving in dealing with percentage problems. In figuring out the minutes for 20%, for example, they doubled the minutes for 10% to obtain the minutes for 20% and again doubled it to identify the minutes for 40%. Compared to doubling, it is identified that halving is utilized more frequently. It is probably because the student usually starts solving percentage problems by looking at or thinking of 100% first, then, halved the percentage to get the smaller percentage, such as having 100% to get 50% and halving 50% to get 25%. They even were able to determine 2.5% by halving...
10% to obtain 5% and then halving 5% to get 2.5%. They knew that if the percentage magnitude is halved or doubled, they also need to do the same on the minute magnitude. Here, they are aware that they need to maintain the ratio on both magnitudes once doing doubling or halving.

The spatial representation of the bar facilitates them to come with and apply the doubling or halving strategy. Such spatial representation stimulates the students to construct the meaning and the impact of the manipulation. When halving, for instance, the student could see that the bar is being divided into two parts equally; meanwhile, the bar is being extended in doubling. The visual image of the bar triggers the student to recognize the connections between division and multiplication where doubling means multiplying by 2, and halving implies division of 2. For example, they recognized that to get half of 140 minutes, they divided 140 by 2. To make the minutes for 20% from the minutes for 10%, the student multiplied 10% by 2; and consequently, they doubled 28 (the minutes for 10%) as well to have the time for 20%.

In addition, the bar shows the spatial visualization of the whole and the parts which lead the students to recognize what the whole and what the parts refer to. For example, when halving 100%, they understood that they had to halve the minutes for 100% (the 280 minutes) to have the minutes for 50% that is 140 minutes. Here, they knew that 50% is one part of the 100% (as the whole). Meanwhile, 140 minutes is one part of the whole (the 280 minutes). At the end of the lesson, they could differentiate the whole in each magnitude (the 100% and the 280 minutes).

**Working with A Ratio by Multiplying or Dividing**

Multiplying and dividing percentages are the other two counting strategies applied by the students in solving percentage problems. The multiplication and the division are based on the doubling and halving since doubling and halving employ the multiplication by 2 (doubling) or the division of 2 (halving) and preserve the ratio on the percentage magnitudes and the magnitude represented by the percentage. The ratio represents the relation between values expressed in numbers to articulate how one is different but related to the other (Walter, 2004). For example, if there are two men and six women in a class, the composition can be expressed in the proportion of 2:6 which is equivalent to the ratio of 1:3. The ratio 1 : 3 is also equivalent to the proportions generated by multiplying or dividing the ratio by a constant number. Such a strategy of establishing equivalent ratios is then called preserving ratio (see Fosnot & Dolk, 2002). The ratio also indicates the quantitative relationship between a magnitude to the whole. For example, the ratio 1:2 indicates that the proportion of 1 to the whole remains 1 over 1+2 (equivalent to 1/3). Meanwhile, the ratio 1:5 shows that the proportion of 1 to the whole is 1 over 1+5 (equivalent to 1/6).

The findings show that the student could identify the idea of ratio and utilize the idea in solving the percentage problem. For instance, in determining the minutes for 10% provided that 100% indicates 280 minutes, they divided both magnitudes (percentage and time magnitude) by 10 (100%: 10 and 280: 10) resulting in that the minutes for the 10% is 28 minutes. Additionally, to determine the percentage for 224 minutes, they multiplied both magnitudes by 8 since 8 × 28 is 224, therefore, the percentage for 224 is 8×10% which is 80%. Looking at the students’ reasoning,
elaborated above, it indicates that the student could recognize the idea of ratio preservation. They understood that, when the percentage magnitude is multiplied or divided by a specific number, the same action is also necessary to be done for the other corresponding magnitude, for example, the minutes, to preserve the ratio of the percentage and the time magnitude.

Working with the ratio in dealing with the percentage problem seems to be inspired by the model used to visualize the problem. The use of the bar model to represent the problem aids in seeing the part-whole relationships of the percentage magnitudes and the referent magnitude. The visualization of the bar model eases the student to see the changes on both magnitudes simultaneously concerning the constant ratio. Figure 11 shows students’ solutions in determining the minutes for 50%, 25%, and 10% knowing that 100% is for 280 minutes. To determine the minutes for 50%, they halved the 100%. The bar indicates that 100% is for 280 minutes, therefore, they also halved 280 and obtained 140 minutes for 50%. A similar strategy was also done to get the minutes for 25% by halving the minutes for 50%. The strategies seem to be triggered by the visualization of the problem in the form of a bar representing double number lines, percentage on one side, and the minutes on the other side. This spatial visualization inspires the students to develop logical thinking and reasoning. For example, if one magnitude is halved, so another magnitude is necessary to be halved as well to preserve the ratio constant on both magnitudes. In addition to doubling and halving, the students also utilized the division by 5. To get the minutes for 10%, for instance, they divided 50% by 5. As 140 minutes represent 50%, it is necessary to divide 140 by 5 as well to get the minutes for 10% (see Figure 12).

![Figure 12: Students’ counting strategies to determine the minutes for 50%, 25%, and 10%](image)

Another example shows that the students are aware of the ratio represented in percentages. In solving the percentage of the minutes for 210 minutes while 280 minutes indicated 100%, the students firstly, solve the minutes for 10% by dividing 280 minutes by 10, and then they said, “I did 28 times 8 which is 224. It (224) is also (for) 80%. Then, I halved this (pointing 28 minutes for 10%) I got 14 (5%). So, I did 224 minus 14 which is 210. So, that is (for) 75%” (See Figure 13).
This complex counting strategy shows that they could think flexibly and meaningfully in dealing with percentages. They simultaneously employed all known computation principles, such as doubling and halving, multiplying, or dividing the magnitudes, and adding or subtracting known percentages to generate other percentages. Here, they used 10% as the benchmark to get 75% by applying the counting principles. Moreover, working with the ratio shows that they treated the bar model as a ratio table. The counting strategies shown in Figure 13 can be represented in the ratio table shown in Table 1. If they applied an algorithm, they would not be able to generate such a flexible, rich, and complex way of thinking. They might just see percentage as a certain procedure of calculation rather than seeing percentage as relations on relations.

It seems that the spatial representation of the percentage in the form of a bar facilitates the students in developing complex but meaningful counting strategies. The visualization triggers students to easily see the possible relationships among the information presented on the bar. For example, the spatial visualization of 100% on the bar triggers the students to do a series of halving to get 50%, and 25%, adding 50% to 25% to obtain 75%, dividing 50% by 5 to produce 10%, doubling 10% to reach 20%, subtracting 50% by 20% to obtain 30%, or multiplying 30% by 3 to acquire 90%. Such a flexible and meaningful way of thinking and reasoning will not be easily developed or thought if the students learn percentages procedurally through memorizing or applying algorithms.

<table>
<thead>
<tr>
<th>Percentage</th>
<th>Minutes</th>
<th>Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>100%</td>
<td>280</td>
<td>280 minutes indicate 100%</td>
</tr>
<tr>
<td>10%</td>
<td>28</td>
<td>Dividing 100% and 280 by 10</td>
</tr>
<tr>
<td>80%</td>
<td>224</td>
<td>Multiplying 10% and 28 by 8</td>
</tr>
<tr>
<td>5%</td>
<td>14</td>
<td>Dividing 10% and 28 by 2</td>
</tr>
<tr>
<td>75%</td>
<td>210</td>
<td>As 80% is subtracted by 5%, 224 is subtracted by 14.</td>
</tr>
</tbody>
</table>

Table 1: Ratio tables for in determining the percentage for 210 minutes

Adding or Subtracting Percentages

It identified that adding or subtracting percentages are among the counting strategies employed by the students to solve the percentage problem. For instance, they added the minutes for 50% and
10% to determine the minutes for 60%. They also used similar strategies to produce other unknown percentages. The minutes for 45%, for example, is generated by subtracting the minutes for 50% by 10% (to get the time for 40%). Then, they added the minutes for 40% to the minutes for 5% obtaining the minutes for 45%. Moreover, to determine the percentage for 42 minutes, they combined 28 minutes (10%) and 14 minutes (5%) to get 42 minutes. Therefore, the percentage for 42 minutes is 15%. The counting strategy indicates that the student could identify the relationships among the known information. It shows that the student could flexibly switch their thinking between the percentage magnitude and the minute magnitude or the other way around. This shows an understanding of the associations between the two magnitudes (ratio relationship) and an understanding of the relationships among the known information within each magnitude (part-whole relationship).

To support the development of such a complex and flexible way of thinking, it requires a proper model that helps the students to visualize and mentally record their reasoning process. Here, the bar model plays a critical role. The findings show that the bar model helps the student to keep them on track when counting which reduces the cognitive load once counting. Sometimes, the student just looked at the bar and then got a solution. For instance, in figuring out the percentage for 42 minutes, the student just observed the bar and did a mental computation. By looking at the minutes for 10%, they counted mentally that adding the minutes of 10% to the minutes for 5% leads to the minutes for 15% which is 42 minutes.

In addition, the spatial representation of the percentage supports the students to see the relationships among the known information to generate new information. The simultaneous visualization of numbers on percentage magnitude and the time magnitude aids the student to see the relationship among the numbers. In findings the minutes for 40% and 60%, for example, the students added 10% to 50% to get 60% and subtracted 50% by 10% to obtain 40%.

The Role of the Spatial Representation

Spatial representation in mathematics is the representation of mathematical ideas into spatial constructs or affairs which triggers the use of spatial reasoning to think and reason about the ideas. For example, the spatial representation of numbers in the form of number line foster students to associate the meaning of the magnitude of numbers as the distance from zero (Fosnot & Dolk, 2001b). Meanwhile, the spatial representation of multiplication in the form of array foster students to think of multiplication as surface area and use the properties of area to explore the multiplication or another way around (Putrawangsa, 2013).

Many studies highlight that the use of spatial representations of mathematical ideas in mathematics learning, such as number line, bar, and array, influence the way students think and the reason that fosters mathematical understanding (Barmby et al., 2009; Fosnot & Dolk, 2001a, 2001b; Putrawangsa, 2013, 2021; Putrawangsa & Hasanah, 2020a, 2020b; Marja Van Den Heuvel-Panhuizen, 2003; van Galen & van Eerde, 2013). In the context of percentage, for example, the spatial representation of percentage in the form of a bar model effectively facilitates students in
making sense of mathematical ideas underpinning percentage (Marja Van Den Heuvel-Panhuizen, 2003; van Galen & van Eerde, 2013; Jannah & Prahmana, 2019). The bar model allows students to imagine, reason, and communicate the proportional or the part-whole relationship represented in percentage (van Galen & van Eerde, 2013). Moreover, the representation supplies a stronghold for estimating the percentage and the relative value represented by the percentage, especially for the problems involving numbers that are not simply converted to a simple fraction or percentage (Marja Van Den Heuvel-Panhuizen, 2003). Therefore, the bar model provides the students with more opportunities to progress.

The bar model which is also considered as a double number line (Fosnot & Dolk, 2002) provides simultaneous information and data of percentage. This model can be used effectively to show the part-whole relationship (proportional relationship) and the ratio relationship underlying percentage. Such characteristics indicate the didactical use of the model to support the students in developing their understanding of percentages.

According to the recent study, the spatial representation of the percentage in the form of bar representation facilitates the students in making sense of the given problems, developing strategies to solve the problems, evaluating their solution, and making connections among the solution. The current study identifies at least seven critical roles of the spatial representation of percentage in the foster mathematical understanding of percentages as part-whole relationships and percentage as an operator, namely:

First, the bar model is an effective model to aid the students in seeing the proportional and ratio relationship underpinning percentage. Recognizing the relationship allows the students to manipulate the percentage by applying multiplication or division on the percentage and its reference. For example, to determine the minutes indicated by 30%, the students multiplied both 10% and 28 minutes (the minutes for 10%) by a constant 3 since 3×10% is 30%. Moreover, understanding the relationships allow the student to add or subtract two percentages. For example, the students subtracted 50% by 10% to generate 40% and at the same time, they subtracted 140 (the minutes for 50%) by 28 (the minutes for 10%) to get 112 (the minutes for 40%).

Second, the bar model helps the student in seeing the part-whole relationship represented through percentage. The visualization of the bar model forming surface area facilitates the students to talk in the terms of part and whole. The surface area allows the student to have a sense that 50% simultaneously is half of 100% and a double of 25%, at the same time, 25% is one-half of 50% and one-quarter of 100%. The students also could see that 10% is one-tenth of 100%, 80% is four times of 20%, or 8 times of 10%.

Third, understanding the part-whole relationship and the proportional relationship foster the students to have a flexibility of thinking and developing various complex counting strategies. For instance, to determine the minutes for 75%, the student divided 100% by 10 obtaining 10%. They then multiplied 10% by 8 to get 80%. Afterward, they halved 10% to get 5%. The last, they subtracted 80% by 5% to get 75%.
Fourth, the visualization of the percentage in the form of a bar promotes the student to do mental computations. It is found that the students looked at the bar while doing mental calculations when dealing with the given percentage problems. For instance, in finding the minutes for 75% elaborated above, they first look at the bar for a while and did calculations verbally.

Fifth, the visualization of the numbers on the bar helps in keeping track of the trajectory of the computation process. The bar provides spaces to write the calculation process. For instance, the students drew a line in the middle of the bar (splitting the bar into two equal parts) to indicate the percentage for 50%. Twenty-five percent was generated by drawing a line splitting the area of the 50% into two equal parts. The track of the splitting allows the students to see the relationship between 25% and 100% where 25% is one-fourth of 100%.

Sixth, As the bar shows two magnitudes of percentages simultaneously, it helps students to switch their thinking, for example, from the percentage magnitude to the minute magnitude. It is found that the student could see that 10% is one-tenth of 100% and, at the same time, see that 28 (the minutes for 10%) is one-tenth of 280 minutes (the minutes for 100%).

Seventh, the bar allows students to make estimations while counting. For example, the students could justify that the minutes for 60% must be greater than the minutes for 50% since not only 60% is greater than 50% but also 60% is closer than 50% to 100% on the bar. Looking at the minutes for 5% (14 minutes), the students could estimate the minutes for 2.5% saying that “the minutes for 2.5% is around 7 minutes as 5% is 14 minutes”.

The didactical use of the spatial representation of percentage is in line with the study by van Galen and van Eerde (2013) on the use of bar model to promote percentage where they advise three advantages of using the bar in thinking of percentage, such as: First, the bar model has a surface area that makes it simpler to talk in the terms of “the part” and “the whole”. Second, the bar model gives a good track to approximate a percentage, especially in cases where the problems involve numbers that cannot be simply converted into a familiar number. Third, the bar model offers more opportunities to progress students’ thinking.

The findings of the current study also support the idea of spatialized instrumentation (Putrawangsa, 2021; Putrawangsa & Hasanah, 2020a) or embodied mathematics (Thom, D'Amour, Preciado, & Davis, 2015) where both ideas highlight the role of spatial reasoning in constructing mathematical understanding. The spatial representation of the percentage stimulates the students to use their spatial reasoning to process the information presented on the bar. For example, the bar representation together with students’ spatial reasoning facilitates the students in seeing the relationship between 50% and 100% by halving the area or the length of the bar presented 100%. Moreover, the complex relationship between the percentage and its relative value can be easily explained through the correlated dual magnitudes presented in the bar model, namely the percentage magnitude and the value represented by the percentage. For example, the students can see that halving 100% into 50% will consequently halve the value represented by the 100%. Here, the spatial representation of the bar model supply relatively spatial mathematical perceptions to
the students (e.g., percentage as an area which can be split or combined) where the perceptions trigger the construction of the intended students’ thinking and reasoning which is articulated in the form of spatial mathematical actions through the spatial representation (e.g., halving or doubling percentage through the bar). The mathematical actions action then stimulates other spatial mathematical perceptions and actions. This reciprocal relationship between mathematical perception and actions contributes to the construction of mathematical knowledge and understanding (Putrawangsa, 2021; Putrawangsa & Hasanah, 2020a; Shvarts, Alberto, Bakker, Doorman, & Drijvers, 2021).

CONCLUSION

The findings of the current study indicate that the design of the learning employed the spatial representation (the bar model) fosters the development of the participating students’ understanding of the concepts of percentage. They could see the meaning of percentage as a proportional relationship (ratio and part-whole relationship) and percentage as an operator. In dealing with percentage problems, the spatial representation of the percentage inspires the students to develop various strategies, such as doubling and halving, working with ratios (multiplying or dividing both magnitudes to preserve the ratio), and adding and subtracting two percentages. However, it is identified that the spatial representation could not be able to help students to make sense of 1%. The smallest percentage the student could go is 2.5% by halving 5%.

It identified that the spatial representation of the percentage problems in the form of a bar is critical in developing students’ thinking and reasoning independently. There are at least seven functions of the bar model in developing students’ counting strategies, namely: First, the bar model becomes a powerful visual representation that facilitates the students in seeing the ratio relationship of the two magnitudes represented by percentage (the percentage magnitude, and the magnitude where the percentage refers to). Second, the bar model aids the student in noticing the part-whole relationship of each magnitude. Third, understanding the part-whole and ratio relationship supports the students to generate flexible and meaningful counting strategies. Fourth, the bar provides visualization that aids the students to do mental computation. Fifth, the bar helps in keeping track of the trajectory of the counting process. Sixth, as the bar shows two magnitudes simultaneously, it helps students to switch their thinking easily and mentally between the two magnitudes (for example the percentage magnitude and the time magnitude). Seventh, the bar allows students to make estimation of their counting.

REFERENCES


Learning Trajectory of Algebraic Expression: Supporting Students’ Mathematical Representation Ability

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Abstract: The mathematical representation ability is essential in solving mathematical problems, especially in algebraic expressions problems. Therefore, it is crucial to have a valid design of learning activities to support students’ mathematical representation ability. Several previous studies claimed that realistic mathematics education is one of the learning approaches that could support this ability. This study is design research aiming to evaluate the RME-based learning trajectory oriented to enhance students’ mathematical representation ability on algebraic expression. The data collected from the documentation of learning trajectories, documentation of students’ answer sheets, and video recording of online teaching and reflection sessions were analyzed descriptively. The findings indicated that the designed RME-based learning trajectory (LT) oriented to support students’ mathematical representation ability in algebraic expression has been valid and could be implemented in the pilot experiment. The implementation of the LT-1 and issues found during the pilot experiment are discussed in the paper. The finding implies that the learning trajectory could be continued to the teaching experiment phase after some revisions and adjustment.

INTRODUCTION

One of the abilities required to learn mathematics is mathematical representation ability (NCTM, 2000), such as simplifying and solving mathematical problems relying heavily on the ability. Representation is the transformation of a problem or idea to a new form, including transforming images or physical models into symbols, words, or sentences (NCTM, 2000). Representation is a means to communicate mathematical problem-solving ideas, and it may be used to facilitate and support conclusions (Pape & Tchoshanov, 2001; Sari & Rosjanuardi, 2018). When learning mathematics, students are suggested to focus more on various forms of mathematical representations to solve mathematical problems well (Afriyani, Sa'dijah, Subandi, & Muksar,
One of the mathematical problems that require mathematical representation skills is algebra.

Algebra is one of the main topics with undeniable importance in mathematics. Comprehension of algebra is important because it is very related and cannot be separated in everyday life, and it is very influential in decision making (Usiskin, 1995). Conceptual comprehension in algebra is defined as identifying functional connections between known and unknown variables, independent and dependent variables, and differentiating and interpreting diverse representations of algebraic concepts (Panasuk & Beyranevand, 2010). Algebraic topics began with basic arithmetics and then advanced to more abstract algebraic operations will be challenging for students (Baroudi, 2006; Sarimanoğlu, 2019). While students might not face difficulties performing arithmetic calculations, Jupri & Drijvers (2016) stated that different things might happen when the calculation involves algebraic expressions.

The emphasis on computation leads to many misconceptions in students’ minds, which will make it more complicated for students (Baroudi, 2006; Sarimanoğlu, 2019). Furthermore, several studies reported various difficulties related to algebraic expression faced by students. Most students have difficulties understanding basic algebraic expressions, especially the meaning of variables in an algebraic expression (Rudyanto, Marsigit, Wangit, & Gembong, 2019). In contrast, the variable in the algebraic expression is the basic concept that must be interpreted to continue learning algebra at a higher level (Booth, McGinn, Barbieri, & Young, 2017). Furthermore, several studies found that most students who still have difficulty understanding the algebraic expression, especially students at the intermediate level, have misconceptions about algebraic prerequisite material (Bush & Karp, 2013) and experience misunderstandings in solving algebraic equations (Sarimanoğlu, 2019).

Often, students’ misconception occurs in understanding the meaning of variables in algebraic equations (Knuth, Alibali, McNeil, Weinberg, & Stephen, 2005). Moreover, Egodawatte (2011) stated that misconceptions often occur in four parts of algebra, namely variables, algebraic expressions, algebraic equations, and story problems. Another misconception in algebraic expressions students face is considering the (+) symbol as an invitation to do something; therefore, they simplified $3x + 4$ as $7x$ and $4 + 3x^2$ as $7x^2$ (Chow & Treagust, 2013). Students' misunderstandings in algebraic calculations occur due to a lack of understanding of the expression of a variable that can be a literal symbol as a label for an object; for example, students mistaking the letter "y" in "addition of 3 and y" as something like yogurt and yum or as the alphabet "D" in David's name that can be mixed (Christou, Vosniadou, & Vamvakoussi, 2007). They might expand $(m + n)^2$ as $m^2 + n^2$, distribute $2(x - 5)$ as $2x - 5$, simplify $3y + 2$ as $5y$ and $3a + 2y$ as $5a^2$ (Al-Rababaha, Yew, & Meng, 2020). It might occur because students consider the procedure of simplifying algebraic expressions similar to that of arithmetic problems, where a final answer is a single-digit number (Herutomo & Saputro, 2014).
Interpreting the meaning of a variable as a symbol or a label as a substitute for an object in algebraic problems is the most fundamental step in minimizing students' misunderstanding of algebraic mathematical modeling. One effort to overcome students' misconceptions is to design learning activities accommodating the fundamental step. Starting the learning process by solving contextual problems is considered will help students develop understanding and overcome the misconception. In line with this opinion, the Indonesian mathematics textbook for Year 7 students published by Kemendikbud (2017) also has contextual problems as the starting point in learning. Figure 1 shows the example of a contextual problem provided by the textbook.

![Problem 3.1](image)

Mr. Erik and Mr. Tohir, who had just bought books from a grocery store, converse as follows.

Erik: “Mr. Tohir, it looks like you bought many notebooks.”

Tohir: “Yes, I am. I bought 2 boxes of books and 3 books, as ordered by my school. What did you buy?”

Erik: “I just bought 5 books for my grade 7 daughter.”

In the conversation, the two persons expressed the number of books with two different units. Mr. Tohir stated the number of books he bought in the boxes unit, whereas Mr. Erik immediately said the number of books unit.

![Alternative of Problem Solving](image)

**Table 3.1 Algebraic Expression of Problem 3.1**

<table>
<thead>
<tr>
<th>Buyer</th>
<th>Mr. Tohir</th>
<th>Mr. Erik</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy</td>
<td>2 boxes of books and 3 books</td>
<td>5 books</td>
</tr>
<tr>
<td>Algebraic Expression</td>
<td>$2x + 3$</td>
<td>5</td>
</tr>
</tbody>
</table>

In Table 3.1 above, the symbol of $x$ represents the number of books in a box.

Figure 1: Contextual Problem and its Problem Solving Alternative Proposed as Introduction to Algebraic Expressions in the Mathematics Textbook (Source: Kemendikbud (2017))

Based on Figure 1, it is visible that after starting the learning activity with contextual problems, the $2x + 3$ symbol is given immediately, without involving students' negotiation and creativity. Students might be wondering why the symbol is needed in the contextual problem and why it must be symbolized by alphabet $x$. Besides, the introduction of algebraic expressions in the textbooks tends to be represented by the number of objects, although it might be associated with the unknown size of a 2D shape, which is part of geometry learning.

Linking algebra learning with other subjects, e.g., geometry, aligns with one of the characteristics of realistic mathematics education (RME), namely intertwinement (Treffers, 1987). Furthermore, it is essential to provide activities requiring students to solve algebraic expression problems without being overwhelmed by abstract terms such as variables, coefficients, and terms at the beginning of the learning. The abstract terms could be introduced to students after they were doing mathematics in solving reasonable or meaningful problems. The utilization of technology, such as online games, might also be inserted in learning algebraic expressions, according to 21st-century learning.
Concerning the issues presented above, this study aims to evaluate the RME-based learning trajectory (LT) oriented to enhance students’ mathematical representation ability on algebraic expression. This paper examines the research question: "how is the design of RME-based LT to develop students’ mathematical representation abilities on algebraic expressions?" However, this paper is limited to the discussion on the first LT.

RESEARCH METHODS

This study is a design research model proposed by Gravemeijer and Cobb that consists of three phases, namely preparing for the experiment, design experiment, and retrospective analysis (Gravemeijer & Cobb, 2013); the three phases that were implemented in a cyclic process is visualized in Figure 2.

![Figure 2: The cyclic process of design research (Fauzan, Musdi, & Afriadi, 2019)](Emerging_Local_Instructional_Theory)

In preparing for the experiment phase, we determined the endpoints of the LT in which the students develop their mathematical representation ability through learning algebraic expression with the RME approach. Therefore, we analyzed the literature related to learning trajectory, realistic mathematics education, students’ mathematical representation ability, Indonesian national curriculum, grade seven mathematics textbook, and algebraic expression. We then designed LTs based on the result of the analysis. The initial LT of the algebraic expression topic was labeled as Prototype-1. The initial idea for Prototype-1 was obtained through an online workshop involving RME research center teams from several universities in Indonesia, including Universitas Syiah Kuala, and several junior high school mathematics teachers in Banda Aceh. A pilot study involving mathematics teachers from two districts in Aceh province was conducted to assess teachers'
responses toward the Prototype-1. Prototype-1 was then revised based on the pilot study result, and we labeled the revised version as Prototype-2.

The data in this study were obtained from the documentation of learning trajectories, documentation of students’ answer sheets, and video recording of online teaching and reflection sessions. The online teaching involved a mathematics teacher and 15 grade 7 students at a private school in Banda Aceh, Aceh Province, Indonesia. The data was then analyzed descriptively.

RESULTS
The Design of Learning Trajectory

The RME-based LT of algebraic expression in this study, labeled as Prototype-2, consisted of three learning trajectories, and was designed to accommodate four lessons related to algebraic expressions. The first LT (LT-1) was designed for introducing the concept of algebraic expression, whereas the other two LTs were for addition, subtraction, and multiplication of algebraic expressions. However, as mentioned previously, this paper will only focus on the first LT of the Prototype-2.

The instructional activities in the LTs were designed based on RME principles and organized according to the level of emergent modeling (Gravemeijer, 1994; Gravemeijer, 2007; Gravemeijer, Lehrer, van Oers, & Verschaffel, 2013; Bos, Doorman, & Piroi, 2021), namely situation level, model-of level, model-for level, and formal level. The instructional activities of the LT-1 is presented in Table 1.

As presented in Table 1, on the situational level, we proposed the context of predicting the maximum number of balls fit inside some closed different-size baskets and dividing rectangular-land based on the Islamic way of distributing inheritance. The context of dividing rectangular land with unknown sizes was intended to develop students' ability to transform word problems into drawing, which is one type of representation. The following three activities in the LT-1 were also designed to promote students' ability in mathematical representations. Through the third and the fourth activities, the students were expected to represent the problem into the mathematical symbol and algebraic expression, while the fifth activity required students to use their own language to describe the meaning of a variable, a coefficient, and a constant.

Prototype-2 was then assessed and validated by validators. As previously mentioned, Prototype-2 was validated by eight validators consisting of 5 lecturers from the mathematics education department and three secondary school mathematics teachers. The lecturers are mathematics education experts who have been involved in realistic mathematics projects for more than ten years. The teachers were selected because of their willingness to participate as validators in this study. Furthermore, they are willing to be teachers in the teaching experiment phase.
### Level of Activities

<table>
<thead>
<tr>
<th>Activities</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson 1</strong></td>
<td></td>
</tr>
<tr>
<td>Topic: Introduction to Algebraic Expression</td>
<td></td>
</tr>
<tr>
<td><strong>Situation</strong></td>
<td>1. Finding the possible total number of balls inside closed different-size baskets.</td>
</tr>
<tr>
<td></td>
<td>2. Solving problem related to dividing rectangular land with unknown size</td>
</tr>
<tr>
<td><strong>Model Of</strong></td>
<td>3. Finding the possible total number of balls inside closed different-size baskets, if x represents the number of balls in a small basket and y represents the number of balls in a large basket.</td>
</tr>
<tr>
<td><strong>Model For</strong></td>
<td>4. Finding the area of rectangles in which variables predetermine sizes.</td>
</tr>
<tr>
<td><strong>Formal</strong></td>
<td>5. Writing the meaning of variables with students’ own words and giving other examples of coefficient and constant.</td>
</tr>
</tbody>
</table>

#### Table 1: Instructional Activities of LT-1

The analysis of the validation sheets resulted in the average validity score reaching 4.54, indicating that Prototype-2 was in the valid criteria. Furthermore, each of the four aspects of the validity, namely content, format, language, and display, also reach valid criteria. The eight validators agreed that the LT-1 of Prototype-2 could be used with minor revisions, as presented in Table 2.

<table>
<thead>
<tr>
<th>LT</th>
<th>Activity</th>
<th>Validator</th>
<th>Suggestions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>SW</td>
<td>The size of balls in every basket should be the same.</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>PJ</td>
<td>Change the figure with scale pictures.</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>TZ</td>
<td>Need to add one activity before activity 1</td>
</tr>
<tr>
<td>4</td>
<td>ST, SW, FH, EM</td>
<td>Change the variables used in the worksheets as a or m or n to avoid confusion with the multiplication symbol, ×.</td>
<td></td>
</tr>
</tbody>
</table>

#### Table 2: Summaries of Validators’ Suggestions to the Prototype-2

The LT-1 was then revised based on the validator’s suggestions. Generally, there were not many modifications have been made to the activities in the LT-1. We just added one activity in the beginning and made minor revisions to typos and the symbols used. The revised version of the LT-1 is presented in Table 3.
<table>
<thead>
<tr>
<th>Level of Activities</th>
<th>Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson 1</td>
<td>Topic: Introduction to Algebraic Expression</td>
</tr>
<tr>
<td>Situation</td>
<td>Activities in Classroom Discussion</td>
</tr>
<tr>
<td></td>
<td>1. Find the possible pairs of father's and son's ages.</td>
</tr>
<tr>
<td></td>
<td>2. Weight scale problems</td>
</tr>
</tbody>
</table>

**Activities in Group Discussion**

1. a. Solving the problem related to weighed scale.

2. b. Find the possible pairs of whole numbers that add up to 10

   \[ \ldots + \ldots = 10 \]

3. Finding the possible total number of balls inside closed different-size baskets.

4. Solving problem related to dividing rectangular land with unknown size

5. Finding the possible total number of balls inside closed different-size baskets, if \( x \) represents the number of balls in a small basket and \( y \) represents the number of balls in a large basket.

6. Finding the area of rectangles in which variables predetermine sizes.

7. Writing the meaning of variables with students' own words and giving other examples of coefficient and constant.

<table>
<thead>
<tr>
<th>Model Of</th>
<th>Formal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6. Writing the meaning of variables with students' own words and giving other examples of coefficient and constant.</td>
</tr>
</tbody>
</table>

Table 3: Revised Version of LT-1 Instructional Activities
The Pilot Experiments

A pilot experiment involving 15 grade 7 students of one private school in Banda Aceh, Indonesia, was carried out to examine how the designed LT works. Considering the Covid-19 pandemic, we conducted the pilot teaching experiment online via Zoom Meeting. As the apperception, the teacher asked students about some mathematics formulas they had learned to remind students that mathematics uses symbols (see Figure 3). Through the classroom discussion, the students said that the symbols were used to simplify mathematical problems.

![Figure 3: Recalling Mathematics Formulas as Apperception Activity](image)

After communicating the learning motivation and objective, the activity continued to classroom discussion about some problems as presented in Figure 4.

![Figure 4: Problem of Possible Ages of Father and Son](image)

It was observed that during the classroom discussion, the students did not have difficulties finding the likely ages of father and son. Students could quickly answer that, for example, if the son is five years old, then the father is 30 years old, which came from $5 + 25$. Similarly, when the question was reversed, the students also did not have trouble determining how old the son is if the father's age is 35 years. However, different cases happen when the teacher asks students questions involving variables. When the teacher asked, "How old is the son if the father's age is $x$?" the students needed more time and intensive assistance from the teacher before they got the answer $(x - 25)$. 
The classroom discussion was then continued with problems presented in Figure 5. It was observed that the students did not face difficulty in solving problems in Figure 5a and Figure 5b.

![Figure 5: Weigh Scale Problems](image)

After the classroom discussion, the students were required to work in a group of two students. The teachers assigned the students worksheet through WhatsApp Group. While still being online in the zoom meeting, the students were asked to discuss the problems with their partners through WhatsApp. As written in Table 3, there were five activities to be solved in the student worksheet.

The first activity was almost like the activity presented during the classroom discussion about the scale balancing problem (see Figure 6a) and possible numbers added to 10. Related to the problem in Activity 1a, based on the students' written answers, all groups gave the correct answer that is 7 kg. However, only two groups wrote the reason for the answer, as displayed in Figure 6b.

![Figure 6: Problem and Students’ Answers of Activity 1a](image)
Related to the questions in Activity 1b, only three groups could provide the correct answer. Even though the other students could answer the first three questions with different pairs of numbers, they could not answer the fourth question containing variable $x$. While some students did not answer it, some assumed the $x$ with a number, e.g., 7, then wrote $7 + (10 - 7) = 10$. The teacher then brought this students' answer to the class discussion until the students understood it.

Activity 2 dealt with the total number of balls placed in two closed baskets of different sizes. Through this activity, after exploring some possibilities of pairs of numbers, students were expected to use symbols to represent the number of balls in a small basket and a large basket. The problem and students' answers are displayed in Figure 7.

Students should solve the problems of Activity 2 through discussion with their friends via WhatsApp chats. However, it was observed that the group discussion did not go well, and only a few students did the discussion, while the other students worked individually. After some time, the students asked for the teacher’s assistance. Thus, the Activity 2 was solved through classroom discussion, as revealed in the excerpt below.

AMS : The [total] number of balls is 26, then it was asked how many balls were in each basket. That means 26 divided by 5.
Teacher : No, no. Take a look at the baskets. Are they the same size?
NZS : Miss, may I answer?
Teacher : Of course. There were two small baskets and three large baskets. If the sizes are different, how about the number of balls in each basket? Give it a try.
SI : Miss, the large basket contain six balls, the small one contain four balls.
Teacher : Ok, how did you get the answer?
SI : Because the large basket automatically has more balls. 26 divided by 8 resulted in a decimal number, 26 divided by 7 resulted in a decimal number, 26 divided by 5 also resulted in a decimal number. However, if, for example, 26 divided by 6 balls, it equals 18 balls, thus the small baskets if 4 balls multiplied by 2, then the remainder is 8.

Based on the above excerpt, the strategy used by SI was trial and error. He tried to find pairs of numbers from the distribution of 26 balls which resulted in integers because it was impossible if the number of balls is a decimal number. So, he found that the number of balls in the three large baskets is 18; thus, each large basket contains 6 balls. Therefore, the number of balls in the two small baskets is 8, so that each small basket contains 4 balls.

The learning continued to group discussion about Activity 2b. However, after some time of no response from the students, the teachers started to give a clue by proposing abbreviations to represent the number of balls in a small basket and a large basket until the students could solve the problems. Figure 8 shows a student’s answer to the problem in Activity 2.

![Strategy 1 and 2](image)

Strategy 1:
\[2 \times \text{the number of balls in a small basket} + 3 \times \text{the number of balls in large basket} = n\]

Strategy 2:
\[2 \times kk + 3 \times kb = n\]

Figure 8: Example of Students’ Answer of Activity 2 Problems

The teacher then assigned students to complete the next activities at home because the time had been up. Figure 9 displays the example of students’ conclusions about variables, coefficients, and constants.
DISCUSSION

The RME-based LT of algebraic expression in this study was designed following the Gravemeijer and Cobb model consisting of preparing for the experiment, design, and retrospective analysis phases (Gravemeijer & Cobb, 2013). We did not accommodate the division of algebraic expression in the designed Prototype-2 because the activities proposed in the Indonesian mathematics textbook have involved vertical mathematization, which was in line with our idea of teaching division of algebraic expressions.

Based on the pilot experiment of LT-1, it was observed that at the beginning of the learning process, in line with the statement of Rudyanto et al. (2019), the students in this study also have difficulty understanding the meaning of variables. While they could easily find pairs of numbers added up to a number, e.g., to 10 as in Activity 2b in the worksheet, they faced difficulty when the question involved a variable. Rather than answering \((10 - x)\), two students assumed the \(x\) with a number, for example, with 7, then wrote \(7 + (10 - 7) = 10\). This case became additional evidence to the statement of Jupri & Drivers (2016) that students might face difficulties when a calculation involves algebraic expression.

It took more time for the students to understand the meaning of the variable. The classroom discussion talked about the solution to Activity 2b that consists of a variable represented by \(n\), which also confused the students. Therefore, rather than directly use the variable \(n\), the teacher proposed abbreviations such as \(kk\) (from keranjang kecil) and \(kb\) (from keranjang besar), respectively represented Indonesian terms for the small basket and large basket. We argued that using abbreviation to represent the number of balls inside the baskets help the students to understand problems consisting of variables because the students could answer the problems in Activity 3b using either general variables such as \(x\) and \(y\) or abbreviation such as \(kk\) and \(kb\). Furthermore, in Activity 4, the students used either \(x\) and \(y\) or \(F\) and \(R\) to represent Farhan and Rusna.

Answer: Variable = alphabet/symbol representing/substituting unknown value.

Answer: \(5a + 2b – 3c + 4d = 9\)
Variables: \(a, b, c, d\)
Coefficients: 1, 2, 3, 4
Constant: 9

Figure 9: Example of Students’ Conclusion about Algebraic Expression
During the learning process, it was observed that the students need more time to understand the symbolic representation rather than verbal and pictorial representation. This case is in line with Novitasari, Usodo, & Fitriana's (2021) study, which stated that most of the students who participated in their study did not reach a good ability in symbolic representation. However, the students' answers to problems in Activity 3 to 5 and the conclusion presented in Figure 9 indicated that, to some extent, the students have been able to achieve the learning objective of LT-1; they have been able to understand the meaning of variables, coefficients, and constants in algebraic expressions.

There were several obstacles encountered related to the implementation of online learning. Firstly, it was challenging to engage students to participate in group discussion actively, whereas one of the principles of RME is the interactivity between every learning component and subject (Treffers, 1978; van den Heuvel-Panhuizen & Drijvers, 2014). The observation of the online learning process indicated that students were more active in interacting with their teachers in class discussions than interactions with their group members. During the reflection session after the learning involving the teacher and the researchers, the teacher stated that the direct approach she often used to teach could be one reason for the students' being less active in discussion, especially in group discussion. This statement is in line with Webb & Peck (2020), stating that "teachers' decisions and actions are influenced a milieu of personal and contextual factors that include teachers’ prior experiences (including the apprenticeship of observation), teachers' beliefs about mathematics and teaching and learning, local curricular policies, available resources, the expectations of the community, and other factors." The next obstacle was in the time allocation used by the teacher. The time allocation for checking students' attendance, assigning group members, and the introductory activities should be managed well so all designed activities may be done together in class with teacher assistance. These obstacles become one remark to revise the starting activity in the LT-2 to ensure that all students had really achieved the learning objective.

CONCLUSIONS

The current study aims at evaluating the RME-based learning trajectory oriented to enhance students' mathematical representation ability on algebraic expression. The results suggest that the LT-1 designed in this study meets the valid criteria and, to some extent, could support students' mathematical representation ability on algebraic expression. The result and discussion indicate that the students who participated in this study were more familiar with verbal and pictorial representations than symbolic representations. It was more challenging for students to express the word problems into symbols than to images. Therefore, some minor activities should be included in the LT so that each level of the emergent model flows more smoothly. This indication became one remark to revise the starting activity in the LT-2, to make sure that all students had really achieved the learning objective.
Despite the conclusion above, we acknowledge some limitations of this study. First, rather than just depending on documentation of learning trajectories, documentation of students’ answer sheets, and video recording of online teaching and reflection sessions, the data of interviews should be included in this study to enhance the data triangulation. Second, as the data in this study was gathered from one group of students, thus we acknowledge that the result could not be generalized. Additionally, we did not conduct a pre-test before the teaching experiment, and we just studied students' understanding of the mathematical representation of algebraic expression through the literature. Consequently, we could not be very sure about how increase was the level of students' understanding. Last, the implication of this study suggests that teachers and educational researchers keep completing and revising learning trajectories related to the mathematical representation abilities.

REFERENCES


Learning Trajectory of Quadrilateral Applying Realistic Mathematics Education: Origami-Based Tasks

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Abstract: There are various misconceptions students have when studied quadrilateral which encourages efforts needed to overcome these misconceptions. This study aims at overcoming misconceptions by designing learning trajectories in the topic of Quadrilateral applying the Realistic Mathematics Education (RME). Design research carried out at one of junior schools in Garut was used in this research in which thirty-one grade VII students took the participation. The data were collected by providing activity sheets and student worksheets, interviews, and classroom observations. The findings suggest that the learning trajectory of quadrilateral consist of four activities, i.e., origami shape, finding the properties, sulid (stacking sticks), and origami puzzle. From these activities, students can understand the concept of a quadrilateral smoothly. In general, the learning trajectory of a series of learning games/activities can help students to understand, develop, and solve problems in various materials.

INTRODUCTION

Geometry is a branch of mathematics (Aydoğdu & Keşan, 2014; Sukirwan, et al. 2018), which has main portion in the education curriculum in Indonesia because geometry is taught from elementary to high level education. One of the geometry topics in school mathematics is quadrilateral. In the elementary school level, quadrilateral topic is taught from 1st grade to 4th grade. The Ministry of Education and Culture (in Darmawati, Irawan, & Chandra, 2017) stated that in the junior high school level, quadrilateral is taught again with standard competencies of analyzing the characteristics of various quadrilaterals based on sides, angles, relationships between sides and between angles and deriving formulas for determine the perimeter and area of a quadrilateral. Quadrilateral knowledge is the requirement knowledge for studying quadrilateral and similarity (Ardianzah & Wijayanti, 2020).

Although the topic has been thought since elementary school, there are many students in junior high school who make misconceptions regarding it (Hartono, 2020; Rahayu & Afriansyah, 2021). Nadjib (2016) suggested that misconceptions of students were due to a lack of understanding of
the parts and characteristics of quadrilateral making them difficult to understand the characteristics of each quadrilateral. Moreover, the misconceptions were due to lack of understanding regarding the concepts and principles of each quadrilateral so that it is difficult to understand the relationship between each quadrilateral and the difficulty of defining each quadrilateral. Furthermore, an observation made by Sopiany and Rahayu (2019) to the MTs Asy-Syifa students suggested that there were still many misconceptions. One of the misconceptions created by students was applying the formula mistakenly and ignoring to write down the units in the answer, for instance length in centimeters (cm).

Based on some descriptions of those misconceptions, a learning innovation is needed in a design of a learning trajectory activities that can support students to understand the concept of quadrilateral. The learning trajectory is a learning design that considers students' thinking levels directly (Andrews-Larson, Wawro, & Zandieh, 2017; Rich, et al., 2018; Widodo, et al. 2019) of which students learn in their way and actively create their knowledge continuously. The learning trajectory describes students' thinking through various activities to achieve learning goals. Through this activity, students are demanded to understand the concept and see the meaning carried in the material being studied and its connection to everyday life (Buelow, et al. 2018; Tanujaya, et al. 2021).

One of the learning innovations in promoting a learning trajectory is the application of Pendidikan Matematika Realistik Indonesia (PMRI) approach. PMRI has a characteristic in the learning process, namely the use of context (Mariani, 2018). It could be Indonesian or cultural context (Fauziah & Putri, 2020). Learning with PMRI gives possibilities for students to rediscover and build mathematical concepts based on realistic problems presented by the teacher (Majid, 2017; Afriansyah, 2021). Realistic situations in learning enable students to use their informal knowledge to solve problems (Sumirattana, Makanong, & Thipkong, 2017). The PMRI approach is one approach that applies a real-world context in the transfer of learning (Edo & Samo, 2017), in which it is expected that students will be highly motivated because they assume that mathematics is strongly connected to the real world. PMRI is an adaptation of the Realistic Mathematics Education (RME) approach which was initiated by Hans Freudenthal from the Netherlands (Zulkardi, Putri, & Wijaya, 2020). Gravemeijer (Arwadi, et al., 2017; Zubair, et al., 2020), RME has five characteristics which are the operationalization of RME principles, namely: 1) the use of contexts; 2) the use of models, bridging by horizontal-vertical instrument; 3) students' contribution; 4) interactivity; and 5) intertwinenment.

By building learning trajectories with this approach, it is expected that students can avoid misconceptions that usually happen when studying quadrilateral. On this paper, the researcher proposed to design the learning trajectory of quadrilateral by applying the RME.
METHOD

This study employs design research (Van den Akker, et al., 2006). Design research can help determine what kind of learning activities need to be designed to help students understand quadrilaterals. Through these three stages of design research (Afriansyah, et al., 2021), we can see a detailed picture of learning in the classroom along with an analysis of the results of students’ answers carried out in each activity. The purpose of this study is to describe the learning trajectory in the topic of quadrilateral using the RME. There are three phases in this design research, namely: preliminary design, teaching experiment, and retrospective analysis (Gravemeijer & Cobb, 2006).

The preliminary design formulates a learning that was applied in the experimental design phase. There were three activities in this phase. Firstly, the school in general as well as, the classroom including the teacher and the students in particular were observed. Secondly, a number of references related to the various difficulties of students in understanding the concept of a quadrilateral were identified. Thirdly, a number of references related to a series of learning activities related to the Realistic Mathematics Education (RME) approach were analyzed. These three activities are used as the basis of information in designing the Hypothetical Learning Trajectory (HLT), consisting of three components: the learning activities, the learning objectives, and the conjectures or the hypotheses in the learning process. This hypothesis serves as one of the frameworks in preparing the design of learning activities and becomes the reason for developing learning activities that have been designed. An overview of the series of learning activities and their assumptions is described in Table 1.

<table>
<thead>
<tr>
<th>Activities</th>
<th>Main Goals</th>
<th>Conjectures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Origami Shape</td>
<td>Encouraging students to know the definition of</td>
<td>Students must arrange each piece of shape into a variety of quadrilaterals that can be formed.</td>
</tr>
<tr>
<td></td>
<td>quadrilaterals</td>
<td>Students draw each quadrilateral on the table provided and provide an explanation for each image that has been found.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Students are only able to recognize quadrilaterals but do not understand the concept definition of a quadrilateral.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Students are wrong in determining the quadrilaterals that are presented, determining whether they are quadrilaterals or not.</td>
</tr>
<tr>
<td>Finding The</td>
<td>Supporting students to find out the properties</td>
<td>Students can write down the properties of quadrilaterals after going through problem-solving so that students are not only based on memorization.</td>
</tr>
<tr>
<td>Properties</td>
<td>of quadrilaterals</td>
<td>Students are confused to distinguish the properties of each type of quadrilateral.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Students assume that the rectangle has only one position, specifically the horizontal position.</td>
</tr>
</tbody>
</table>
Sulid Activity
Supporting students to find the formula for the perimeter of a square and a rectangle

Students do the activity of sticking sticks that have been cut the same length on each edge of the square and rectangular images.

Students are directed to find the concept of the perimeter of a square and a rectangle by themselves before solving the problems presented.

Students do not understand the perimeter problems presented so that when solving these problems students are confused about what formula to use.

Students do not write perimeter units in solving the problems presented.

Origami Puzzle
Assisting the students to find the formula for the area of a square and a rectangle

Students do the activity of pasting origami paper cut into small squares on each square and rectangular image.

Students are still confused about solving the area problems presented.

Students are wrong in writing the unit area in solving the problems presented.

Table 1: The Overview of Activity and Conjecture of The Learning Process

The teaching experiment was carried out in two cycles, namely the teaching experiment and the pilot experiment. In the previous step, the HLT which had been designed in an experimental experiment was applied in a small group learning process consisting of six students, was selected purposively. The aim was to see how far the learning series that had been designed could explore students' strategies and understanding. Then, the HLT was refined and improved based on the findings from the first cycle. In the second cycle, namely the teaching experiment, the HLT revision was implemented in a natural class setting. Data collection techniques were carried out through classroom observation using videos and student worksheets. In addition, other group discussions by recording to describe students' understanding during the learning process.

In the retrospective analysis stage, all data were obtained, collected, and analyzed. The hypotheses developed in the initial HLT were compared with the results of the implementation of the learning trajectory. Next, an investigation was conducted on the role of learning in analyzing how students gain an understanding of the quadrilateral concept. This HLT revision is applied in the next cycle and analyzed based on the implementation results. This analysis activity was carried out repeatedly depending on the number of cycles carried out, and in this study, only two cycles were carried out.

The research was carried out at one of junior schools in Garut in which thirty-one students participate as the research subject. They were alternately taught in a schedule setting. The whole
schedule of the research activities is presented in Table 2. Three students were selected as the following participant subject to arrange the interview. The three students were chosen based on the difference of their abilities, i.e.: one student each with high, medium, and low abilities.

<table>
<thead>
<tr>
<th>No</th>
<th>Date</th>
<th>Activities/Topics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>November 21, 2020</td>
<td>Quadrilateral definition</td>
</tr>
<tr>
<td>2.</td>
<td>November 25, 2020</td>
<td>Quadrilateral properties</td>
</tr>
<tr>
<td>3.</td>
<td>November 28, 2020</td>
<td>The perimeter of square and rectangle</td>
</tr>
<tr>
<td>4.</td>
<td>December 2, 2020</td>
<td>The area of square and rectangle</td>
</tr>
<tr>
<td>5.</td>
<td>February 6, 2021</td>
<td>Conducting interviews with the selected participants</td>
</tr>
</tbody>
</table>

Table 2: Schedule of Activities

The research was carried out with limited face-to-face learning because it was still in the Covid-19 pandemic condition. The data was collected by providing activity sheets and student worksheets and doing interviews and observations. The given student worksheets are in the form of tests consisting of questions about the description of quadrilateral. The interviews are designed to enable the researchers in obtaining information directly from students. Meanwhile the observations are applied to observe the learning process of Realistic Mathematics Education approach.

RESULTS

The learning trajectory design in this study is a description of student activities in learning the topic of quadrilaterals applying Realistic Mathematics Education approach. The learning trajectory design includes four activities carried out for four meetings covering the definition of quadrilaterals, the properties of quadrilaterals, the perimeter of a square and a rectangle, and the area of a square and rectangle.

Activity 1: “Origami Shapes” Game

The learning goals of activity 1 are that students can identify and understand quadrilaterals and are able to represent quadrilaterals. In this activity, firstly, teacher gave contextual problems by giving examples of rectangular images, such as images of windows, kites, and diamonds. Next, teacher assigned the activity Sheet 1 which contained the steps of the origami shape game which aims at identifying and understanding rectangular shapes and represent quadrilaterals. This activity was performed by six groups consisting of 5-6 students. Each group arranged the required tools and materials, such as origami paper, scissors, rulers, and stationery. Next, they returned to the origami papers of several quadrilaterals that are drawn with calculated sizes. Then each group arranged the pieces of the quadrilaterals into various other quadrilaterals (see Figure 1).
Figure 1: Activity 1 “Origami Shapes” Game

In this activity, the findings of each group are illustrated in the given table in which the students give an information for each image (see Figure 2). Students drew all quadrilaterals on the table, identified whether each image is quadrilateral, and wrote the name of each quadrilateral.

Let's Answer

1. From the activity above, draw your findings in the following table and give a description!

<table>
<thead>
<tr>
<th>No.</th>
<th>Figure Two-Dimensional Shape</th>
<th>Quadrilateral/Not a Quadrilateral</th>
<th>Two-Dimensional Shape Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Quadrilateral</td>
<td>Square</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Quadrilateral</td>
<td>Kite</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Quadrilateral</td>
<td>Rhombus</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Quadrilateral</td>
<td>Rectangle</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Not a Quadrilateral</td>
<td>Triangle</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Quadrilateral</td>
<td>Trapezoid</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2: Example of Student Work Results on Activity Sheet 1
After the activity was finished by all groups, the teacher then distributed Student Worksheets (see Figure 3) as a reinforcement of the initial understanding of rectangular shapes that must be done in groups. The answer from all groups at the problem number 1 i.e., mentioning all the shapes of figures in the picture, were not complete because the triangular shape is not mentioned. However, in the problems numbers 2 and 3 about quadrilaterals, all groups gave correct and complete answers.

![Figure 3: Example of Student Worksheet Answers 1](image)

After that, the teacher shared problems related to quadrilaterals (Figure 4). This time the students worked independently.

![Figure 4: Activity Problem 1](image)
After this problem is presented, the teacher begins to open a discussion with the students:

Teacher: “Well, which one do you think is a quadrilateral?”
S-17: “Which (b) and (d)”
Teacher: “Anything else?”
S-8 and S-26: “No”
Teacher: “Why are (b) and (d) quadrilateral?”
S-17: "Because it has four sides"
S-8: "Because it has four angles, four sides"
S-26: "Because it has four sides and has four right angles"

From the above conversation, the S-17 and S-8 have a good understanding of quadrilaterals, especially the S-8. While S-26 appears to have been misunderstood and upon closer inspection, in the image of S-26, the rectangle is square. In the discussion, the understanding of S-26 was successfully clarified by his friends.

At the end of the activity, it is likely that students already know and understand the definition and kinds of quadrilaterals as illustrated in the following interview fragment:

Teacher: "What is the definition of a quadrilateral?"
Students: "A quadrilateral is a shape that has four sides and four angles".
Teacher: "Mention the kinds of quadrilaterals!".
Students: "The kinds of quadrilaterals include square, rectangle, rhombus, parallelogram, kite, and trapezoid".

In the last activity, the teacher and the students together create conclusions about the activities that have been carried out in accordance with the learning goals to be accomplished at this first meeting.

**Activity 2: “Find the Properties” Game**

The learning goals of this activity are that students can understand and explain the properties of quadrilaterals and are able to solve problems in daily life linked to the properties of quadrilaterals. In this activity, firstly, the teacher presented several problems associated with the properties of rectangular shapes by presenting some examples of quadrilaterals on paper with squares. Next, the teacher assigned Activity Sheet 2 containing the steps of the game to obtain the properties of a quadrilateral which aims to understand and explain the properties of a quadrilateral. The game was performed by six groups of 5-6 students. Each group arranged the required tools and materials, such as origami paper, scissors, rulers, and stationery. Next, the students drew on origami paper a square measuring and a rectangle measuring. Then each of corner of the paper were named ABCD (see Figure 5).
In this activity, the findings of each group are illustrated in the table given in which the students presented information for each image (see Figure 6, see Appendix 1, 2, & 3 for English version of students’ answer). In the answers of this students’ group, it appears that in the first question, students can mention the properties of squares and rectangles, namely there are parallel lines, diagonal lines, symmetry’s axes, and angles. In the second question, students can also write down the properties of other quadrilaterals, such as: rhombus, parallelogram, trapezoid, and kite.

After the game was completed by all groups, the researcher then distributed Student Worksheets as a strengthening of understanding about the properties of quadrilaterals that must be done in groups (see Figure 7, see Appendix 4 & 5 for English version of students’ answer). Based on the student worksheet, it can be suggested that the answers given by students are correct and complete.
Students are able to describe the properties of various quadrilaterals, namely: square, rectangle, trapezoid, rhombus, parallelogram, and kite.

![Figure 7: Example of Student Worksheet Answers 2](image)

After the activity is complete, the teacher shares problems related to the properties of the quadrilateral (Figure 8) and students are required to work independently.
After this problem is presented, the teacher and students begin to discuss:

Teacher: “Well, now what kind of rectangles are built according to these characteristics?”
S-17: “Rectangle”
S-8 and S-26: “Square”
S-17: “Uh, Square”
Teacher: “Now try to draw a square shape!”
(All students draw the square correctly)
Teacher: “Show me which sides are parallel!”
S-17: (S-17 shows two pairs of parallel sides)
Teacher: “Are all the sides the same length?”
S-8: “Equal length”
Teacher: “Then, does it have two diagonals that are perpendicular to each other? Try Showing!”
S-17: “Yes, there are two diagonals”
(All students draw two perpendicular diagonals)
Teacher: “Finally, are every corner, right?”
S-8 & S-26: “Yes, right corner”
Teacher: “How big is the angle?”
All students: “90 degrees”
From the conversation above, all students have a good understanding of the properties of quadrilaterals, especially the properties of squares.

At the end of the activity, it is likely that students have understood the properties of each type of quadrilateral as described in the following interview fragment:

Teacher: "What are the properties of a square?"
Students: "The properties of a square include all the sides are the same length, have two pairs of parallel sides, have two diagonals that are perpendicular to each other and all angles are right angles".

In the last activity, researchers and students together make conclusions about the activities that have been carried out in accordance with the learning goals to be achieved at this second meeting.

Activity 3: “Sulid (Arrange Sticks)” Game

The learning goals in this activity are that students can understand and determine the perimeter of a square and a rectangle and are able to implement the concepts of the perimeter of a square and a rectangle to solve problems in everyday life. In this activity, firstly, the teacher gave some problems linked to the concept of the perimeter of a square and rectangle. Next, the researcher assigned Activity Sheet 3 containing the steps of the Sulid game (stacking sticks) which aimed at understanding and determining the perimeter of squares and rectangles. The game was performed by six groups of 5-6 students. Each group arranged the required tools and materials, such as sticks, paper glue, rulers, and stationery. Next, they arranged the unit sticks that have specific size. In this game, the number of sticks arranged in each shape is called the perimeter (see Figure 9).

![Figure 9: Activity 3 “Sulid (Arrange Sticks)” Game](image)

From these activities, the findings of each group are illustrated in the table given by presenting information for each image found (see Figure 10, see Appendix 6 & 7 for English version of students’ answer). Based on the example of this activity sheet, students arrange sticks that have
been cut with the same size, which is 2 cm, on the edges of the square and rectangular shapes. Through this activity of arranging sticks, students can understand the meaning of the perimeter in squares and rectangles.

![Figure 10: Example of Student Work Results on Activity Sheet 3](image)

After the game was completed by all groups, the researcher then distributed Student Worksheets as a strengthening of understanding about the concepts of the perimeter of a square and a rectangle that must be done in groups (see Figure 11, see Appendix 8, 9, & 10 for English version of students’ answer). Through the student worksheet, it can be seen that students can find the formula for the perimeter of a square and rectangle so that students are able to apply the formula to the given problem.
After the activity is finished, the teacher shares problems related to the perimeter of the rectangle (Figure 12), and students are required to work independently.

After this problem is presented, the teacher and students begin to discuss:

Teacher: "Well, what do you know and ask about from this question?"
S-17: "The area of a rectangle 130cm^2 and its width 10cm. Asked about the perimeter of the rectangle"
Teacher: "What is the formula for the perimeter of a rectangle?"
S-26: "Lxp×k"
Teacher: "What are Lxp×k?"
S-26: "Length, width, perimeter"
Teacher: "You are asked perimeter. Eh but, is that the correct formula?"
S-26: "Oh, so Lxp"
Teacher: "Are you sure?"
S-26: (Silent)
S-17: "No, it should be 2×(p+l)"
Teacher: "Well, okay. Now try to explain how to do it!"
S-17: “From Luas = p \times l. The area 130cm², Panjang = luas: lebar = 130:10=13. So, the length is 13cm”.
Teacher: “After knowing the length, what is the next step?”
S-8: “To the perimeter formula”
\[ K = 2 \times (p + l) \]
\[ K = 2 \times (13+10) \]
\[ K = 2 \times 23 \]
\[ K = 46cm² \]
Teacher: “Why the unit cm²?”
S-8: “Because the width 10cm and the length 13cm, so that cm + cm = cm²”
Teacher: “Oh, I see. Does anyone have another answer?”
S-17:
\[ K = 2 \times p + 2 \times l \]
\[ K = 2 \times 13 + 2 \times 10 \]
\[ K = 26 + 20 \]
\[ K = 46 \]
Teacher: “Okay, it's different in the unit. So, the correct unit is?”
(Most of the students say cm)
Teacher: “Good, cm yes. Let's continue, from that question, why don't you just use the formula for the perimeter of a rectangle?”
S-17: “Because the length is unknown”

From the conversation above, S-8 has a pretty good understanding of the perimeter of a rectangle, it's just that it's wrong to mention the unit. The S-17 has perfect understanding and is not selfish. S-17 pays attention to his friend's answer and always responds to his friend's answer. Meanwhile, S-26 seems not to understand, it can be seen from his presentation about the perimeter formula. Unfortunately, the teacher could not find out more about the answer.

At the end of the activity, it is likely that students already know and understand the concepts of the perimeter of a square and a rectangle as illustrated in the following interview fragment:

Teacher: “Bu Sin plans to fence the flower garden with wire. The length of the flower garden is 7 m and the width is 5 m. What length of wire does Mrs. Sin need? What concept is used to solve the problem?”
Students: “To solve this problem, use the concept of the perimeter of a rectangle, so that the length of wire needed to fence Mrs. Sin's flower garden can be known”.

In the last activity, researchers and students together drew conclusions about the activities that have been carried out in accordance with the learning goals to be achieved at this third meeting.
Activity 4: “Origami Puzzle” Game

The learning goals in this activity are that students can understand and determine the area of squares and rectangles and are able to apply the concepts of square and rectangular areas to solve problems in daily life. In this activity, the teacher gave problems related to the concept of the square and rectangular area. Next, the researcher distributed Activity Sheet 4 which contains the steps of the origami puzzle game which aims to understand and determine the area of squares and rectangles. The game is performed by six groups of 5-6 students. Each group provided the required tools and materials, such as origami paper, scissors, paper glue, ruler, and stationery. Next, they drew on 6 square origami paper squares and arrange the unit squares in square and rectangular shapes. In this game, the number of unit squares that make up each shape is called the area (see Figure 13).

Figure 13: Activity 4 “Origami Puzzle” Game

From these activities, the findings of each group are illustrated in the table given by presenting information for each image found (see Figure 14, see Appendix 11 & 12). Based on the example of the activity sheet, students stick some origami papers that have been cut in a square shape 2 cm x 2 cm in length, on the square and rectangular pictures. Through this activity of sticking origami paper, students can understand the meaning of square and rectangular area.
After the game was completed by all groups, the teacher then distributed Student Worksheets as a strengthening of understanding about the concept of the square and rectangular area that must be done in groups (see Figure 14, see Appendix 13 & 14 for English version of students’ answer). In the worksheet, it can be seen that students can find the formula for the area of a square and rectangle so that students are able to apply the formula to the given problem.

Figure 14: Example of Student Work Results on Activity Sheet 4
When finished, the teacher distributes problems related to the area of the rectangle (Figure 16) and students are required to work independently.

Through this problem, the teacher invites students to discuss:

Teacher: "Well, what do you know and ask about from this question?"
S-26: “Bu Tan has a rectangular rice field with a length 10meter and width of 4meter. Bu Tan wants to expand her rice field by size 8meter × 8meter”
Teacher: “Do you understand what the question means?”
S-26: “Understood, have to draw”

(S-26 illustrates a rice field after it is expanded)
S-8:

(S-8 illustrates an expanded rice field)
S-17:

(S-17 illustrates a rice field after it was expanded)
Teacher: “Okay, S-17, why is the size became 18meter × 12meter?”
S-17: “You want to expand the rice field with a size of 8meter×8meter. So, from 10meter extended 8meter to 18meter and 4meter extended 8meter to 12meter”
Teacher: “Ohh.. What do the others think, which is the correct answer?”
(Most of the students said the answer was S-8 or S-26)

From the conversation above, the S-17 had a mistake in understanding the word expansion. In simple terms, the S-17 thought that this expansion could be solved by addition. In fact, if the area is calculated, different results will be obtained. From this discussion, all students can understand the true broad meaning.

At the end of the activity, it is likely that students already know and understand the concepts of the perimeter of a square and a rectangle as described in the following interview fragment:

Teacher: “Amir wants to replace the living room floor tiles with new tiles. The living room floor measures 10m×10m. What concept was used to determine the tiles Amir needed?”. Students: "To solve this problem, use the concept of a square area, so that it can be seen the number of tiles needed for the living room floor".

In the last activity, researchers and students together drew conclusions about the activities that have been carried out in accordance with the learning goals to be completed at this fourth meeting.
DISCUSSION

A series of learning activities regarding the topic of quadrilaterals that the students went through consisted of four activities, namely the origami shape activity, the activity of find the properties game, the solid activity (stacking the sticks), and the origami puzzle activity (sticking the papers). Each of these activities has its own purpose and is of course interrelated with one another. The following are the objectives of each activity: 1) origami shape is aimed at encouraging students to know the definition of quadrilaterals, 2) finding the properties game is aimed at supporting students to find out the properties of quadrilaterals, 3) solid activity is aimed at supporting students to find the formula for the perimeter of a square and a rectangle, and 4) origami puzzle is aimed at assisting the students to find the formula for the area of a square and a rectangle. Giving activity sheets and student worksheets supports the learning process, where each activity presents the characteristics of RME (Subekti & Prahmana, 2021).

A series of activities through the learning process can assist students to understand the concept of quadrilaterals. This is in line with research carried by Puspasari, Zulkardi, and Somakim (2015) which suggests that a series of learning processes with the RME, in this case, the plotted Tangram, can support students to find the broad concept of polygons. In this study, a series of learning consisting of four activities with RME approach is designed and can support students to find the concepts of perimeter and area of squares and rectangles, and avoid students having misconception in understanding quadrilateral topics in detail. This is also in line with the results of research conducted by Afriansyah (2017) which reveals that a series of RME learning activities can create student-teacher candidates no longer mistaken in understanding the topic of fractions in detail. Therefore, the learning trajectory of a series of learning activities can help students to understand, develop, and solve problems in various materials (Prahmana, Kusumah, & Darhim, 2017; Confrey, et al., 2017; Putra & Vebrian, 2019; Nursyahidah, et al., 2020; Sunedi, 2021).

CONCLUSIONS

Through this research, researchers have succeeded in designing a series of learning activities using the RME approach to study quadrilaterals. The learning trajectory in the topic of quadrilaterals using the RME consists of four activities. Firstly, origami shape activity, which can evoke mathematical ideas of the definition of what a quadrilateral is and what types of quadrilaterals. Secondly, the activity of finding the properties of a quadrilateral which can make students come up with the properties of a square, rectangle, parallelogram, rhombus, kite, and trapezium. Thirdly, solid activity (stacking sticks), which can make students know how to formulate the perimeter of a square and rectangle and the application of the concepts of the perimeter of a square and rectangle in daily life. Lastly, origami puzzle activity which has mathematical ideas about how to form the
area of a square and rectangle as well as the application of the concept of area of a square and rectangle in daily life.

This research can provide contribution for other researchers to be able to design other activities with the same topic. Also, expanding this activity can be a good option if it can cover even better goals. Because what matters most is the contribution of our research to teachers and students in schools.

REFERENCES


Nasional Mahasiswa Kerjasama Direktorat Jenderal Guru dan Tenaga Kependidikan Kemendikbud.


Appendix 1. English Transcript for Figure 6 Part 1.

Figure 6: Example of Student Work Results on Activity Sheet 2

Tools and Materials: Origami Paper, Ruler, Scissors, Stationery

Do this activity in groups!

Let’s do it

Instruction

1) Prepare tools and materials
2) Draw on origami paper a square with a size of 6 cm x 6 cm and a rectangle with a size of 9 cm x 9 cm
3) Cut out the shapes that have been drawn
4) Name each corner of the paper with ABCD

Let’s Answer

1. From the activity above, answer the questions below!

Take a square piece of paper!

1) Measure the lengths of AB, BC, CD, and AD using a ruler
   AB = 6 cm, BC = 6 cm, CD = 6 cm, AD = 6 cm

2) Are the lengths of AB, BC, CD, and AD the same? Mention! Are the lines parallel to each other? If there are parallel lines, name the pair of lines!
   AB = BC = CD = AD = 6 cm
   Line AB with line DC and line BC with line AD

3) Draw the diagonals of the square ABCD. Measure the length of the diagonals AC and BD!
   AC = 8 cm, BD = 8 cm
Appendix 2. English Transcript for Figure 6 Part 2.

Figure 6: Example of Student Work Results on Activity Sheet 2

4) What is the length of the diagonals AC and BD?
The lengths of the diagonals AC and BD are the same length
5) Mention other properties of rectangles that you can find (Example: a measure of angle, an axis of symmetry)
The property of a square has an axis of symmetry A and has a right angle of 90 degrees
Take a rectangular piece of paper!
1) Measure the lengths of AB, BC, CD, and AD using a ruler
   AB = 9 cm, BC = 6 cm, CD = 9 cm, AD = 6 cm
2) Are the lengths of AB, BC, CD, and AD the same? Mention! Are the lines parallel to each other? If there are parallel lines, name the pair of lines!
   AB = CD = 9 cm, BC = AD = 6 cm
   The parallel lines AB and CD are 9 cm, and the lines BC and AD are parallel that is 6 cm
3) Draw the diagonals of the rectangle ABCD. Measure the lengths of the diagonals AC and BD.
   AC = 9.4 cm, BD = 9.4 cm
4) What is the length of the diagonals AC and BD?
   Diagonals AC and BD are the same lengths
5) Are opposite sides parallel?
   Yes, i.e. AB is parallel to DC and BC is parallel to AD
Appendix 3. English Transcript for Figure 6 Part 3.

6) Mention other properties of squares that we can find (Example: a measure of angle, an axis of symmetry) The property of a rectangle has 2 axes of symmetry and has a right angle of 90 degrees

2. Observe the flat shapes below! (1) Rhombus; (2) parallelogram; (3) Trapezoid; (4) Kites

Side length:
(1) same length; (2) There are two lengths of parallel sides; (3) 1 pair; (4) 2 Pair side

Parallel sides:
(1) 2 pairs of parallel sides; (2) 2 pairs of parallel sides; (3) 1 pair of parallel sides; (4) –

Diagonals:
(1) 2; (2) divide equally; (3) has 2 diagonals; (4) 2 perpendicular to each other

Angle:
(1) 4; (2) 4; (3) 4; (4) 4
Appendix 4. English Transcript for Figure 7 Part 1.

From the image of the wake, the cloud must be grouped according to its properties. Find the length of each side, the length of the diagonal, the measure of the angle, and the six shapes above according to the following properties by marking (√)!  

<table>
<thead>
<tr>
<th>No</th>
<th>Note</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Has exactly one pair of parallel sides</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Has two pairs of parallel sides</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Each pair of opposite sides is the same length</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>All sides are the same length</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>The two diagonals bisect each other length</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>The two diagonals are perpendicular to each other</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Both diagonals are the same length</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Each pair of opposite angles is equal</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Every angle is a right angle</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>The sum of the angles is 360 degrees</td>
<td></td>
</tr>
</tbody>
</table>

2. From number 1, can you conclude the properties of each shape? Write your answer in the box below!

Figure 7: Example of Student Worksheet Answers 2
### Appendix 5. English Transcript for Figure 7 Part 2.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Attributes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Square</strong></td>
<td>1. Has two pairs of parallel sides</td>
</tr>
<tr>
<td></td>
<td>2. The two diagonals are perpendicular to each other</td>
</tr>
<tr>
<td></td>
<td>3. Both diagonals bisect each other.</td>
</tr>
<tr>
<td></td>
<td><strong>Length</strong></td>
</tr>
<tr>
<td></td>
<td>4. Every angle is a right angle</td>
</tr>
<tr>
<td></td>
<td>5. The sum of the angles is 360 degrees</td>
</tr>
<tr>
<td><strong>Rectangle</strong></td>
<td>1. Has two pairs of parallel sides</td>
</tr>
<tr>
<td></td>
<td>2. The two diagonals are perpendicular to each other</td>
</tr>
<tr>
<td></td>
<td>3. Both diagonals bisect each other.</td>
</tr>
<tr>
<td></td>
<td><strong>Length</strong></td>
</tr>
<tr>
<td></td>
<td>4. Each pair of opposite sides is the same length</td>
</tr>
<tr>
<td></td>
<td>5. Every angle is a right angle</td>
</tr>
<tr>
<td></td>
<td>6. The sum of the angles is 360 degrees</td>
</tr>
<tr>
<td><strong>Kite</strong></td>
<td>1. Has exactly one pair of parallel sides</td>
</tr>
<tr>
<td></td>
<td>2. Has two pairs of parallel sides</td>
</tr>
<tr>
<td></td>
<td>3. The two diagonals are perpendicular to each other</td>
</tr>
<tr>
<td></td>
<td>4. Both diagonals bisect each other.</td>
</tr>
<tr>
<td></td>
<td><strong>Length</strong></td>
</tr>
<tr>
<td></td>
<td>5. Every angle is a right angle</td>
</tr>
<tr>
<td></td>
<td>6. The sum of the angles is 360 degrees</td>
</tr>
<tr>
<td><strong>Parallelogram</strong></td>
<td>1. Has two pairs of parallel sides</td>
</tr>
<tr>
<td></td>
<td>2. The two diagonals are perpendicular to each other</td>
</tr>
<tr>
<td></td>
<td>3. Both diagonals bisect each other.</td>
</tr>
<tr>
<td></td>
<td><strong>Length</strong></td>
</tr>
<tr>
<td></td>
<td>4. Every angle is a right angle</td>
</tr>
<tr>
<td></td>
<td>5. The sum of the angles is 360 degrees</td>
</tr>
<tr>
<td><strong>Trapezoid</strong></td>
<td>1. Has two pairs of parallel sides</td>
</tr>
<tr>
<td></td>
<td>2. Both diagonals bisect each other.</td>
</tr>
<tr>
<td></td>
<td><strong>Length</strong></td>
</tr>
<tr>
<td></td>
<td>3. The two diagonals are perpendicular to each other</td>
</tr>
<tr>
<td></td>
<td>4. The sum of the angles is 360 degrees</td>
</tr>
<tr>
<td><strong>Kite</strong></td>
<td>1. Has two pairs of parallel sides</td>
</tr>
<tr>
<td></td>
<td>2. Both diagonals bisect each other.</td>
</tr>
<tr>
<td></td>
<td><strong>Length</strong></td>
</tr>
<tr>
<td></td>
<td>3. The two diagonals are perpendicular to each other</td>
</tr>
</tbody>
</table>

---

Figure 7: Example of Student Worksheet Answers 2
Appendix 6. English Transcript for Figure 10 Part 1.

**Figure 10:** Example of Student Work Results on Activity Sheet 3

**Aim:**
Understanding the perimeter of squares and rectangles

**Determine the perimeter of a square and a rectangle**

**Tools and Materials:** Sticks, Ruler, Paper glue, Stationery

**Do this activity in groups!**

**Let's do it**

**Instruction:**
1) Prepare tools and materials
2) Make a stick measuring 2 cm
3) Arrange the unit sticks on the edges of the shape (a) and the shape (b)
4) The number of unit sticks arranged in each shape is called the circumference

**Let's Answer**
1. From the activity above, arrange the unit sticks in the shapes (a) and (b) below!

   **Rectangle**
Appendix 7. English Transcript for Figure 10 Part 2.

![Image of student work]

Rectangle

2. How many unit sticks are arranged in shape (a)?
12 sticks

3. How many unit sticks are arranged in shape (b)?
16 sticks

4. What is the perimeter of the shape (a)?
12 sticks unit

5. What is the perimeter of the shape (b)?
16 sticks

Figure 10: Example of Student Work Results on Activity Sheet 3
Appendix 8. English Transcript for Figure 11 Part 1.

Pay attention to the square in the image below!
1. Calculate:
   - Length AB = 4 units
   - Length BC = 4 units
   - Length CD = 4 units
   - Length AD = 4 units
2. From number 1, we can know that:
   - Length AB = Length BC = Length CD = Length AD
3. Based on the results obtained from numbers 1 and 2, complete the blanks below!
   - Perimeter of Square ABCD = AB + BC + CD + AD
   - Perimeter = 4 (AB)
   - Perimeter = 4 (4) units of length
   - Perimeter = 16 units of length
   - If the length of AB = s units of length, then in general the perimeter of the square is:
     \[ K = AB + AB + AB + AB = 4s \]
You managed to find the formula for the perimeter of a square, then you can solve the problem above. How many meters did Mr. Lim travel? Write your answer in the box below!
\[ K = 4 \times s = 4 \times 18 = 47 \text{ m} \]
So, the distance that Mr. Rico travels is 47 m.

Figure 11: Example of Student Worksheet Answers 3
Appendix 9. English Transcript for Figure 11 Part 2.

Problem 2
Pak Lim owns a rectangular flower garden. The length of the flower garden is 20 m and the width is 15 m. The plan, Mr. Lim wanted to fence the flower garden with barbed wire. What length of wire does Mr. Lim need to fence the garden?

Task 2:
Pay attention to the rectangle in the image below!
1. Calculate:
   - Length AB = 5 units length
   - Length BC = 3 units length
   - Length CD = 5 units length
   - Length AD = 3 units length
2. From number 1, we can know that:
   - Length AB = Length DC
   - Length BC = Length AD
3. Based on the results obtained from numbers 1 and 2, complete the blanks below!
   - Perimeter of Rectangle ABCD
     = AB + BC + CD + AD
     = AB + AD + AB + AD (because AB = BC and BC = AD)
     = 2AB + 2AD
     = 2(AB + AD)
     = 2(5 + 3)
     = 2 x 8
     = 16 units of length
   - If the length of AB = p units of length and BC = l units of length, then in general the perimeter of the rectangle is:
     - K = 2p + 2l
     - K = 2(p + l)
Appendix 10. English Transcript for Figure 11 Part 3.

**Figure 11: Example of Student Worksheet Answers 3**

You managed to find the formula for the perimeter of a rectangle, then you can solve problem 2 above. What length of wire does Mr. Lim need to fence the garden? Write your answer in the box below!

K = 2p + 2l
2 x 20 + 2 x 15
= 40 + 30
= 70 m

So, the wire needed Mr. Lim is 70 m
Appendix 11. English Transcript for Figure 14 Part 1.

Origami Puzzle Game

Aim:
Understanding the area of squares and rectangles
Determine the area of a square and a rectangle
Tools and Materials: Origami paper, Paper glue, Stationery, Scissors, Ruler

Do this activity in groups!
Let’s do it
Instruction!
1) Prepare tools and materials
2) Draw a unit square on origami paper measuring 2 cm
3) Arrange the unit squares in shape (a) and shape (b)
4) The number of unit squares that make up each shape is called the area

Let’s Answer
1. From the activity above, arrange the unit squares in the shapes (a) and (b) below!

(a) square

Figure 14: Example of Student Work Results on Activity Sheet 4
Appendix 12. English Transcript for Figure 14 Part 2.

Figure 14: Example of Student Work Results on Activity Sheet 4

(b) Rectangle

2. How many unit squares make up the shape (a)?
9 Square

3. How many unit squares make up the shape (b)?
15 Square

4. What is the area of figure (a)?
9

5. What is the area of figure (b)?
15
Appendix 13. English Transcript for Figure 15 Part 1.

Task 1:
Pay attention to the square in the image below!
1. Calculate:
   Length AB = 4 units length
   Length BC = 4 units length
   Length CD = 4 units length
   Length AD = 4 units length
2. From number 1, we can know that:
   Length AB = Length BC = Length CD = Length AD
3. Based on the results obtained from numbers 1 and 2, complete the blanks below!
   The area of Square ABCD
   = Length AB $\times$ Length BC
   = 4 $\times$ 4
   = 16 units of area
   If the length of AB = s units of length, then in general the area of the square is:
   $L = \text{Length AB} \times \text{Length CD}$
   $L = s \times s$
   You managed to find the formula for the area of a square, then you can solve the problem 1 above. How much wallpaper does Wendi need to cover the entire wall surface?
   Known: $s = 12$ m
   $L = s \times s = 12 \times 12 = 144$ m$^2$
   So, Wendi needs wallpaper to cover the entire surface is 144 m$^2$

Figure 15: Example of Student Worksheet Answers 4
Appendix 14. English Transcript for Figure 15 Part 2.

Problem 2
Susan's rectangular bathroom floor will be covered with some tiles. If the length of the bathroom is 6 meters and the width is 4 meters, how many tiles does Susan need?

Task 2:
Pay attention to the rectangle in the image below!
1. Calculate:
   - Length AB = 5 units length
   - Length BC = 3 units length
   - Length CD = 5 units length
   - Length AD = 3 units length
2. From number 1, we can know that:
   - Length AB = Length DC
   - Length BC = Length AD
3. Based on the results obtained from numbers 1 and 2, complete the blanks below!
   - The area of Rectangle ABCD = Length AB x Length DC = 5 x 3 = 15 units of area
   - If the length of AB = p units of length and BC = l units of length, then in general the area of the rectangle is:
     \[ A = p \times l \]
4. You managed to find the area of the rectangle, then you can solve Problem 2 above. How many tiles does Susan need? Write your answer in the box below!
   - p = 6 m
   - l = 4 m
   - \[ A = p \times l = 6 \times 4 = 24 \text{ m}^2 \]
   - So, Susan need number of tiles is 24 m$^2$
Learning Design on Surface Area and Volume of Cylinder Using Indonesian Ethno-mathematics of Traditional Cookie maker Assisted by GeoGebra

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Abstract: "Kue Putu" is a traditional Indonesian cake that is well-known to the students. Besides, the "Kue Putu" mold is made from cylinder-shaped bamboo culm and possesses the potency for learning geometry in a meaningful way. In addition, Indonesian Realistic Mathematics Education (IRME) is a mathematical learning strategy in which students study using context relevant to learners' life as beginning points. The article examines a mathematics learning design on the area and volume of cylinders using ethnomathematics context carried by traditional cake mold assisted by GeoGebra. This material is designed to help students understand the relationship between surface area and volume by examining the diameter and height of the cylinder. In developing the design, a research method, namely Design research, was applied. Following Design Research from the Gravemeijer model, we tested design research in terms of Hypothetical learning trajectory on area and volume of cylinders in the three phases, namely preliminary design, teaching experiment, and retrospective analysis. The study resulted in the theoretical design and practical instruments based on the method that contributes to instructional theory and supports student learning on the area and volume of the cylinder. Students show excellent reasoning on how increasing cylindrical radius gives a more significant effect than increasing its height. The student also construes the design of the cylinder that provides the most considerable volume by expanding its base or radius.

INTRODUCTION

Since Realistic mathematics education (RME) developed in the Netherlands, it is now widely spread worldwide, including in Indonesia. RME has a context in its characters, and it plays a significant role in learning. A good context can be developed from mathematics as it can be imagined or derived from local situations. In Indonesia, RME has livened up Indonesian teaching and learning activities as an alternating approach (Wijaya, 2012; Prahmana et al., 2020). Even
RME is originally from the Netherlands (Van den Heuvel-Panhuizen & Drijvers, 2014) it becomes Pendidikan Matematika Realistik Indonesia (PMRI) as it Indonesian version that uses local context in it designs (Zulkardi, Putri, & Wijaya, 2020). As context plays a significant role in learning mathematics, a local context close to the student is chosen as part of PMRI design (Putri, Dolk, & Zulkardi, 2015; Nursyahidah, Saputro, & Albab, 2020). Many of them using traditional games (Putri, 2012; Nursyahidah, Putri & Somakim, 2013; Prahmana et al., 2012) and other using Indonesian national heritage like batik (Cici et al., 2014; Widada et al., 2019a; Widada et al., 2019b). One of the local contexts that can also be promoted as a context in mathematics learning is ethnomathematics. Ethno-mathematics itself defines as activities using mathematics that is used by particular ethnic groups such as farmers, anglers, etc. (Nursyahidah, Albab, & Saputro, 2019).

Categorizing ethnomathematics as an activity means that ethnomathematics exhibit a human work using mathematics. If it is not an activity, it is excluded from ethnomathematics. In line with this definition, Freudenthal said that mathematics is a human activity. The teacher needs to introduce Mathematics in the same way: using activities (Sembiring, Hadi, & Dolk, 2008).

The philosophy and local heritage are excellent for learning design in the mathematics classroom (Nursyahidah, Saputro, & Albab, 2020). Local heritage offers an eminent starting point in learning mathematics. It serves as an initiator for the student to develop their concept knowledge, algorithm, or tools. Later, when students generalize their informal knowledge to formal knowledge, context can be less used anymore (Van den Heuvel-Panhuizen, 2020). Interesting ethnomathematics can be selected from the activity conducted by traditional food vendors like Putu Bambu, a traditional Indonesian cake made from rice and palm sugar. The context is about the mold of the cake in cylindrical form. As the mold is made from bamboo trees, it is frequently ended with non-identical mold. There is a diameters variation in its molds. This situation will be a perfect starting point for the student to construe the relation between the volume and diameter of the cylinder. Learning surface area and volume of the cylinder in this way is lacking in Indonesian textbooks (Khoironisyah, 2007). This topic and other similar topics are taught in a traditional setting where students do not understand mathematics.

Even many activities are identified as ethnomathematics, not all of them can be a good context in learning mathematics. In this issue, Van den Heuvel-Panhuizen (2005) suggested criteria on how context will serve as a proper context: context must be able to make the problem transparent and increase student accessibility to the problem, context exposes all sides of the problem, and context provides an essential strategy for students. In this study, we use the wisdom from Indonesian culture, “Kue Putu”. The mold of Putu cake is made from the bamboo culm. However, not all of the bamboo culm is identical. Putu cake makers tend to make the mold differ based on the diameter of the bamboo culm. Because the mold is not similar, this case can be an excellent ethnomathematical context to examine the volume of different mold producing the same amount of Putu.
After choosing the proper context, one step of using context as a starting point in mathematics learning, developing a model, is crucial. Mathematics in RME will not be taught as formulas that students are ready to keep in their memory. The concept of the surface area and volume of the cylinder is developing using a design heuristic represented in the iceberg hierarchy from informal to formal mathematization (Gravemeijer & Bakker, 2006). Ethno-mathematics in the local community often grows as informal mathematics or just an intuitive phase without any confirmation. Learning sequence designed using ethnomathematics drawn in Hypothetical Learning Trajectory (HLT) helps students understand the situation. Furthermore, they use it in formal mathematization.

HLT is a set of the aim of design, set of activities, and student thinking conjectures. HLT proposed evocation of decisive aspect of designing mathematics lesson. Simon and Tzur (2004) suggest that HLT should fulfill three elements: the aim of student learning activities, mathematical activities that support student learning, and conjecture about how the student will respond to the task.

To help the students understand the cylinder material in this study, GeoGebra was used in students’ activities. By using GeoGebra can have a positive impact to support students understanding of the properties of a geometry object and the concepts of volume and surface area of 3-D shapes like a cylinder (Putra et al., 2021). In line with this, Dogruer & Akyus (2020) stated that students’ understanding of 3D shapes enhanced simultaneously with argumentation and dynamic geometry software of GeoGebra assisted them in visualizing and reasoning in that concept. In addition, using GeoGebra can help stimulate the emergence of several aspects in solving the problem and demonstrate understanding of the concepts being studied (Sukirwan, et al. 2018; Hernández, Díaz, & Machín, 2020).

This study seeks to determine whether or not a mathematics learning design on the area and volume of a cylinder utilizing ethnomathematics context using traditional cake mold assisted by GeoGebra enhances students’ knowledge of the relationship between surface area and volume of a cylinder.

**METHODOLOGY OF RESEARCH**

To examine the instructional design that aids students in problem-solving, we propose the Gravemeijer Design Research Model (Gravemeijer & Cobb, 2006). Design research has five traits: intervention character, process-orientedness, reflecting component, cyclical nature, and theory orientation (Gravemeijer, 2004; Prahmana, 2017). Additionally, design research has been selected for this study since it is a systematic and adaptable strategy for enhancing the quality of classroom instruction through collaboration among researchers and educators to construct a learning design (Gravemeijer, 1994). The design research is divided into three stages: preliminary design, design experiment, and retrospective analysis (Bakker, 2004; Gravemeijer & Cobb, 2006).
Preliminary research

The preliminary design stage focuses on developing learning activities and instruments to assess the learning process. A literature survey was undertaken in this research on the concepts of surface area and volume of a cylinder, PMRI, and analysis of a cylinder material in the Indonesian mathematics curriculum to create a conjecture of students' thinking. Besides, in this phase, Hypothetical Learning Trajectory (HLT) is made as a design and instrument. HLT as design contains activity goals, activities, and conjecture of student response to activity done by them. In this research, there are four activities in understanding the concept of the cylinder. In the second role, HLT guides the direction and focus of research analysis (Alim et al., 2020). HLT is improving during this step using a thought experiment, an experiment in which we guess and check the possibility of student thinking from the experienced teacher's perspective. In this phase also, HLT can be adjusted from the findings at each stage (Nuraida & Amam, 2019).

Design experiment

The design experiment step comprises two cycles. The first is a pilot experiment that attempts to evaluate and enhance the intended learning trajectory in a small group. The second is a teaching experiment in which a learning trajectory is implemented and assessed during the actual learning in the classroom. The students of grade IX SMP were involved in the classroom teaching experiment. Six students participated in the pilot experiment, and 26 students participated in the classroom teaching experiment. This research was assisted by a model teacher and was conducted in the first semester of the 2020/2021 school year at SMPN 38 Semarang, Indonesia.

The data collected in this study were students' strategies in carrying out activities. The research instruments were class observation sheets that included field notes, student activity sheets, and interviews with students and teachers. Student difficulties and their strategy in solving a problem in the activities are used to improve HLT. Techniques pursued in this phase are focus group observation, video-typing, and walkthrough.

Retrospective analysis

The final phase is a retrospective analysis. The data obtained during the design experiment step are assessed by comparing conjecture and HLT to the learning trajectory implementation outcomes during the experiment phase's design (Gravemeijer & Cobb, 2006). Moreover, student difficulties and strategies found from the study are then analyzed findings and notes on how students respond or student strategy on the situation in design. By comparing student thinking conjecture and results in teaching experiments, research questions are solved. The following is an overview of the research framework in this study.
RESULT AND DISCUSSION

As a result of this research, we acquired a learning trajectory description of the cylinder's surface area and volume utilizing the Putu cake mold assisted by GeoGebra. Furthermore, in this chapter, the researchers describe the results obtained throughout each study stage as follows.

Preliminary Design

In this phase, the researchers implemented the initial idea of learning the surface area and volume of the cylinder utilizing the context of the Putu cake mold. Putu's cake mold was picked as the context in this study since its shape represents a cylinder and is familiar for students, enabling students to grasp the ideas of surface area and volume of a cylinder. Additionally, there is an exciting aspect to the situation of creating Putu cake molds from bamboo trees. Because the sizes of the bamboo culms are not identical, the variation that arises is that the molds have different diameters. To keep each mold holding the same filling, Putu's cake-cutter craftsman makes a few manipulations. The variations involve making short culms of bamboo with a large diameter and making long culms of bamboo small in diameter. It will be an intriguing topic to discuss with pupils.
Moreover, the development of HLT in every learning activity is the most crucial part of designing student learning activities. The design is inseparable from the learning trajectory, which contains a lesson plan for teaching the material. In this case, the learning trajectory is a concept map that students will pass during the learning process. In addition, the HLT planning process was done by conducting a literature review, conducting observations, and designing a learning trajectory for the surface area and volume of the cylinder as a series of instructional learning. Also, a curriculum review to ensure that lessons based on the mathematical standards were appropriate for students in the ninth grade. The analytical process involves establishing instructional materials, objectives of the lesson, and learning indicators. The learning process developed by HLT includes four activities for three meetings. Each activity is simple, engaging, and delightful. For detail, Table 1 summarizes the activities and the student's conjectures in this study.

<table>
<thead>
<tr>
<th>Name of activity</th>
<th>Aim</th>
<th>Conjecture of student response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Creating the &quot;Kue Putu&quot; mold</td>
<td>To identify the components of the cylinder</td>
<td>- Students are familiar with and capable of describing the shape of the &quot;Kue Putu&quot; mold; afterward, they create the model using paper</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Students make different types of cylinder nets</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Students are puzzled when attempting to determine the elements of a cylinder</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Students attach the cylinder model's net recklessly and without regard for the proper size to create a cylinder model's net</td>
</tr>
<tr>
<td>Determining the formula for a cylinder's surface area</td>
<td>To devise a formula for the cylinder's surface area through cylinder nets that find in the previous activity</td>
<td>- Students investigate determining the cylinders' surface area by adding the surface area of the cylinder net</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Students locate if the cylinder's surface area equals the sum of the base area multiplied by two and the curved surface area</td>
</tr>
</tbody>
</table>

Figure 2. Putu's cake in the bamboo mold
Students find the area of cylinder nets thru employing their knowledge of circle and rectangular areas
- Students discover the curved surface is equal to a rectangle with a length is the cylinder base circumference \(2\pi r\), and width is \(t\) (height of cylinder)

| Discovering the formula of cylinder volume | To discover a formula for the volume of the cylinder using the assistance of students' worksheets by identifying the formula principle of the prism's volume | - Students can recall the volume formula of the prism
- Students might conclude by determining the prism volume formula if the volume formula of the cylinder equals the base area times height |
| Resolve problems related to volume and surface area of cylinders | - To investigate whether the different dimensions of cylinders have the same or almost the same volume or not
- To explain why changes in height give a more negligible impact on the cylinder volume than changes in volume
- To assess that same surface area provides variation in volume
- To maximize the volume of a cylinder using fixed surface area | - Students can discern the effect of each height and radius on the volume
- Students think that the *Putu* mold is unfairly filled
- Students can comprehend the instructions
- Students believe that both designs can produce different amounts of volume |

Table 1: The outline of the learning process's activities and hypotheses

**Design Experiment**

At this stage, the researcher implemented a learning trajectory designed for ninth-grade junior high school students on the cylinders' surface area and volume using the *Putu* cake mold. However, in the mids of the covid-19 pandemic, learning takes place online. We efficiently track student work progress through the use of Geogebra Classroom and Google Meet conferencing. Figure 3 illustrates the GeoGebra Classroom activity panel.
Besides, there are four activities carried out in the design experiment stage, including:

**Activity 1: Creating the "Kue Putu" mold to identify the components of the cylinder**

Before beginning the class, the teacher activated the audio of the cake putu vendor, which sounded like the shrill sound typical with the steam boiler "Ngiiingggg". When the teacher turned on the audio, the students appeared eager to learn about the material that would be discussed. Following that, the teacher offered a salutation and questioned the pupils, "Do you know what sound I played earlier?" Most students said that the sound was that of a Putu cake vendor. In contrast, others responded that it was the sound of chimneys, steam boilers, and broken machines. The teacher then confirmed that the voice was indeed that of a cake Putu vendor.

Additionally, the instructor inquired about the Putu cake. The teacher addressed clarifying questions to ascertain pupils' understanding of the Putu cake mold, which will serve as the context for the learning process. Students can make several references to the Putu cake mold, as demonstrated in Dialogue 1.

**Dialogue 1.**

**Teacher**: Have you bought a putu cake from a peddler?
**Students**: Yes, I had.
**Teacher**: What do you know about Putu cake?
**Students**: Putu cake form a small cylinder with a brown sugar filling and a coconut topping. This made me starve.
**Teacher**: When ordering Putu cake, did you consider Putu's cake mold? Could you elaborate on what you've observed?
**Students**: The mold is made from bamboo, the shape of the mold is a cylinder, the mold is like a small drum.

Dialogue 1 demonstrates that students were acquainted with the Putu cake mold and could determine if the mold represents a cylinder. It will make the learning process more fascinating for
pupils to learn the surface area and cylinder volume since the context is relevant. The instructor explains the context of the *Putu* cake mold, which serves as a jumping-off point for the learning process. Following that, the teacher notifies pupils of the learning objective, which is to determine the cylinder's elements by making a *Putu* cake mold model using paper. Additionally, the teacher informs pupils about classroom activities with discussions and presentations.

The teacher then divides the pupils into several small groups of 4-5 students using the Goggle Meet breakout room to facilitate student discussion. The teacher instructs learners to construct a paper model of *Putu's* cake mold. When the teacher observed student discussion activities in the Google Meet breakout room, the students appeared engaged and attempted to create various versions of cylinder nets as *Putu's* cake molds. After successfully creating a putu cake mold model out of tube nets, students were asked to identify the elements of the cylinder. In this activity, students may quickly determine that the tube element has a circular top and bottom side and a rectangular curved side.

After the students have completed Activity 1, the teacher asks one group to share the outcomes of their discussion. One group employed Goggle Meet to present the discussion results, demonstrating the cylinder nets successfully, made for the *Putu* mold model and mentioning the cylinder elements. The teacher subsequently invites other groups to raise questions or request clarification on the results that have been presented. Following that, the teacher reinforces material topics related to nets and elements of the cylinder. Additionally, it could be concluded that students perceive the idea of the cylinder and its parts appropriately in light of the first activity's objectives.

**Activity 2: Determining the formula for a cylinder's surface area**

In activity 2, students are provided student worksheets and GeoGebra applets that are organized to assist them in grasping the notion of a cylinder's surface area. Students are instructed to determine the surface area of the cylinder using the cylinder nets made in the preceding activity to solve questions in the student worksheet. Then, students have drawn a net of the cylinder to identify the elements. Additionally, students can infer that the cylinder surface area equals the area of the cylinder nets. When the teacher validates the students' replies, students explain that the surface area of the cylinder is the same as the nets area because when the cylinder is opened, it forms a cylinder net and vice versa.

Students can understand that to get the formula for a cylinder's surface area, and they must first determine the area of each element. Students can readily calculate the area of the top and bottom sides of the cylinder when discussing in groups through the Goggle Meet breakout room. On the other hand, students frequently struggle to find the length and width of a curved side. Students thought that the length of the rectangle, which is the curved side was the diameter of the side of the base. Following that, the teacher asks students to try to prove it by using cylinder nets that have been made. The student then realized that their statement was wrong. Next, the teacher instructs pupils to determine the area of the curved side using the GeoGebra applet (see Figure). Students
use the Geogebra applet to open the cylinder into a net of cylinders, allowing them to visualize that the length of the rectangle on the curved side of the cylinder equals the circumference of the base side of the cylinder.

Figure 4: GeoGebra Applets to assist students in finding the surface area of the cylinder

Furthermore, based on this activity established aided by student worksheets, the cylinder nets of the Putu cake mold model, and the GeoGebra applet, students can deduce that the formula for a cylinder’s surface area equals the sum of its base, top side, and the curved side.

**Activity 3: Discovering the formula of cylinder volume**

The activity carried out at this stage is the activity to discover the formula of cylinder volume. The teacher shared the student worksheet link via the Google Meet chatbox. Next, students work in groups to solve questions on the student worksheet. Students were instructed to determine the volume of a cylinder by comparing it to the volumes of a triangular prism and a cuboid. However, some students struggle to discover the connection between the volume of a cuboid, triangular prism, and a cylinder. Hence, the teacher poses several questions to the students to elicit assistance in determining the connection related to three of it, as illustrated in Dialogue 2. However, after discussing with the teacher, students could discover that the cylinder volume formula is the base area multiplied by height, where the base is a circle.

**Dialogue 2.**

Teacher : What is the correlation between the (p) length and (l) width in the cube volume formula?
Activity 4: Resolve problems related to volume and surface area of cylinders

To give students a progressive understanding of the surface area and volume of a cylinder, in the last activity teacher gave two problems related to the effect of variation in height and radius to Putu’s mold volume and examined different volumes from the same surface.

The first problem is related to the local wisdom of Putu bamboo makers. This problem would assist students in investigating whether the different dimensions of cylinders have the same or almost the same volume and explain why changes in height have less impact on the cylinder volume than changes in radius. Students face situations on Putu bamboo mold, traditional food made from rice flour and palm sugar steamed in Bamboo culm mold. The molds are not identical. Students need to investigate whether the molds will give the same volume. This investigation was carried out using a manipulative applet adapted from GeoGebra sources, consisting of a cylinder model with surface area and volume and buttons to help students to be able to change the height and radius of the cylinder (see Figure 5). Students can explore the impact of height and radius variation on volume by dragging red buttons. Information related to surface area and volume of cylinder appears beneath the cylinder model. In these activities, students need to find the relation between height and radius variation to the volume. Then, students need to conclude that Putu makers are fair enough in making the volume the same or almost the same filling.

Figure 5: Problem-related to height and radius variation effect on volume

<table>
<thead>
<tr>
<th>Base area = 12.87 cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume = 38.6 cm³</td>
</tr>
<tr>
<td>Total area of curved surface = 63.86 m²</td>
</tr>
</tbody>
</table>

Show/hide the surface area
Show/hide the volume
Show/hide the base area
### Common Issues

**Students cannot distinguish the effect of each height and radius on the volume**

“Try to change 1 unit in height. How is the volume? Now try to change 1 unit in radius. How is the volume? Which changes give much effect?"

**Students think that the *Pulu* mold is unfair in filling**

“Assume height and radius of ‘thin’ and ‘fat’ mold, calculate the volume, and compare the volume! Is it significantly different or almost the same?”

<table>
<thead>
<tr>
<th>Tabel 2: Question to prompt the students to conjecture in the first problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Responses to this activity vary, but most share ideas that the impact of variation in height and radius gives a different result. Figure 6 explains the student’s argument that radius variation gives a more significant effect on the volume.</td>
</tr>
</tbody>
</table>

**Variation in height and radius give different (impact) because formula of cylinder volume is** \( V = \pi r^2 t \). Radius of cylinder is squared, while \( t \) (height) does not. Adding 1 cm in radius gives greater than adding 1 cm in cylinder height.

---

Variation in height and radius give different (impact) because formula of cylinder volume is \( V = \pi r^2 t \). Both \( r \) and \( t \) are not linear correlated.

---

Variation in height and radius give different (impact), because formula of cylinder volume is \( V = \pi r^2 t \). Radius of cylinder is squared, while \( t \) does not.

---

**Figure 6: Sharing idea of student related to the variation of height and radius of cylinder to the volume**

Based on Figure 6, students are reasoning in a progressive mathematics way. It is fascinating, in any case. They compare ‘r’ and ‘t’ forms in the cylinder formula, which is ‘r’ is squared, so ‘r’ addition gives quadratic growth in volume. Different from ‘r’, adding 1 unit in ‘t’ only gives linear volume growth.
From this reasoning, students start to understand Putu’s mold height differentiation. They agree that artisans make short culms of bamboo with a large diameter and long culms of bamboo with a small diameter to maintain the same filling volume. Furthermore, Figure 7 tells us how Putu’s mold craftsman idea supports student thinking.

![Figure 7: Student thinking Putu’s Bamboo mold manipulation](image)

To equalize the size of the volume produced by bamboo molds with a larger diameter or radius with a shorter and bamboo prints with a smaller diameter or radius but higher

<table>
<thead>
<tr>
<th>To equalize the size of the volume produced by bamboo molds with a larger diameter or radius with a shorter and bamboo prints with a smaller diameter or radius but higher</th>
<th>To equalize volume or filling of putu cake, bamboo mold with large diameter cut into short size, whether bamboo mold with narrow radius cut in long size</th>
<th>To produce the same volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sony Haryanto</td>
<td>Yanuar Fitrianto</td>
<td>Sindi Nur Aini</td>
</tr>
</tbody>
</table>

Furthermore, the second issue linked the aluminum plate. This challenge could help students understand why the same surface area produces different volumes and maximize the volume of a cylinder with a fixed surface area. In this problem, students face situations on two different canned beverage packaging made of aluminum plate sheets. The packaging is slim and fat even though it is made of the same size plate. Students need to explore and investigate which packaging gives the most significant volume. This investigation was carried out using a manipulative applet adapted from GeoGebra Sources, consisting of an aluminum plat model with surface area and volume and buttons to help students to be able to change the height and radius of the cylinder (see Figure 8). Then, the students need to answer a question “to make a packaging that can have the most considerable volume, which side should be coincided?”
Figure 8: Activities examining different volumes from the same surface area

<table>
<thead>
<tr>
<th>Common Issues</th>
<th>Suggested question to prompt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students don’t understand the instruction</td>
<td>“Take a sheet of paper. Coincide short side. What cylinder do you have? Now consider coinciding the long side of your paper. What cylinder do you have? Are both cylinders identical? How do you compare which design gives the most significant volume?”</td>
</tr>
<tr>
<td>Students think that the two designs can give the</td>
<td>“Set the plate in swapped position—the height of the fat can serve as the perimeter of cylinder base. Compare the volume?”</td>
</tr>
<tr>
<td>same volume</td>
<td></td>
</tr>
</tbody>
</table>

Tabel 3: Question to prompt the students to conjecture in the first problem

Students are asked to deepen their understanding of the relationship between radius, height, and surface to cylinder volume in the second problem. How if the mold of Putu is made from the aluminum board. With the same surface area (identic size board), does a cylinder with the base made by coinciding the longest side give a more significant volume than a cylinder with the base made by coinciding the narrow side of the plate. Figure 9 shows the variation of cylinder design. This task facilitates students to examine different volumes that may be produced from the same
surface area. The question to this task is which design of cylinder (coinciding narrow or longest) the most significant volume gives?

Figure 9: Designs of two cylinders, coinciding longest side and narrow side

Students respond to this task by taking an example of measure 8 × 10. Figure 10 tells how students explore applets using the exact size of the aluminum board to make the different designs of cylinders.

Figure 10: Applet of comparing two designs of cylinders using the same aluminum board

Most students answered that cylinder of mold that gives the most significant volume is the fat one. This design coincides with the longest side of the aluminum plate. Figure 11 shows an example of a student’s response.
Answer

If the plate area is the same (8 × 10) and (10 × 8), the surface area of the cylinder is the same. However, the volumes differ. From the calculation above, (design) that gives the biggest volume is a cylinder that coinciding the longest side of the board (10 × 8)

Figure 11: Student argues that fat model gives the most significant volume

In this response, students take 10 × 8 and 8 × 10 as measurements of the cylinder. The most significant volume comes from a cylinder coinciding with the longest side, making it the base of the cylinder. The most extensive mold design is the fat model.

**Retrospective Analysis**

The HLT shown in Table 1 serves as a guideline for responding to study objectives. HLT is contrasted about what occurs during the learning process to research and describes how learners might transfer informal methods such as surface area and volume to formal strategies. Moreover, the HLT was compared to the data gained in the design experiment to explain learners' methods and thinking processes in comprehending surface areas and the volume of a cylinder using the context given.

As predicted by the student conjecture, the first problem given demonstrates that students used excellent reasoning. Students get that the purpose of Putu mold height differentiation is to achieve the same volume. The learner recognizes that a large mold diameter equals the volume of long bamboo culms with a tiny diameter. All of the students' thoughts are predicted in the learning conjecture and no confusing question. It can be seen from the student's answer. They did well on the question.

The second problem in Activity 4 indicates that the student realizes that surface area gives different volume amounts. Students inferred that even though these plates have the same size, the volume produced was different. However, in this activity, we lack information that the base of the cylinder made from the position was not measured. For the second cycle, we need to improve the size of plates and the base made after construction. We gave them more support because students lacked in assuming plate size. In the next cycle, the plate should be fixed-sized and placed in a portrait-like and landscape-like position.

Finding from this study indicates that ethnomathematics plays a good starting point to understanding mathematical ideas. The situation in ethnomathematics close to the student is compatible with Freudenthal (1991) views. The student also realizes and respects the wisdom of local communities. Mathematics also exists in the traditional cake. Even mathematics in this community is not practiced as formal mathematics, and they struggle to solve the problem using
mathematics (Nursyahidah, Albab, & Saputro, 2020). It is recommended to explore more local wisdom to be a context in mathematics. This idea is compatible with the Van den Heuvel-Panhuizen (2005) thought that the proper context could make the problem transparent and increase student accessibility to the problem; context exposes all sides of the issue. Context provides an essential strategy for students. It is supported from the finding in Figure 6 that students realizing bamboo mold differ in height is to anticipate reasonably in the volume of cake.

Besides, based on activities 1 to 4, all RME characteristics applied throughout the learning process, such as learners' involvement and interaction, have been depicted. This includes pupils finding the surface area and volume of the cylinder by their ideas, as well as pupils interacting with each other during discussion groups and demonstrations. Furthermore, intertwining occurs when students utilize the circle and rectangular area concepts to get the cylinder's surface area and the prism volume idea to determine the cylinder's volume. Additionally, the traits of model-based learning are demonstrated. At the same time, learners use paper as a medium for modeling the Putu cake mold to locate the cylinder net and the elements, and when students used GeoGebra applet to solve problems related to surface area and volume of a cylinder. Additionally, the use of context was incorporated by utilizing a Putu cake mold to represent a cylinder.

It is essential to teach students using RME because it makes students construe the high ability of reasoning. Student deliberately uses their inferences using the connection of the formula they get from the previous grade and the fact they found from the applet. Learning mathematics using the RME approach also arms students with mathematical modeling to the situation and develop student mentally to generalize pattern using advanced modeling call model. This mathematization process brings ethnomathematics far beyond its origins. Mathematics is well known and taught in school because it is presented formally proven theory. If HLT of many ethnomathematics of Indonesian origin is established, it will give an outstanding contribution to teaching and learning of RME, or PMRI in Indonesia. It also helps culture and national heritage be safe from extinction.

CONCLUSIONS

The design learning developed in this study consists of four activities, namely: creating the "Kue Putu" mold to identify the components of the cylinder, determining the formula for a cylinder's surface area, discovering the formula of cylinder volume, and resolve problems related to volume and surface areas of cylinders. Besides, using ethnomathematics as a context for learning mathematics support student understand of the relation between surface areas, height, and radius to the volume of a cylinder. The cake mold of Putu Bamboo helps the student to be able to make the problem transparent and increase student accessibility to the problem, context exposes all sides of the issue, and context provides an essential strategy for students. Students show excellent methods and reasoning during the classes and how increasing the cylindrical radius will give a
more significant effect than increasing its height than that height does. The student also construes the design of the cylinder that gives the biggest volume by expanding its base or radius.

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Students’ Ability to Solve Mathematical Problems in The Context of Environmental Issues

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Abstract: The environment is a global issue that must be addressed in a variety of ways, including education. Environmental topics will be included into mathematics, which is allowed to boost students’ awareness of environmental concerns. This is a qualitative study conducted in a qualitative technique. The objectives of this study were to (1) describe students' mathematical abilities in solving problems related to environmental issues and (2) determine students' knowledge and responses to the environment. The research was performed on 26 students in class VIII at Villanova Catholic Middle School in Manokwari, West Papua, during the even semester of the 2020/2021 academic year. The results indicated that students were classified into four categories based on their ability to solve mathematical problems relating to environmental issues. Students who are capable of solving mathematical problems while also being environmentally conscious. Students that are capable of solving mathematical problems but have no interest in environmental issues. Students who are unable to answer mathematical tasks but are concerned about environmental issues, and students who are not able to solve mathematics problems at the same time don’t care about environmental problems.

INTRODUCTION

Education plays a critical role in developing humans with positive attitudes, specifically concern for and understanding of the environment (Velemponi 2016; Susanti, et al. 2018; Mumu, et al. 2020; Tanujaya, et al. 2021). Education contributes to the development of students' favorable attitudes toward the environment. This is confirmed by Samani and Hariyanto (2012) and Suparno (2015), who argue that the educational community at all levels of education is the appropriate focus for fostering a positive attitude toward the environment.

Integrating environmental education into classroom mathematics instruction can help students develop environmental sustainability characteristics. This is understandable because mathematics is a required topic at every stage of education, from elementary to university, for it to develop into...
a "habitat" that can continually cultivate an environmental stewardship character (Jianguo, 2004). However, in Mathematics disciplines, the thematic - integrative approach that includes components of environmental education to build environmental stewardship traits is almost non-existent (Sutriani, et al. 2020). Additionally, Panjaitan (2020) noted that teachers' difficulty incorporating environmental factors into mathematics learning was owing to a dearth of mathematics teaching materials that may serve as a resource for mathematics teachers. This investigation confirms the findings of Spiropoulou et al. (2005) and Litner (2016) that there is a dearth of literature on environmental integration in mathematics.

Connecting mathematics with the environment is not novel or difficult. Mumu et al. (2020) argued that mathematics is a tool that the environment uses to describe a variety of environmental events, including problem solving. Several experts have been documented that global and national environmental problems can be brought into the mathematics classroom to be solved and solutions discovered (Byrne, 2009; Schwartz, 2010; Gutstein & Peterson, 2013; Barwell, 2013; Widodo, et al. 2019). These include problems with pollution, water, deforestation, natural disasters, garbage, oil and energy, and other local environmental problems. Students must be aware of and sensitive to national and global environmental concerns, possess the information necessary to understand them, and be familiar with a variety of viable solutions (Schwartz, 2010). Thus, integrating the environment into mathematics can contribute to the achievement of the Sustainable Development Goals (SDGs).

The context of the environment and environmental problems can be utilized to explain and guide students' comprehension of abstract mathematical objects. The realistic context of the learning environment enhances the understanding of mathematics instruction, which is one of the characteristics of Realistic Mathematics Education (Hadi, 2017; Hendriana, et al. 2019). Some other characteristics of Realistic Mathematics Education (RME) are the use of model, students' creation, and contribution, interactive of teaching process, and intertwining of various learning strands (de Lange, 1987; Gravemeijer, 1994).

RME is a Netherlands theory of mathematics instruction, developed by Freudenthal Institute (Van den Heuvel-Panhuizen & Drijvers, 2020). This theory views mathematics as a human activity (Freudenthal, 1973; Gravemeijer, 1994), so learning mathematics should be connected to reality. Consequently, in RME, studying mathematics entails doing mathematics, which includes addressing real-world problems (contextual problems) (Freudenthal, 1991). Students should be encouraged to develop mathematical concepts, and the teaching-learning process should be much more interactive (Fauzan, et al. 2002).

Therefore, RME is a possible instruction approach for increasing students' knowledge of mathematics (PMRI Team, 2010). This is due to the fact that mathematics is an abstract discipline that is difficult to understand. The abstract of mathematics should be adjusted to make it more tangible so that students could visualize it through a contextual situation (Swanson & Williams, 2014). Additionally, Gravemeijer (1994) believe that students should be allowed to rediscover Mathematics in their own way, guided by adults, and that the process should begin with the exploration of diverse issues and real-world situations. Mathematics may be used to investigate
waste, population expansion, global warming, flooding, pollution, and the destruction of biodiversity. Students will feel better because of experiencing what they are learning and realizing the importance of mathematics in their lives when they learn from everyday happenings.

Students' behaviors and responses when confronted with an issue and seeking answers vary depending on a variety of circumstances, including their necessary abilities and the level of their cognitive thinking. Similarly, students' perceptions on the environment and its problems vary. Panjaitan (2020) discovered that students were unable to address environmental essay task throughout her investigation. The student reasoned that the question did not correspond to the provided example. This confirms Daryanto and Karim (2017) assertion that students can solve difficulties only if the problem is identical to the one has been provided by the teacher and that students will fail if the problem's context is different from the example problem. According to Clement (2009), such impediments are classed as "epistemological" barriers, i.e., impediments that develop because of one's knowledge being contextualized. On the other hand, one of the hurdles that students face when attempting to solve story problems is a lack of problem-solving abilities.

In terms of the curriculum, problem solving is one of the objectives of the learning process. Problem solving is critical because it helps make Mathematics more relevant and sustainable (Guzman, 2018; Stacey 2005). Problem solving is one of the components of 21st century learning that must be mastered (Szabo, et al. 2020). Students with high problem-solving abilities will be able to navigate the intricacies of global concerns in the future. Problem solving ability is a necessary skill for students to possess to address mathematical problems, problems in other fields, and challenges encountered in daily life (Risdiyanti & Prahmana 2020). Students with a high-level problem-solving ability will be able to resolve issues. Numerous issues that will always arise throughout life.

The National Council of Teachers of Mathematics (NCTM) has stressed the importance of problem solving in learning. According to NCTM (2000), mathematics learning involves five primary standard competencies: problem solving ability, reasoning capacity, connection ability, communication ability, and representation ability. This inability will result in low-quality human resources, as shown by the inability to solve problems. This is because so far, education has not provided opportunity for students to build their abilities to solve problems, and students are therefore less acclimated to problem solving (Hesti & Setiawati, 2016; Widodo, et al. 2019; Muhammad & Pujiastuti, 2020). Additionally, problem solving questions are frequently assigned towards the conclusion of class hours, ensuring that they are not discussed in class by the teacher (Alison, 2017). This method of instruction does not facilitate students in developing their problem-solving abilities.

Based on the statements above, the researchers are interested in the students’ competence to solve mathematical problems when challenged with essay questions about environmental issues. This study is the initial research a series of research that will culminate in the development of an environment-based mathematics teaching material. As a result, the findings of this study are intended to guide academics in developing mathematical learning materials that are integrated with
environmental concerns. As such, this study will investigate students' mathematical problem-solving abilities and their awareness of environmental issues.

METHOD

This is a qualitative study using a descriptive technique. The objectives of this study were to describe students' mathematical abilities in solving problems related to environmental issues and determine students' knowledge and responses to the environment. The research was conducted on 26 students in class VIII at Villanova Catholic Middle School in Manokwari, West Papua, during the even semester of the 2020/2021 academic year. They are students who have studied environmental topics in the elementary school and the grade VII junior high school. They have studied the environment issue in elementary school through thematic subjects, and in junior high school through natural sciences. This is in accordance with the Regulation of the Minister of Education and Culture of the Republic of Indonesia Number 24 of 2016 concerning the basic competencies of mathematics lessons in the 2013 curriculum.

Data collecting procedures included observation, essay tests, and unstructured interviewing. An essay test consists of three numbers was used as the research instrument. Environmental issues such as clean water, flooding, and garbage is employed as realistic contexts. Mathematical concepts related to environmental issues include decimal numbers, distances, exponent, unit conversions, fractions, and sets. Details of the context of environmental issues and mathematical concepts for each item are presented in Table 1.

<table>
<thead>
<tr>
<th>Task Number</th>
<th>Environmental Issues</th>
<th>Mathematical concepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Toxic Waste</td>
<td>Addition of decimal number and distance</td>
</tr>
<tr>
<td>2</td>
<td>Clean Water</td>
<td>Exponent and Unit Conversion</td>
</tr>
<tr>
<td>3</td>
<td>Flooding</td>
<td>Fraction and Sets</td>
</tr>
</tbody>
</table>

Table 1: Context of environmental issues and mathematical concepts

The first problem is developed using two mathematical concepts, distances, and addition of decimal number using the context of hazardous waste. In other words, this assignment is designed to measure their ability to solve mathematical problems as well as their awareness of hazardous waste (see Figure 1). The second question is based on the concept of exponent numbers and converting units in the context of clean water, to evaluate students' awareness of the environment, particularly about clean water. Meanwhile, the third item was established in consideration of the waste that contributes to flooding. This item contains a mathematical concept involving the set and addition of fractions. It is predicted that students who possess knowledge of organic and inorganic waste will be able to correctly solve this question.
Task Number 1

The three questions consist of four indicators of the problem solving, namely understanding the problem, planning a strategy, solving the problem, and re-examining. The four indicators are used to classify students’ problem-solving abilities and their environment awareness into four types of abilities, as stated in Table 2.

<table>
<thead>
<tr>
<th>Type</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Students are able to solve mathematics problems correctly and have a positive response to the environment</td>
</tr>
<tr>
<td>II</td>
<td>Students are able to solve mathematics problems correctly but have a negative response to the environment</td>
</tr>
<tr>
<td>III</td>
<td>Students are able to solve mathematics problems correctly but have a negative response to the environment</td>
</tr>
<tr>
<td>IV</td>
<td>Students cannot solve mathematics problems correctly and have a negative response to the environment</td>
</tr>
</tbody>
</table>

Table 2: Type of Students Problem-Solving Ability and Environment Awareness

Furthermore, the research subjects were grouped based on the four types of problem-solving abilities and environment awareness on each item. From these groupings, one of students’ answer was selected that represents each type of students’ ability for further analysis.
RESULTS

Students are grouped into four problem-solving abilities and environmental awareness categories based on the outcomes of the essay test, except for type II for the second item. Table 3 summarizes the percentages of students' ability for each category.

<table>
<thead>
<tr>
<th>Item Number</th>
<th>Type of Students' Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>1</td>
<td>26,92</td>
</tr>
<tr>
<td>2</td>
<td>23,07</td>
</tr>
<tr>
<td>3</td>
<td>15,38</td>
</tr>
</tbody>
</table>

Table 3: Distribution of Students’ Problem-Solving Ability and Environment Awareness

Based on Table 3, most students assessed are classified as belonging to group fourth. This category of students is comprised of those that are unable to solve problems correctly and have a bad attitude toward environmental issues. This shows that generally students have low ability to solve mathematical problems as well as their lack of care for environmental issues. However, there is a second percentage that is extremely significant for pupils who are environmentally conscious and possess strong problem-solving abilities. This group consists of students classified as the first type.

Furthermore, an analysis of students' abilities based on each item's type of problem-solving ability. The student answers to task number one and number three are vary and can be grouped into 4 types of students. Students' answers to the second question can only group them into three types of students, without students having characteristics that can be categorized into the second group. The following are some instances of student responses as presented on figure 1 to figure 4.

Based on the student's statements in Figure 2, it appears that students comprehended the issue of choosing the "most appropriate" pathway to the grocery while also taking out the rubbish. Garbage that needs to be disposed of is classified as hazardous waste and deposited in a red garbage container. The student then determines the shortest route home. Additionally, the student demonstrates her understanding of the concept of decimal number addition.
A sample of student answer

Translation

Route 1: Home ➔ Green Can ➔ Grocery
8.2 + 14 + 5 = 27.2 m
Route 2: Home ➔ River ➔ Grocery
14 m
Route 1: Home ➔ Red Can ➔ Grocery
11.3 + 14.1 + 10.6 = 36 m
Garbage that needs to be discarded (used masks, unused disinfectant bottles, used baygon bottles, and batteries) is classified as hazardous waste, which is why I chose the red trash can.

a. The route chosen is route number three, which is 36 meters long.
b. Return home using route number 2, as it is the shortest.

Figure 2: Type I students' answers to the first task

The following is student answer as an example for those who are capable of accurately solving mathematical problems but has a negative attitude about the environment, as presented on Figure 3.

A sample of student answer

Translation

a. Left path: house + red box + supermarket
11.3 + 14.1 + 10.6 = 36.0 m
To exercise, take the far more distant path.
b. Return home along that road.

Figure 3: Type II students' answers to the first task

The responses of student in Figure 3, provided information that the student appear to understand how to correctly do decimal number addition operations as well as the concept of distance. Student, on the other hand, is unaware of the issue of hazardous waste and its relationship to the red trash can. This is reflected from student' stated reasons for choosing the route, notably "to be able to exercise." Student make the connection between distance and health. Student' environmental sensitivity is not evident in their responses.

Based on the two answers, students' ability to solve mathematical problems appears to be unrelated to their care about environmental issues. Students with a high-level mathematics problem-solving ability could have completely opposite perspectives on environmental issues. Mathematics instruction do not influence pupils' attitudes on environmental issues. This is consistent with the findings of interviews with students, who reported that teachers hardly ever discuss environmental issues while teaching mathematics or other lessons. Another student mentioned that he gathered...
knowledge of environmental issues outside of the classroom from television, social media, and the internet. On the other hand, according to Jianguo (2004), incorporating environmental topics into mathematics classroom can help students develop characteristics associated with environmental awareness.

The absence of a relationship between students' ability to solve mathematical problems and their concern for environmental issues is also noticeable in the results of students' answers as presented in Figure 4, Figure 5, Figure 6, and Figure 7.

A sample of student answer

![Translation](image)

Figure 4: Type III students' answers to the second task

Figure 4 presented information that the student doesn’t understand the problem of the task. The student is unable to recognize any information of the provided questions. Despite his failure to overcome the problems posed, the student is concerned about the issue of clean water, which is required by a large number of people. Other student demonstrated the same results, as presented in Figure 5.

A sample of student answer

![Translation](image)

Figure 5: Type III students' answers to the third task

Based on the student’ responses in Figure 5, it appears as though students has already comprehension the problem. Students understand that garbage can be classified as organic or inorganic. The grouping is an approach for simplifying the calculation of the result of fraction addition. However, due to the error of included the instant noodle packet in the organic waste group, the fractions addition was not completed correctly. However, pupils have an awareness of the garbage that contributes to flooding. Don’t throw the trash in the ditch.

The following are samples of students’ answers that are unable of solving mathematics problems but are unconcerned about environmental issues.
A sample of student answer

Translation

100 x 10^{-1} \times 24 = 100124

Plenty/over water

Figure 6: Type IV students' answers to the second task

In the sheet answer as presented in Figure 6, student could not identify the problem and incorrectly planned the problem-solving strategy. The student is unable to perform multiplication procedures accurately. Additionally, student doesn’t believe that the issue of clean water is a major one. This is assumed to be influenced by the pupils’ daily lives. The student lives in an area with bountiful mountain water, ensuring that he is never without water.

A sample of student answer

a. leaves, food, instant noodles
b. plastic bottles
c. burnt

Figure 7: Type IV students' answers to the third task

Figure 7 presented information about students who do not understand the problem in the task. Students attempted to classify organic and inorganic garbage, but did so incorrectly due to a lack of information about the many types of waste. This error prevented pupils from progressing to the next step of problem solving. Furthermore, students realize that rubbish might cause flooding, but the solutions proposed are unsatisfactory. It is advised that the garbage be burned.

DISCUSSION

Based on the data collected and the analysis results, it appears that students' problem-solving capacity is still somewhat limited. In general, students do not comprehend problems involving mathematical concepts and environmental situations. They are unable to devise effective solutions for resolving mathematical issues that are constructed in terms of their environmental setting. As a result of this condition, they will have a low level of care about environmental issues. This is consistent with Henderina's (2018) assertion that students' ignorance of the environment is a determinant of environmental consciousness. Additionally, Szabo et al. (2020) state that the primary requirement for solving a problem is to understand the problem, what is known, what is
being asked, whether there is a requirement to answer what is being asked, and whether the requirement is sufficient or additional conditions are required.

Students must have a basic understanding of the environment in order to solve environmental problems. Similarly, students must be able to comprehend mathematical concepts in order to solve mathematical problems. In other words, students must possess environmental knowledge and a proper understanding of mathematical concepts in order to answer environmental-related mathematics issues. Mathematics problems involving environmental issues can be handled only if students possess adequate and accurate understanding of the environment and the challenges that surround it. As a result, they will have a favourable reaction to the environment, as well as a knowledge of and concern for the environment. According to Zheng et al. (2018), adequate environmental knowledge is required to influence students' environmental behaviour. Their research demonstrates a positive correlation between environmental knowledge, environmental behaviour, and environmental attitudes. Furthermore, education is crucial in fostering positive attitudes in humans, particularly a concern for and awareness of the environment (Yumusak et al., 2016; Leksono, 2017).

One aspect contributing to students' lack of understanding about environmental issues is the scarcity of mathematics textbooks that incorporate environmental issues. According to Spiropoulou (2005), including environmental elements in mathematics textbooks can help students have a better understanding of ecological, social, technological, and historical environmental challenges. Additionally, He was said that when considering society's changing needs and beliefs, education must take this into account and provide opportunity for students to learn more about environmental issues. Furthermore, Matilde et al. (2020) noted that mathematics plays a critical part in reaching the Sustainable Development Goals (SDGs) while also allowing students to interact with real-world scenarios in mathematics topics, which promotes active learning.

CONCLUSION

Environmental-based mathematics learning and the availability of environment-based mathematics textbooks must be supported by professional mathematics teachers. This is because the teacher plays a critical part in the instruction process. The teacher serves as the lesson's planner, implementer, and evaluator. These three responsibilities position the teacher as the primary determinant of the success of environmental-based mathematics instruction. They should be able to integrate environmental issues into their mathematics instruction. According to Rosenshine (2012), the effective teachers ensured that pupils were acquired, practiced, and connected to other knowledge in an efficient manner.

Finally, environmentally oriented mathematics instruction can assist students in developing their capacity to address mathematical problems involving the environment. The teacher is the agent of
this learning's accomplishment. Mathematics teachers must be able to incorporate environmental education into their classes in order to teach mathematics successfully. Mathematical teachers should incorporate more examples of mathematics issues that are related to the environment into their classroom instruction. Environmental issues can be used as a realistic context in an effort to improve students' understanding of mathematical concepts. In order to realize this, it is necessary to develop environment-based mathematics textbooks

REFERENCES


Realistic Mathematics Learning on Students’ Ways of Thinking

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Abstract: This research is motivated by the importance of developing ways of thinking (WoT) in geometry learning at the Sambas District Elementary School, West Kalimantan, Indonesia. In general, this study aims to describe realistic mathematics teaching to WoT elementary school students in completing geometry material related to the design of the Sambas Malay traditional house. The method used in this research is a qualitative method with a hermeneutic phenomenology approach. The subjects of this study were fifth-grade students from one of the public elementary schools in Sambas district with a purposive sampling technique. Data collection techniques used include observations, tests, and interviews. The results showed that students’ thinking in solving geometry problems in schools that applied realistic mathematics learning was described in high-ability students, the mental acts displayed were interpreting, problem-solving and inferring. Then, for students with moderate ability, conduct mental behavior explanation and mental behavior reasoning. For students with low mental abilities, the acts displayed are interpreting and problem-solving. After researching the literature on realistic mathematics learning, her goal is to make math learning meaningful and diversify the way students think about solving geometric problems.

INTRODUCTION

Realistic mathematics learning is an adaptation of realistic mathematics education adapted to the social environment, culture, and characteristics of the Indonesian people. Realistic learning of mathematics comes from the Dutch language, namely "zich realiseren", which means "imagine" or "imagine" (Van Den Heuvel-Panhuizen, 2003). In this case, the realistic learning of mathematics is based on Freudenthal's view that mathematics is a human activity (Gravemeijer, 1994). Therefore, Gravemeijer (1994) suggests three key principles of realistic mathematical learning, namely, guided reinvention / progressive mathematization, didactic phenomenology, and self-developed models. This is similar to the realistic process of mathematical learning, in general, it has five characteristics (Wahyudi et al., 2017) that include (1) the use of context problems, (2) the use of models, (3) the use of contributions. of students, (4) the occurrence of interactions in the learning
process, and (5) the use of different learning theories that are relevant, interrelated, and linked to other learning topics.

Realistic mathematics learning occurs through contextual linking of real and real problems. Realistic mathematics learning refers to problems whose situations are related to the real world and allows the incorporation of mathematical concepts, methods, and results in the solving process known as mathematical context problems (Bliss, et al. 2019; Blum & Niss, 1991; Blum, 2002; Cojocariu, et al. 2014; Niels Jahnke, 2016; Risdiyanti & Prahmana, 2020; Prahmana, et al. 2021). Realistic mathematics learning presents students’ reality and real-life experiences in everyday life as a starting point for learning and makes math activity for students. Several previous studies that involved realistic mathematics learning include research (Le, 2006; Van Den Heuvel-Panhuizen, 2003; Tanujaya, et al. 2021) claiming that reality plays an important role in realistic mathematics learning. In realistic mathematics learning, students can apply mathematical concepts to solve mathematics problems. Mathematical concepts are shown in Figure 1.

![Figure 1: De Lange's Matematization Concept](image)

In this case, students have the opportunity to rediscover mathematical concepts. According to the research of Gold et al. (2017), good mathematics education must observe how students acquire mathematical concepts.

The integration of culture as a context in the mathematics learning process was examined in the research of Ambrosio (1985). Local cultural elements that can be linked to realistic learning of mathematics are language, knowledge, technology, equipment, art, livelihoods, religion, relatives, customs, traditional buildings, and buildings. Community organizations. There are several studies on the integration of local culture in the learning of mathematics (Brandt & Chernoff, 2015; Maryati & Prahmana, 2019; Muhtadi, et al. 2017; Owens, 2012; Rosa & Orey, 2011; Torres-Velasquez & Lobo, 2004). Reinforced by the research of Widada et al. (2018) that mathematics, which takes into account local quantitative, relational, and cultural aspects of society, in this case, is integrated into contextual problems that can be observed or understood by students through a mathematical process. The benefits of the local culture are one of the
Mathematics learning that makes use of the environment is intended to generate thoughts and give students the greatest possible opportunity to understand mathematical material. Based on the characteristics of realistic mathematics learning, including the use of contextual problems, the use of models or bridges as vertical tools, use of student contributions, interactivity, and integration with other learning topics. This gives students the greatest opportunity to learn math in a real-world context, which is very important in building a solid foundation for a math topic (Hill, et al. 2008). The relevant topic is geometry, especially in the form of flat structures that use the daily life of students, especially for students in rural areas with limited options (Sukirwan, et al. 2018). Using the existing daily context. One of the contexts related to the local culture of the area is the traditional home. In this way, the ability of the student's brain to understand a mathematical problem is formed and mental actions involving ways of thinking (WoT) and ways of understanding (WoU) are well developed. In other words, realistic mathematics learning embedded in the local culture helps students with developmental acts involving ways of thinking and understanding to solve the problems that students experience when solving mathematical problems.

Students start with a real context and then use informal language and symbols to define the problem (Guler & Gurbuz, 2018). Students begin to work with mathematical symbols and can achieve formulas by building relationships between concepts (Gravemeijer, 1994). Furthermore, knowledge is built when students study real-world context problems, and then knowledge is built mathematically after mathematization (Guler & Gurbuz, 2018). When solving a math problem, students have a variety of ways of thinking (Harel, 2008). This fact is reinforced by the research of Jupri and Drijvers (2016) that found differences in students' work when formulating equations, diagrams, or diagrams since mentalities are relevant for student understanding. It is reinforced by several expert studies showing ways of thinking about and understanding mathematics learning (Harel, 2008; 2020; Hunting, 1997; Ikhwundin, et al. 2019; Ikhwandin & Suryadi, 2018; Lockkwood & Weber, 2015; Syamsuri, et al. 2016; Widodo, et al. 2019). Furthermore, Harel found that there is a connection between ways of thinking and understanding (Harel, 1998; 2001).

In other words, the way students understand certain concepts affects their way of thinking and vice versa (Çimer & Ursavaş, 2012). This is in the process of building students' knowledge and students are at the center of the learning process, which aims to develop students' learning skills that focus on developing mindsets and ways of understanding the mental actions of the students. When students learn the desired way of thinking and understanding through repetitive thinking, it allows them to integrate them into the real and academic life of the students (Öflaz & Demircioğlu, 2018). At this point, the dimensions of mathematical didactics are considered; The mental and psychological needs of the students must be taken into account so that the learning objectives can be successfully achieved.
RESEARCH METHOD

The research method used in this study is a qualitative method with a hermeneutical phenomenological approach. The qualitative method with the hermeneutical approach was chosen because the research carried out is a study carried out to interpret a meaning acquired by someone from experience as well as an understanding of the hermeneutical phenomenology itself (Lindseth & Rn, 2004). Phenomenology and hermeneutics complement each other. This means that a phenomenon cannot be understood without an interpretation of the subject's experiences. The definition of phenomenology was formulated by Grbich (Kafle, 2013) as "An approach to understanding the hidden meanings and essences of an experience collectively". Hermeneutics according to (Kakkori, 2020) is the "art of interpretation".

This research was conducted at 2 Sambas State Primary School, West Kalimantan Province, Indonesia for the 2020/2021 school year. Research locations and objects were determined according to the purpose-specific random sampling procedure. Intentional sampling is a sampling technique performed intentionally by researchers based on the quality of the location or research topic (Tanujaya, et al. 2017). The subjects of this study were students of classes VA, VB, and VC with a total of 92 students. Then, 3 students analyzed the results of student responses regarding the thinking tool based on student abilities, namely 1 high-ability student, 1 moderate-ability student, and 1 low-ability student. The grouping criteria based on the mean and standard deviation are shown in Table 1.

<table>
<thead>
<tr>
<th>No</th>
<th>Student ability</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>high ability</td>
<td>$X \geq \bar{X} + SD$</td>
</tr>
<tr>
<td>2</td>
<td>moderate ability</td>
<td>$\bar{X} + SD &lt; X &lt; \bar{X} - SD$</td>
</tr>
<tr>
<td>3</td>
<td>low ability</td>
<td>$X \leq \bar{X} - SD$</td>
</tr>
</tbody>
</table>

Table 1: Criteria for grouping high, moderate and low ability students

Description: $\bar{X}$ is the average value of the initial mathematical ability
SD is the standard deviation of the initial mathematical ability value

This study uses various tools that are used to obtain the necessary data, including observation, tests, interviews. The data analysis stages, instead, used research stages based on the stages of hermeneutical phenomenological analysis according to Ricoeur (Tan, et al. 2009) as follows: explanation, naive understanding, and deep understanding.

RESULTS AND DISCUSSION

This study aims to describe the understanding of students' ways of thinking in realistic mathematics learning based on local Sambas culture based on students' learning experiences on
geometry material. In the primary stage, the researcher discovered the getting to know procedure within the 5th grade which was carried out by the teacher in teaching geometry material using realistic mathematics learning. At the start of the lesson, the teacher offers a gap greeting, assesses scholar attendance and scholar readiness in following the lesson, and prepares geometry fabric, particularly flat shape. The trainer offers a miniature of the traditional Malay Sambas house. Then ask students to observe and discuss what flat shapes are discovered within the traditional Malay house of Sambas.

The teacher then distributes a student worksheet containing contextual questions related to local traditional samba culture, especially geometric materials. In the process of learning realistic mathematics, teachers provide students with a foothold in the form of exploratory questions to build their knowledge to find answers to geometric problems. In a realistic math learning process, the teacher acts as a moderator, and students actively participate in discussions, process questions, communicate the results of discussions, exchange ideas, and complete friends' answers. At the end of the lesson, the student completes the lesson and the teacher refines the outcome of the conclusion. A three-question written exam was then conducted, including indicators of interpreting, problem-solving, explaining, and inferring.

This is the result of an analysis that tracks how students think about geometry. The researchers asked 92, 5th-grade elementary school students questions about the essay test and traced them back to different ideas based on student answers. Figure 2 shows the distribution of student reactions to the idea of geometric materials.

![Figure 2: The distribution of student responses related to ways of thinking](image-url)
In Figure 2, it can be seen that in working on ways of thinking questions on geometry material, overall students can solve problems on interpreting indicators, namely as many as 21 students. Furthermore, on the problem-solving indicator, 19 students can do well. Then as many as 16 students were able to solve the problem of solving ways of thinking on geometry material by explaining indicators. Meanwhile, as many as 10 students were able to solve the questions of ways of thinking with inferring indicators. In addition, from the results of the analysis of students' ways of thinking test questions with indicators, it was found that 14 students did not answer the questions given, and as many as 12 students answered with other answers.

Students can explore and develop the knowledge of students related to the local culture of the Sambas region on the problems posed by a combination of realistic mathematics learning based on the local culture. Table 2 shows the percentage of student ways of thinking test tool results based on the indicators.

<table>
<thead>
<tr>
<th>No</th>
<th>Indicator</th>
<th>Percentage</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Interpreting</td>
<td>76.8</td>
<td>Good</td>
</tr>
<tr>
<td>2</td>
<td>Problem solving</td>
<td>53.67</td>
<td>Sufficient</td>
</tr>
<tr>
<td>3</td>
<td>Explaining</td>
<td>52.19</td>
<td>Sufficient</td>
</tr>
<tr>
<td>4</td>
<td>Inferring</td>
<td>54.6</td>
<td>Sufficient</td>
</tr>
</tbody>
</table>

Table 2: Recapitulation of the percentage of students' Ways of thinking scores obtained from the test results made based on the indicators.

Table 2 shows that students who obtained the highest percentage of 76.8% longed to the good category in making meaning from symbols or images with various interpretations. Furthermore, 53.67% of students who can solve the given problem by paying attention to alternative or geometric problem-solving strategies are classified into sufficient categories. In addition, the percentage of student scores of 52.19% longs to the sufficient category, students can make explanations of geometric problems related to real contexts. While a percentage score of 54.6% with sufficient category, students can make conclusions from solutions to solving geometric problems.

Then an analysis of the results of students' answers to the tests of students' ways of thinking will be described in realistic mathematics learning. Questions 1a and 1b are questions with indicators of interpretation presented in Figure 3.

<table>
<thead>
<tr>
<th>No</th>
<th>Indicator</th>
<th>Test questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>Interpreting</td>
<td>Write and sketch (draw) what types of flat shapes you know from your observations on the sambas Malay house!</td>
</tr>
<tr>
<td>1b</td>
<td>Interpreting</td>
<td>Do you remember how to find the area of a square and give an example!</td>
</tr>
</tbody>
</table>

Figure 3: Questions number 1a and 1b interpreting indicator.
Observe the shape of the roof, windows, and doors of the Malay Sambas house in Figure 4.

Figure 4: Sambas Malay traditional House

The question of ways of thinking that has been made by the researcher in Figure 4 aims to describe in depth how students make meaning from symbols or images with various interpretations. The results of the analysis of students' answers with high, moderate, and low abilities related to interpreting indicators are presented in Table 3.

<table>
<thead>
<tr>
<th>Student ability</th>
<th>Answers to test questions</th>
<th>Translate</th>
</tr>
</thead>
<tbody>
<tr>
<td>High ability student (ST04)</td>
<td><img src="image_url" alt="Image" /></td>
<td><img src="image_url" alt="Image" /></td>
</tr>
</tbody>
</table>

1b. Example
A house has square windows
What is the area of the square

Answer

\[ L = 80 \times 80 \]
\[ L = 6400 \text{ cm}^2 \]

1a. how to find the area of a square by measuring the length and width of the sides of the square

The area of a square is length \( \times \) width or side \( \times \) side

example

length of square = 2 cm
width of square = 2 cm
area of square = 2 x 2 = 4cm²
1a. answer flat shape found from observations of the traditional Malay house of sambas

1b. How to find the area of a square. area of square = side x side
Please explain with an example

area of square = 4 x 4
area of square = 16

Table 3: Recapitulation of student answers on the interpreting indicator

Based on Table 3, the results of the analysis of the answers with interpreting indicators are presented, in students with high, moderate, and low abilities, including subjects are high ability students, SS63 subjects are moderate ability, and SR25 subjects are low ability students. Using the test analysis of Question 1a, the three students were able to interpret the shape of the roof, windows, and doors of the house in a flat shape. Even though the SR25 theme doesn't write down the names of all the flat shapes you're working on. Regarding question 1b, all three students can correctly give an example related to the area of a square together. Both high ability students (ST04) and low ability students (SR25) carefully solve the given problem to obtain the square area, that is, the side x side formula. This shows that students can understand and find the geometric concept of flat shapes and the broad concept of flat shapes. Next, a moderate ability student (SS63) does not write down how to get the square area formula, but immediately gives an example of a question related to the square area. In this case, the SS63 topic is only limited to memorizing formulas and using them.
Next, the answer to question number 2 will be discussed with an indicator explaining, presented in Figure 5.

<table>
<thead>
<tr>
<th>No</th>
<th>Indicator</th>
<th>Test questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>explaining</td>
<td>Explain the reason why the formula for the flat shape of the traditional house is $\frac{1}{2} \times \text{base} \times \text{height}$?</td>
</tr>
</tbody>
</table>

Figure 5: Questions number 2 are explaining indicator

The questions of ways of thinking that have been made by the researcher shown in Figure 6 aims to describe in depth how students explain the relationship related to geometric material, especially the area of a flat triangle with a square or rectangular shape. Furthermore, the results of the analysis of student answers with high mathematical ability, medium and low regarding indicators explaining are presented in Table 4.

<table>
<thead>
<tr>
<th>Student ability</th>
<th>Answers to test questions</th>
<th>Translate</th>
</tr>
</thead>
<tbody>
<tr>
<td>High ability student</td>
<td>Triangle in my opinion the result is cut in half with oblique cuts</td>
<td></td>
</tr>
<tr>
<td>(ST04)</td>
<td>Then the area of the triangle is $\frac{1}{2} \times \text{base} \times \text{height}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A triangle is $\frac{1}{2}$ of a rectangle.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The formula for a rectangle is length $\times$ width</td>
<td></td>
</tr>
<tr>
<td></td>
<td>area of triangle $= \frac{1}{2} \times$ area of a rectangle</td>
<td></td>
</tr>
<tr>
<td></td>
<td>area of triangle $= \frac{1}{2} \times$ (length $\times$ width)</td>
<td></td>
</tr>
</tbody>
</table>
Table 4: Recapitulation of student answers on the explaining indicators

Based on table 4 obtained the results of the analysis of student answers show that students are high abilities (ST04) even though they have written the area of the triangle correctly, but the process for obtaining the area of the triangle is not correct. Students are wrong in explaining the process to obtain the area of a triangle. This is because students do not understand the meaning of the triangle relationship in a square. Furthermore, students with moderate abilities (SS63) can understand questions and answers using mental act explaining through pictures and writing and provide systematic and precise explanations. This means that students can explain the relationship between rectangular flat shapes and triangular flat shapes. The low-ability student (SR25) cannot explain the relationship between the area of a triangle and the area of a square. The students quickly drew the triangle and the area of the triangle but did not follow the correct procedure to find the area of the triangle. This means that students do not correctly understand and recognize the concept of geometric materials. This is because students learn by learning formulas rather than understanding concepts. Finally, Question 3 is a question about problem-solving indicators and inferring indicators. The student responses analyzed were those of high, moderate, and low abilities shown in Figure 6

<table>
<thead>
<tr>
<th>No</th>
<th>Indicator</th>
<th>Test questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Problem-solving</td>
<td>Fulanah's house is one of the traditional houses that is still preserved. Has an equilateral trapezoidal roof</td>
</tr>
<tr>
<td></td>
<td>inferring</td>
<td>The front side of the roof of Fulanah's house has a parallel side length of 22 meters and 12 meters respectively and a height of 8 meters. Determine the area of the roof of the front side of the house and write the conclusion!</td>
</tr>
</tbody>
</table>

Figure 6: Questions number 3 are a problem-solving and inferring indicator
Figure 6, discusses the ways of thinking test questions with problem-solving indicators and inferring indicators. Furthermore, the results of the analysis of student's answers with high, moderate, and low abilities related to problem-solving indicators and inferring indicators are presented in Table 5.

<table>
<thead>
<tr>
<th>Student ability</th>
<th>Answers to test questions</th>
<th>Translate</th>
</tr>
</thead>
<tbody>
<tr>
<td>High ability student (ST04)</td>
<td>Is known to the front side of an isosceles trapezoid, the length of the parallel sides is 22 meters and 12 meters, and a height of 8 meters asked the front side of the roof Write the conclusion answered Area of trapezoid = 1/2 x (length of side a + length of side b) x height area of trapezoid = 1/2 x (12 + 22) x 8 area of trapezoid = 34 x 4 area of trapezoid = 136 The total area of the roof of the house is 136 meters Then the roof area of the front side of the house = 1/2 x the area of the trapezoidal roof of the house as a whole</td>
<td></td>
</tr>
</tbody>
</table>
the front side of the roof = 1/2 x 136
the front side of the roof area = 68 meters

Conclusion
So, the roof area of the Fulanah’s house which is in the form of an isosceles trapezoid is 136 meters. While the roof area of the house on the front side is 68 meters

Area of trapezoid = 1/2 x (number of parallel sides) x height
area of trapezoid = 1/2 x (length of side 1 + length of side 2) x height
area of trapezoid = 1/2 x (22 + 12) x 8
area of trapezoid = 1 x (22+12)x8
area of trapezoid = (11 + 6) x 8/2
area of trapezoid = 17 x 4
area of trapezoid = 68 meters
So the area of the trapezoid which is the shape of the roof of the fulanah’s house is 68 meters
The lengths of the parallel sides are 22 meters and 12 meters.

\[ \text{area of trapezoid} = \frac{1}{2} \times \text{number of parallel sides of trapezoid} \times \text{height} \]

\[ \text{area of trapezoid} = \frac{1}{2} \times (22 + 12) \times 8 \]

\[ \text{area of trapezoid} = \frac{1}{2} \times 34 \times 8 \]

\[ \text{area of trapezoid} = 17 \times 8 \]

\[ \text{area of trapezoid} = 136 \text{ meters} \]

\[ \text{area of the front side of the trapezoid} = \frac{1}{2} \times \text{the total area of the trapezoid} \]

\[ \text{area of trapezoid} = \frac{1}{2} \times 136 \]

\[ \text{area of trapezoid} = 68 \text{ meters} \]

Table 5: Recapitulation of student answers on the problem solving and inferring indicator

Based on Table 5, the results of the related analysis are obtained from the problem-solving strategy used by students with high ability (ST04) in solving problems where the process to determine the area of the trapezium applies the right procedure. Students understand each stage in the work to get a solution related to the area of the trapezium. In addition, the ST04 subject wrote the correct conclusion to find the answer to the geometry problem. Furthermore, students with the moderate ability (SS63) are already able to carry out strategies in the process of solving the trapezoid area problem, even though the answers obtained are correct. However, the calculation process was wrong and made an error in the division operation so that an error occurred in the student's answer. This is because students do not pay much attention to the calculation process for obtaining the area of the trapezoid. Using the results of the low-ability student's answer (SR25), the student uses solution strategies and procedures to solve the correct trapezoidal area, but in the SR25 topic, the conclusion is as an answer to question number 3 not described.

Student results analysis ways of thinking about applying realistic mathematics learning to solve geometric problems at school tend to help high abilities students understand the indicators of interpretation, problem-solving, and inferring. This indicates that there is. In addition, students with moderate abilities tend to understand indicators ways of thinking, namely explanation, and
inferring. On the other hand, low abilities students tend to understand interpreters and problem-solving indicators. Demetriou (2004) states that students have more diverse ways of thinking (WoT), more strategies to solve problems, and the opportunity to become flexible thinkers in dealing with new situations and problems. Consistent with the study of.

In addition, an analysis of the student's answers to the three questions revealed that the material for the flat-shaped area was not understood, the procedure was applied incorrectly, and the prerequisite material was not mastered. The above findings are in line with the results of the research by Ikhwanudin et al. (2019) which states that the patterns of student errors in learning mathematics include lack of understanding of fractions, insufficient understanding of fractions, and the denominator equation and the method of adding fractions are mistakenly applied. Reinforced by the results of the study of Mazzocco et al. (2011) which states that students with mathematics learning disabilities state that they make mistakes in comparing and estimating numbers.

The results of this study can be used by teachers of mathematic subjects who apply realistic mathematics learning to achieve their learning goals. Teachers need to be confident that they can guide students to learn and practice mathematics, change their attitudes from passive recipients to active individuals, and develop mathematical reasoning. In addition, realistic mathematics learning will be the mainstay for helping low abilities students develop and improve their mathematical skills while answering mathematic problems.

CONCLUSIONS

Realistic mathematics learning makes mathematics learning meaningful and students' ways of thinking in solving geometric problems are increasingly diverse. The student's ways of thinking found were various visual interpretations and symbols, ways of explaining (explaining), approaches or strategies in problem-solving, and ways of concluding. The more diverse ways of thinking that are raised by students, the implications for the higher the student's ability score. This shows that the more diverse students' ways of thinking, the higher their geometric problem-solving ability. Furthermore, several errors were found in the students' work, including lack of understanding of the area of flat shapes, incorrectly applying the procedure (concept error), and lack of proficiency in the required materials.

REFERENCES


Analysis of Students' Abilities in Solving Realistic Mathematics Problems Using “What-If”-Ethnomathematics Instruments

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Abstract: The research aimed to analyze the students’ abilities in solving realistic mathematics problems using "What-If"-Ethnomathematics Instruments with content focused on plane and space materials. The "What-If"-Ethnomathematics instruments are instruments that enable educators to analyze various errors and obstacles experienced by students in solving realistic problems by prioritizing applied and culture mathematics and questions that test students' mathematical thinking skills. The research design was descriptive qualitative. The subjects of the research were 46 students of SMP Widiatmika in the 2020/2021 academic year. The data collection method using a test, interview, and documentation. The data was then analyzed using qualitative descriptive data analysis with the following stages: data reduction, data presentation, and drawing conclusion and verification. The result showed that the students' abilities in solving realistic ethnomathematics problems using "What-If"-Ethnomathematics Instruments are still lacking which include: errors in understanding the problems, errors in representation, errors in reasoning, errors in answering "What-If" Questions. The highest errors were errors in reasoning, which was 69.56% of all students, followed by errors in answering "What-If" Questions of 65.21%, then errors in understanding the problems 43.47%, and finally errors representation as much as 34.78%. From the results of an interview, teachers tend to provide learning that focuses on the delivery and use of formulas and ignores the understanding of concepts and improved thinking skills of students based on realistic mathematics problem-solving.

INTRODUCTION

Mathematics is an important subject in the development of education of a student. Mathematics is a universal science that underlies the development of modern technology today, this is because mathematics has an important role as a means of solving life problems (Graciella, 2016; Dewimarni, 2017). Following the National Council of Teachers Mathematics, mathematical reasoning and problem-solving are important for the development of mathematics education. There are various types of problems in mathematics, but one of the problems that are the main developments in mathematics is realistic problems. Gravemeijer, et al (2004) describe realistic problems as a problem that constructed from a concrete or paradigmatic situations that are experientially real for students and allowed the students to communicate reasoning strategies. Realistic problems are very important because the purpose of learning mathematics requires students not only to understand the concept but also to apply the concept to solve everyday problems. Realistic Mathematics Education (RME) is an approach to learn mathematics that was
developed in 1971 by a group of mathematicians from the Freudenthal Institute, Utrecht University in the Netherlands. This approach is based on the assumption that Freudenthal (1973) emphasizes that mathematics is a human activity. According to this approach, the mathematics classroom is not a place to transfer mathematics from the teacher to students, but a place where students rediscover mathematical ideas and concepts through exploring real problems. In realistic mathematics learning, before students are brought to an 'informal situation', starting with real problems first, then students with the help of the teacher are allowed to rediscover and construct their concepts and then apply them in everyday problems or other fields (Romadoni & Rudhito, 2016).

The low mathematical ability of students can be seen from the assignments and students' difficulties with the material being studied. Learning difficulties are students' inability to master facts, concepts, principles, and skills (Waskitoningtyas, 2016). Students' problem-solving abilities are still relatively low. As many as 73% of students still have relatively poor problem-solving skills (Sumartini, 2016). Students have difficulty solving non-routine mathematical problems that contain many concepts and procedures (Mawaddah & Anisah, 2015). Students are always confused when given problems related to cases in everyday life so that many errors occur. done by students and causes learning to be not optimal. Therefore, understanding what mistakes are made by students in solving real problems is very important (Moru, 2014).

Errors that are usually made by students in working on description problems are caused because students find it difficult to understand the problem solving contained in the problem. The results of Moru, et al (2014) suggest that error analysis can increase knowledge in teaching, recognition of student errors, and error analysis of language, because some errors in mathematics are interrelated, and make an effort to gain an understanding of learning theories because they are related to how knowledge is learned and built by students.

To support the success of the analysis of students’ ability in solving realistic mathematics problems, educators must use appropriate and effective instruments so that the results obtained are more detailed and accurate. One of the right instruments to use is "What-If"-Ethnomathematics Instruments which is based on the concept of “what-if” and ethnomathematics questions. The "What-If" question was first developed by Payadnya, et al where this question allows students to go through two levels of problem posing, namely "accepting the problem" and "challenging the problem" (Payadnya et al, 2016). At the level of "challenging the problem", new questions can arise from the problem. The Ethnomathematics is defined as mathematics practiced by cultural groups, such as urban and rural communities, labor groups, children of certain age groups, indigenous peoples, and others (Rachmawati, 2012). Ethnomathematics can also be considered as a program that aims to study how students can understand, articulate, process, and ultimately use mathematical ideas, concepts, and practices that can solve problems related to their daily activities.

Research by Lubis, et al (2017) found that students' ability to solve mathematical problems in terms of planning and re-examining answers is very low. Meanwhile, Sipayung (2020) found that in solving math problems, students have not been able to solve math problems.: detailed mathematical ideas, apply concepts and algorithms in solving problems in detail, and understand the concepts or algorithms in solving problems in detail. Although it has analyzed students' problem-solving abilities, previous studies have not specifically addressed students' ability to solve
real problems and still tend to use RME as a course approach. Therefore, it is very important to conduct an in-depth analysis of students' ability to solve realistic math problems using appropriate instruments that in this case "What-If"-Ethnomathematics Instruments.

Because of the importance of realistic problem-solving skills that students have for the success of math learning, the importance of knowing what are the obstacles experienced by students in answering realistic problems, researchers conducted research that focused on the analysis of students' abilities in solving realistic mathematics problems using "What-If"-ethnomathematics instruments that can help researchers to provide an in-depth description of students' abilities. Some of the questions proposed in this study are: 1) What are the abilities of student’s abilities in solving mathematics realistic problems? 2) What are the mistakes of students and obstacles that occur in learning related to realistic problem solving and its causative factors? The data collection method using a test, interview, and documentation. The data was then analyzed using qualitative descriptive data analysis with the following stages: data reduction, data presentation, and drawing conclusion and verification.

RESEARCH METHOD

Research Subject
The place and time of the research were carried out at SMP Widiatmika Jimbaran in the 2020/2021 academic year. The research subjects were 46 students with details of 26 students from class 8.1 and 20 students from class 8.5. Students already have basic knowledge about material from problems that will be presented in instruments. Students have studied the material and perimeter of flat wakes as well as the surface area and volume of building rooms during grade 7.

Research Design
This study uses a qualitative research method that aims to show more accurately the students' mistakes in working on mathematical description problems with the subject of straight-line equations with "What-If"-Ethnomathematics Instruments. A qualitative approach was chosen to reveal more carefully about students' errors in solving mathematical description problems. In addition, with a qualitative approach, researchers can communicate directly with respondents to find out students' mistakes in solving story problems.

The type of research that will be conducted is descriptive qualitative research. Qualitative descriptive research was used to obtain data directly from data sources through tests and interview guidelines. This study is described to collect information about the analysis of student abilities in solving realistic problems using "What-If"-Ethnomathematics Instruments. The purpose of this research was to find out how students’ ability in solving realistic problems using "What-If"-Ethnomathematics Instruments. This research will provide an overview of the problems faced by students in solving real problems that will be used basically by teachers in preparing the appropriate learning design in the future.
Data Source

Data sources are sources from which data can be obtained. In this study, researchers used primary data sources and secondary data sources.

Primary Data

Primary data is data obtained directly from the source or object of research. In this study, researchers obtained primary data from the results of students' answers in doing questions and interviewing students that done on-line.

Secondary Data

Secondary data is data that has been published or used by other parties. In this study, researchers obtained secondary data from the literature, websites, and documents in the form of students' math scores in the previous semester from mathematics teachers.

Data Collection Techniques

In this study, several methods of data collection were used, namely as follows:

Test

The test used in this study is in the form of an essay test. Data collection methods are the results of students' answers in working on realistic problems with adapted materials. In this study, the test used was realistic math problems in the form of "What-If"-Ethnomathematics as many as 3 questions on shape and space materials. This means that the real questions presented contain mathematical concepts which are then expanded by "What-If" questions related to the problem. The ethnomathematical concepts presented in these 3 questions are ethnomathematical concepts contained in traditional Balinese culture, especially in terms of religious ceremony facilities and traditional buildings. The test is done online using the Google Classroom and Zoom apps.

Interview

The interview guide used in this study was an unstructured interview. Interviews were conducted on student representatives from the three categories (high, medium, and low) who made mistakes in solving problems. This interview aims to find out the factors that cause students to have difficulty in solving the given problems. Interviews are also conducted with teachers to find out how learning has been going on so far. Interviews are conducted online using the Zoom app at the end of the classroom activities.

Documentation

Documentation is looking for data about things or variables in the form of notes, transcripts, books, grades lists, student attendance lists, and so on. This documentation method is a technique used to obtain data on the students' mathematics test scores in the previous semester, the number of students in class VIII of SMP Widiatmika Jimbaran.
Data Analysis Technique

The qualitative descriptive data analysis technique used in this study with the following stages.

Data Reduction

Data reduction is a form of analysis that sharpens, categorizes, directs, discards unnecessary data, and organizes data in such a way that conclusions can be drawn and verified. This activity leads to the process of selecting, focusing, simplifying, and abstracting the raw data written in field notes. The stages of data reduction in this study are: 1) Correcting the results of student work which is then ranked to determine students who will be used as research subjects, 2) The results of student work which are the subject of research which are raw data are transformed into notes as material for interviews, 3) The results of the interviews that have been carried out are simplified into a good and neat language arrangement, then translated into notes.

Data Presentation

Data presentation is a structured collection of information that provides the possibility of drawing conclusions and taking action. At this stage, the data in the form of student work are arranged according to the object of research.

Drawing Conclusions or Verification

Verification is part of a complete configuration activity so that it can answer research questions and research objectives. Comparing the results of student work and the results of interviews, conclusions can be drawn about the location and causes of student errors in working on realistic problems.

Data Validity Check

After the existing data is analyzed to find answers to the research problems, then check the validity of the findings. To determine the validity of the findings (credibility) an examination technique is needed. Examination of the validity of the findings in this study using triangulation techniques. In this study, the type of triangulation used is source triangulation, which is to compare and check back the degree of trustworthiness of information obtained through different times and tools in qualitative methods. The source triangulation stage carried out in this study was to compare the results of student work with the results of interviews.

Student Activities

Students were involved in learning activities related to ethnomathematics that conducted in whole class settings and individually. In general, these activities are divided into three steps, namely: opening activity, doing math, and interview. The total time allocation for these three activities is 150 minutes which are conducted on Saturdays outside of regular learn hours. Here's the explanation of each activity.

Opening Activity

Opening activity is carried out in a whole class setting where students in both classes follow online learning using the Zoom application. Students are given explanations by teachers assisted by
illustrations using learning videos and power points on the concept of ethnomathematics and examples of ethnomathematics in their environment. Teacher then explained that there are many concepts of ethnomathematics in Balinese culture as well.

The teacher then gives examples of mathematical concepts and problems related in Balinese culture. In this case, the example given by the teacher focuses on the Balinese ceremonial tools as well as traditional building structures containing mathematical concepts. The concepts and problems provided by teachers will be similar to the problems presented in the next activity, doing math.

Doing Math

In this activity, students are given three ethnomathematical problems related to Balinese Culture. These problems are as follows:

1. Gede will make a Klakat for a religious ceremony. Gede was given 12 pieces of bamboo that were ready to be assembled to make a Klakat. The length and width of the bamboo being 25 cm and 1 cm. How many holes can be in the Klakat designed made by Gede? What is the area and circumference of the Klakat and what is the total area of the holes in the Klakat?

“What-If” Question Number 1:

a. What if Gede is asked to make two Klakat from the pieces of bamboo provided? Can Gede make it? State your reasons.

b. What if Gede wants to strengthen the middle of his Klakat? What size bamboo is needed by him? How many? State your reason
2. Sintia wants to make Ituk-ituk with sizes like the one below.

![Image of Ituk-ituk](image)

If Sintia has coconut leaves that are 1 m long each, how many Ituk-ituk can be made from one of these leaves?

“What-If” Question Number 2

a. What if Sintia’s mother asked Sintia to make 100 Ituk-ituk, how many coconut’s leaves does Sintia need?

b. How can the Ituk-ituk be made to be twice as big as before? How many Ituk-ituk can be made from one coconut’s leave?

3. Mr. Made will build a Pelinggih like the picture below on his land measuring 10x10 m.

![Image of Pelinggih](image)

If the height of the Pelinggih roof is 1 m and the volume of the Pelinggih roof is 4 m$^3$, how many Pelinggih can be lined up on the ground?

“What-If” Question Problem Number 3:

a) What if only the land area is known, which is 100 m$^2$, how do you find the number of Pelinggih that can be lined up?
b) If Mr. Made wants to build a Sanggah/Merajan, how many Pelinggih roofs does Pak Made need, what are the variations in size? If necessary, draw the plan.

Students are then given the opportunity to answer these problems individually using their basic knowledge of shape and space as well as ethnomathematics. The time allowed for this activity is 60 minutes. Students then write their answers on a piece of paper and then take a photo or scan of the answer and post it in Google Classroom.

Interview

At the end of the learning activities, interviews were conducted with 3 student representatives to find out student responses to the activities that have been carried out as well as student opinions regarding problems related to ethnomathematics in Balinese culture. In addition to students, interviews were also conducted with teachers to find out the teacher's response regarding ethnomathematics and learning situations that have taken place so far. This activity lasts for 30 minutes. The interview topic is around but not limited to these following questions:

What do you think about the problems given?

What's the hard part of solving ethnomathematics problems in your opinion?

Do ethnomathematics be interesting and challenging?

How do you respond to what-if questions given in each problem?

RESULTS

In collecting data, the researcher gave realistic mathematics test questions in the form of "what-if"-ethnomathematics to 46 students of SMP Widiatmika. Each student's answers were then analyzed and checked to find out what types of errors were made by the students in answering the three "what-if"-ethnomathematics problems given. From all student answers, there were 4 types of common errors made by students, namely: errors in understanding the problems, errors in representation, errors in reasoning, and errors in answering “what-if” questions. From each type of error, the number of students who make errors according to type then collected. The number of students who make mistakes in each type of error is then divided by the total number of students and then the percentage of the number of students in each type of error is obtained. The results of the students' problem-solving ability tests are shown in Table 1.

<table>
<thead>
<tr>
<th>No</th>
<th>Error Type</th>
<th>Number of Error</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Understanding the problems</td>
<td>20</td>
<td>43,47%</td>
</tr>
<tr>
<td>2</td>
<td>Representation</td>
<td>16</td>
<td>34,78%</td>
</tr>
<tr>
<td>3</td>
<td>Reasoning</td>
<td>32</td>
<td>69,56%</td>
</tr>
<tr>
<td>4</td>
<td>Answering “What-If” Question</td>
<td>30</td>
<td>65,21%</td>
</tr>
</tbody>
</table>

Table 1: Type of Error of The Subject's Answer
From the Table 2, it can be seen that the most types of errors made by students were errors in reasoning, where there were 32 students (69.56%) who made this type of error by not giving appropriate reasoning to the written answers. The second most common type of error made by students was answering “what-if” questions with a total of 30 students and a percentage of 65.21%, followed by the type of error in understanding the problems where there were 20 (43.47%) students who failed to understand the problems so that the answers made by students are incorrect. The last most common type of error is representation where there were 16 (34.78%) students who fail to make appropriate representations in answering the problems.

<table>
<thead>
<tr>
<th>No</th>
<th>Category</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>High</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>Middle</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>Low</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 2: Classification of Students' Problem-Solving Ability Categories

**Analysis of Student Errors in Problem No. 1**

**Explanation:** In problem Number 1, students are asked to solve the problem of making "Klakat" which is one of the Balinese Hindu ceremonial facilities which is usually square and used as a base for "Banten" or offerings. In this case, students are expected to be able to make good use of the number of rectangular bamboos provided and make a Klakat with the maximum size and number of holes. Some of the students' mistakes in answering this problem are presented in Figure 2 below.

![Figure 1. Examples of Student Understanding Errors in Problem No. 1](image-url)
Translation:
Knowing: 12 bamboos with length and width is 25 and 1 cm
Question: How many holes can be made?
What is the area and circumference of the Klakat and what is the total area of the Klakat hole?
Answer: Area = Length x Width = 25 cm
Circumference = 2 (Length + Width) = 2 (25 + 1) = 2(26) = 52 cm
what is the area of the Klakat holes = 25 + 12 = 37 holes
Area = Length x Width = 52 x 25 = 1300 cm

Explanation: In Figure 1, it can be seen that students have difficulty understanding the problems presented. In this case, students misinterpreted the size of the bamboo sticks that were presented as a measure of the length and width of the Klakat made so that in calculating the area and circumference students used \( p = 25 \) cm and \( l = 1 \) cm which is the length and width of each bamboo stick to be used. arranged into a Klakat. The final step that should be taken by students is to use 12 bamboo sticks to form a Klakat as shown in the picture where the 12 bamboo sticks can only be used to make 1 Klakat with 9 holes and the length and width of the bamboo sticks are used to determine the size of each hole. formed and calculate the area.

The next type of error is an error in reasoning as shown in Figure 2.
Translation:

Knowing = Bamboos’ length = 25 cm, amount of bamboos needed for 1 Klakat = 12

Bamboos’ width = 1 cm

Question = circumference?

Answer: circumference = S x 4 = 25 x 4 = 100 cm

Question = area?

Answer: area = S x S = 25 x 25 = 625 cm²

Explanation: In Figure 2 it is clear that students give answers without providing appropriate reasoning why students choose the solution. Students only provide raw answers without any appropriate reason or analysis why the calculation of the circumference and area of the Klakat is calculated using these methods and measures. The hope to be achieved is that students can analyze the problem well, and provide solutions with appropriate explanations, such as why only one Klakat can be made, why only 9-hole Klakat can be formed, and so on. Doing a representation by describing the design of the Klakat will also greatly support the reasoning that is done by students.

In addition to the main problems, this instrument also presents "What-If" questions which aim to see the extent to which students' ability to use their thinking skills to solve problems is also presented. Here are the “What-If” questions given.

Explanation: In “What-If” Question problem No. 1, students are expected to be able to give the right answer with appropriate reasoning even though the answers from students will vary but it doesn't matter if the reasoning given is correct. However, of the many answers, students are still not able to give proper reasoning and only give short answers. The following are examples of student answers.

![Figure 3. Example of Student Answers to the "What-If" Question No. 1](image)

Translation:

- a. It is can because the bamboos are still left
- b. 1 cm, 5 pieces
**Explanation:** From Figure 3 above, it can be seen that students cannot provide reasoning and explanations for why they give such answers. Students only give short answers that seem answered without doing detailed analysis or calculations so that the answers given are less precise.

**Analysis of Student Answers on Question No. 2**

**Explanation:** In question No. 2, students are asked to make one of the Hindu prayer facilities called Ituk-ituk which is in the form of a triangle. In this problem, students are given material in the form of coconut leaves with a length of 1 m and are asked to make as many tucks as possible. The thing that students have to pay attention to is that not all parts of the coconut leaves can be used as Ituk-ituk because there are parts of the ends of the leaves that tend to be small. The following are examples of student answers.

**Translation:**

Knowing: coconut leaves length 1 m, Ituk-ituk high = 7 cm

Question: How many Ituk-ituk that can be made from 1 coconut leave?

Answer: Circumference of The Triangle = A + B + C (S x 3) = 7 + 7 + 7 = 21 cm

= 1 m = 100 cm

= 100 cm : 21 cm = 4

**Explanation:** From Figure 4 it can be seen that students make mistakes in solving the given problems. Students are less able to think realistically so they use all the leaves given to make "Ituk-ituk". The answer should be that students are able to imagine that not all parts of the leaves can be used as "Ituk-ituk" because the ends are too small so that at least students can subtract one fruit from the total "Ituk-ituk" obtained from calculations on each leaf.

**Explanation:** In the "What-If" question problem No. 2 students were asked to develop a problem with the condition that if the "Ituk-ituk" to be made amounted to 100 pieces and what if the size of the "Ituk-ituk" was doubled. To answer this "What-If" question, students must be able to analyze
problems well and use their thinking skills to find appropriate solutions. The following are examples of student answers.

![Students' Answers to the "What-If" Question Number 2](image)

**Translation:**

a) *If we made 100 Ituk-ituk, how many coconuts leaves are needed?*
   
   We need 25 coconut leaves.

b) *Ituk-ituk is made with 2 times bigger*
   
   Circumference = 21 cm, So = 21 cm x 2 + 42

**Explanation:** From Figure 5 it can be seen that students are less able to analyze the problems given. Students only give simple answers without appropriate reasoning. Students also experience errors in answering problems so that they provide answers that do not solve the problems given. This also shows that students are also not able to understand the problem well.

**Analysis of Question Number 3**

In question Number 3, the material switches to building space. In this problem, ethnomathematics related to traditional Balinese buildings in Hindu places of worship are in the form of rectangular pyramids. Here is question Number 4.

In answering problem Number 3, most of the students did not answer the problem correctly. Students do not provide detailed explanations and make mistakes in giving answers. This is because students do not make representations when answering problems. Representation is the key in solving problem Number 3.
Knowing: Pelinggih 10 x 10 m
Roof height 1 m, Roof volume = 4 m³

Question: How many Pelinggih can be lined up on the ground?

Answer: 10 x 10 = 100, 4 m³ = 64 m, 1 m + 64 m = 65 m, 100 m – 65 m = 35 m = 15 Pelinggih

Explanation: From Figure 6 it can be seen that students make mistakes in solving problems. Students should find the size of the roof base first using the volume and height measurements given. After that, students can draw a map of the placement of the Pelinggih using the size of the roof obtained. However, students made the mistake of not calculating the size of the base of the pyramid-shaped roof and also not making representations to solve the problem. This indicates that students cannot understand the questions well and also cannot provide appropriate reasoning.

Analysis of the “What-If” Question Number 3

Translation:

a. 100 m² divided by 64 m = 16 Pelinggih
b. 5 Pelinggih with size 2 m or 64 cm
**Explanation:** From Figure 7 it can be seen that students still make the same mistakes in answering the "What-If" questions by not reasoning the answers given. Students still tend to give short answers without an explanation of how to get the answer. This causes students to be wrong in answering the "What-If" questions given.

After working on realistic ethnomathematics problems, students are interviewed by teachers to get students' responses to the problems given and can understand more about the student's abilities. From interviews conducted, students mostly admitted that it is difficult to understand the problem and it is difficult to imagine it for real. Here is one of the interviews of students and teachers.

Teacher: How did you respond to these problems?

Student: Difficult sir

Teacher: Why is it so hard?

Student: It's hard to imagine that, sir.

Teacher: Why is it hard to imagine? You should know these concepts as Balinese.

Students: Yes sir, because we rarely engage in religious and cultural activities directly, we usually accept so and just follow what adults do.

From this answer, it appears that there is less able to understand the surroundings which is the basis in solving realistic problems. If associated with the RME principle (Zulkardi, 2002), students will not be able to go through the Guided Reinvention stage where students are unable to understand a contextual or realistic problem that subsequently through activities students are expected to rediscover traits, theorems, definitions, or procedures. Thus, students have difficulty in Didactic Phenomenology as well as Self Developed Models (self-model development) so that students are unable to connect their knowledge of real situations to abstract situations or from informal to formal mathematics and create their models in solving problems, with a process of generalization and formalization, the model eventually becomes a model according to mathematical reasoning.

From the results of interviews conducted with 3 student representatives selected from each student who scored in the high, medium, and low categories, it was found that most of the students expressed difficulty in answering realistic type questions. Students from the high group tend to be able to understand the questions, but always have difficulty in reasoning or do not think to do the appropriate reasoning. Students from the medium group tend to experience errors in writing calculations in solving realistic problems given and make mistakes in determining appropriate strategies to solve problems. Students from low groups tend to express difficulties in understanding the problems given so that they cannot solve problems properly.
DISCUSSION

This study found that students make many mistakes in solving realistic math problems from various aspects. The following will be explained in more detail about the mistakes found in this study and explained further about how students' ability to solve realistic math problems.

Students are often misunderstanding the problem given. The researcher found that the first of the students’ difficulties in solving realistic problems is that the students still lack the ability to understand the surrounding environment, including realistic concepts contained in their own culture, which causes students to be unable to understand realistic ethnomathematical problems properly (see Figure 1). Many students misunderstand the problems given and have difficulty in interpreting the meaning of the problems so that they are wrong in writing what is known from the problems presented. This is very strange because realistic problems are problems that are arranged based on students' daily lives so that they are easily understood by students, especially realistic problems with ethnomathematical themes which should be close to students because they have the theme of community culture. This is following the opinion of Triyas (in Sulistiyorini, 2015) which states that the difficulty of students in solving story or realistic problems in the aspect of understanding the problem is the difficulty of understanding what the meaning of the question is and the difficulty of students distinguishing shapes/symbols from what is known. This is fatal because, in RME, the use of context includes understanding it not as a form of application of a concept but as a starting point for the development of a concept (Wijaya, 2012). Understanding contextual problems are very important in learning using RME. Ahlfors (Wijaya, 2012) states that the extraction of appropriate concepts from a concrete situation, generalizations to observed cases, inductive arguments, arguments with analogies, and an intuitive basis in formulating a conjecture are forms of mathematical ways of thinking.

The second finding is that the students still show errors in making representations in solving problems. Students still have difficulty in doing appropriate work in solving realistic problems, even many students who consider representation unimportant (see Figure 6). This is following Suryowati’s (2015) opinion which also revealed that students still do not understand how to represent real-world problems into representative mathematical problems. In addition, Syafitri et al (2021) stated that factors causing difficulties in the mathematical representation ability are non-visual aspects, representational aspects of mathematical expressions, and aspects of word representation or written text is a non-cognitive learning factor. Efforts can be made by teachers so that students have representational abilities by choosing and using the right learning approach so that the learning process takes place optimally and can develop mathematical representation abilities.

The third finding and the most common mistakes found in this research is the students' mistakes in reasoning in solving problems (see Fig. 2, 3, 5, 6). Most students answer the problem briefly without an explanation of why the answer appears and how the steps are taken to obtain the answer. This is following the opinion (Sulistiaiawati, 2014) also revealed that the majority of students' answer errors were in determining the work steps caused because students were less accustomed to working on mathematical reasoning questions. In addition, Ario (2016) stated that the various errors made by students were misunderstanding the meaning of the questions, errors using formulas, errors in performing arithmetic operations, not understanding concepts, and difficulties in writing reasons in written form.
The last is students also make an error in answering the “What-If” Questions. This research found that many students did not understand the realistic problems given. Students tend to answer simply without the appropriate explanation and reasoning (see Figure 3, 5, 7). Students can’t show how the flow is until the answer is found and verify the answers obtained. This shows that students still do not understand the "What-If" questions given and are unable to use their reasoning abilities to imagine developing problems through the new situations presented. This is following the opinion of Payadnya et al (2016) which states that many students are confused in answering the "What-If" questions and have difficulty doing appropriate reasoning.

Of the errors found, researchers conducted observations and interviews with teachers. As a result, it was obtained that teachers have been applying learning that still focuses on the delivery and use of mathematical formulas. Teachers lack learning that focuses on mastering mathematical concepts and improving students' mathematical thinking skills which are the basis for solving realistic problems. In learning, teachers rarely give realistic problems and more to regular problems that only require the application of formulas with various variations. This causes students to be less able to delve into the material provided and often do not think critically of the acquired problems. This is following the opinion of Simon (1995) who states that students who tend to learn by applying mathematical formulas and procedures tend to be not well-examined conceptually. Seeing from this, a teacher needs to structure learning that is more oriented to understanding concepts and improving students' thinking skills. Learning must also present more realistic and projected problems so that students can better connect mathematical concepts with the real world. This is the basis of the success of Realistic Mathematics Education.

CONCLUSIONS

It was found that the student's abilities in solving realistic ethnomathematics problems using "What-If"-Ethnomathematics Instruments” is still lacking which include: errors in understanding the problems, errors in representation, errors in reasoning, errors in answering "What-If" Questions. The highest errors were errors in reasoning, which was 69.56% of all students, followed by errors in answering "What-If" Questions of 65.21%, then errors in understanding the problems 43.47%, and finally errors representation as much as 34.78%. Students also have difficulty in the Guided Reinvention stage where students are unable to understand a contextual or realistic problem that subsequently through activities students are expected to rediscover traits, theorems, definitions, or procedures an also Didactic Phenomenology as well as Self Developed Models so that students are unable to connect their knowledge of real situations to abstract situations or from informal to formal mathematics and create their models in solving problems, with a process of generalization and formalization, the model eventually becomes a model according to mathematical reasoning.

From the results of the interview, teachers tend to provide learning that focuses on the delivery and use of formulas and ignores the understanding of concepts and improved thinking skills of students. Therefore, to improve students' mathematical problem-solving skills and RME success, teachers must design learning that more often presents more realistic and projected problems so that students can better connect mathematical concepts with the real world.
ACKNOWLEDGEMENTS

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Pedagogical Content Knowledge of Teachers in Teaching Decimals through Realistic Mathematics Education

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Abstract: Pedagogical Content Knowledge (PCK) as combines a teacher's knowledge of teaching and content so that the specific content is easy for students to understand. This study aimed to investigate the teacher's PCK in teaching decimals after mentoring by the research center of the Indonesian Realistic Mathematics Education (PRP-PMRI) team. This qualitative study involved two teachers teaching fourth grade at the school partner of PRP-PMRI, Aceh Province, Indonesia. The participation was voluntary, and the teachers had attended training related to the implementation of RME. Data were collected by observing during learning, interviews, reflection, tests, and interviews. This study showed that teachers’ PCK in teaching focused more on the reality principle, activity principle, interactivity principle, and guidance principle of RME. They paid less attention to the other RME principles: level principle and intertwinement principle. The teacher also lacked experience making students’ conjectures and did not prepare strategies to anticipate them. The teacher also paid less attention to identifying students’ ways of thinking. In addition, the teacher’s knowledge about the content was also unsatisfactory. This study also revealed that teachers’ content knowledge also influenced teaching strategies, leading to students’ misconceptions. This research implies that mentoring to improve teachers’ PCK needs to be carried out continuously.

INTRODUCTION

Professional teachers carry out their teaching activities through special skills to create active, innovative, creative, effective, and fun learning (Iru & Ode, 2012). Teachers’ work is not just to convey material but must carry out educational activities and help students with learning difficulties (Shulman, 1987). Shulman (1986) defined pedagogical content knowledge (PCK) as ways to represent and formulate specific content so that it is easy for students to understand. Good quality teachers are teachers who know the content and know how the specific content is taught (Kennedy 1998, Pavinee, Jari, & Kalle, 2013; Shulman, 1987).

PCK is a special fusion of content and pedagogical knowledge built from time to time through various experiences to produce professional teachers (Pavinee, Jari, & Kalle, 2013). Research by Loughran, Berry, and Mulhall (2012) showed that one of the possible factors increasing the effectiveness of teachers’ work is to enrich their PCK. Observing and analyzing the PCK of a
teacher, either during the learning process or when planning the lesson, can provide an overview to examine and understand the teacher's competence (Depaepe, Verschaffel, & Kelchtermans, 2013). Therefore, there is a need for a more in-depth investigation of the teacher's PCK in preparing learning tools and their implementation in the classroom.

Teachers having problems with PCK can cause students’ misconceptions or the emergence of student reluctance to learn mathematics (Fianga, Khabibah, Amin, & Ekwati, 2020). One of the topics that students often hold misconceptions about is decimals. Steinle and Stacey (2004) found misconceptions of Grade 4 to 10 students who consider 0.73 less than 0.6 by focusing on the size of the denominator. Their knowledge that one hundredth is less than one-tenth is incorrectly generalized to any number of hundredths less than any number of one-tenth. Desmet, Grégoire, and Mussolin (2010) and Sackur-Grisvard, and Léonard (1985) presented that many students of Grade 4 to 7 decided that 4.25 > 4.3 because 25 >3. Ubuz and Yayan (2010) reported that the most common mistakes in decimal addition problems were reading scales, ordering numbers, and operating decimals. When adding the last digit after the point, for example, adding 0.1 to 4.256 and 6.98, the student then gave an incorrect answer of 4.257 and 6.99, instead of 4.356 and 7.08.

One of the efforts to overcome students’ misconceptions about decimals is to implement Realistic Mathematics Education (RME). RME is an approach that starts learning with challenging realistic problems. Students solve these problems informally, and then gradually, the teacher scaffolds them to reach the formal thinking stage through horizontal mathematization and vertical mathematization processes (Gravemeijer, 1994; van den Heuvel-Panhuizen and Drijvers, 2014). Freudenthal (1991), the founder of RME, explained that, in RME, students should reinvent mathematical ideas, such as concepts, strategies, procedures, formulas, definitions, etc., with teacher guidance because mathematics is a human activity, not a ready-made.

Treffers (1987) and van den Heuvel-Panhuizen and Drijvers (2014) stated six principles of RME, namely activity principle, emphasizing that students are treated as active participants in the learning process; reality principle, emphasizing the importance of learning mathematics that begins with problems in everyday life or real problems, students carry out the mathematization process to solve them; level principle, emphasizing that students learn mathematics through various levels of understanding, from informal understanding related to the situation gradually through model of, model for, and formal knowledge; intertwinement principle; emphasizing the interrelationships between topics in solving rich problems such as numbers, geometry, measurements, and data; interactivity principle, views mathematics not only as an individual activity but also as a social activity; guidance principle, emphasizing the active role of teachers in developing scenarios of learning activities that can facilitate or guide students to achieve the expected level of understanding.

Several studies have been conducted on increasing the PCK of prospective teachers or teachers in overcoming students’ misconceptions about decimals through RME. Widjaja (2008) designed learning instruction theory (LIT) assisted linear arithmetic block (LAB) for teaching decimals for preservice teachers. LIT made substantial improvements in both content and PCK of preservice teachers. Pramudiani, Zulkardi, Hartono, and van Amerom (2011) compiled a learning trajectory along with weight and volume measurement activities, in which a number line is used as a model. The students could discover decimals by themselves and develop ideas to come to the number line.
as a model for placing the value. Furthermore, Wirda, Johar, and Ikhsan (2015) adapted these activities to suit the 2013 Indonesian curriculum using thematic learning and made video lessons involving expert teachers. Yet, research examining teachers’ PCK in teaching decimals by video lessons and implementing RME principles is limited. The research question of this study is how is the primary school teachers’ PCK in teaching decimals related to RME principles?

RESEARCH METHODS

This qualitative study involved two of the teachers participating in the PMRI teacher workshop-assisted video lessons. Both teachers teach at partner schools of the research center of Indonesian Realistic Mathematics Education (Pusat Riset dan Pengembangan Pendidikan Matematika Realistik Indonesia/PRP-PMRI). The teacher was involved voluntarily and had attended training related to the implementation of RME. They hold an undergraduate education; teacher 1 (T1) is an alumnus of the mathematics education department, while Teacher 2 (T2) graduated from the Islamic Education Department. T1 has been teaching for ten years while T2 for 21 years. The two teachers attended training on decimals learning through RME.

The PMRI workshop assisted video lessons were carried out by researchers as the PRP-PMRI team in Aceh Province, Indonesia. The video lesson was obtained from the recording of decimals learning by the expert teacher. Also, researchers adapted the lesson plan and student worksheet designed by Wirda, Johar, and Ikhsan (2015). The workshop was held for two days (12 hours) with the following activities: a) Pre-test on ordering decimal numbers, b) Discussion of teachers' experience in teaching decimals, c) Discussion on the principles and characteristics of RME, d) Discussion on the scope of decimal topic for Grade 4, e) Watching a video lesson of decimals learning through RME, f) Discussion on critical moments in the video associated with RME principles, and g) Discussion on the improvement of lesson plans for implementation. Furthermore, the teachers implement decimals learning with the RME approach as in the video lesson in their classes.

Teachers copied the files of two video lessons to their laptops so they could watch them many times. Video lesson 1 was about exploring the meaning of decimals, and video lesson 2 was about ordering decimals. The activities on video lessons are presented in Table 1.

<table>
<thead>
<tr>
<th>Day</th>
<th>Content</th>
<th>Goals and Activities</th>
</tr>
</thead>
</table>
| 1   | Exploring meaning of decimals | (i) Understanding text containing decimals

The text of the Muara Takus Temple is found in the student textbook with the theme My Hero. In the text, it is written that the length of the temple fence is 1.5 km. This activity allows students to predict the meaning of 1.5 km and estimate how long the 1.5 km is.

(ii) Measuring the length of objects in the classroom

This activity gives students the experience that the length of an object is not always a natural number. If the object's length is not in integer,
the students write in their language, for example, 21 cm less or 21 cm more than two small lines.

(iii) Measuring the weight of items

A total of five nutmegs were weighed using two types of scales. On the first scale, the weight of the nutmeg is 500 grams, while on the second scale, it is 0.5 kg. This activity stimulates students to find the relationship between 0.5 kg and 500 grams. Next, in groups, students weigh an imitation of nutmeg, small stones, each weighing 100 grams or 0.1 kilograms. Students are free to determine how many stones will be weighed.

(iv) Rewriting the length of the object in an activity (ii) using the decimals symbol

This activity allows students to use an analogy between the length of the object ‘21 cm and two small lines’ and the number symbol on the scale so that with the teacher’s guidance, students are expected to write down the length of the object to be 21.2 cm.

2 Ordering decimal number

(i) Sorting the distance of a location written on the signpost

This activity allows students to predict which locations are far or near the sign

(ii) Weighing and sorting the weight of students using a digital scale

This activity leads students to order their friend weigh in decimals

(iii) Hanging the decimal number card and doing the exercise problem

This activity leads students to order decimals using cards and write them on the number line

2 Convert fraction to one-digit decimal and vice versa

(i) Shading part of a whole

These activities lead students to remember a fraction of the tenth, such as 1/10, 3/10, etc. Students shade some parts then write them in the fraction with denominator 10

(ii) Weighing several pieces of green bean packets

This activity guide students to find the relationship between fraction and one-digit decimals so that they convert fraction to one-digit decimals and vice versa

Table 1: Decimal number activities

There were two additional activities other than the video lesson such as using imitation of scale face and pouring seeds into a block glass.
The implementation was carried out by the teachers and mentored by the researchers. Mentoring was done when the teachers made the preparations and after implementation in the classroom through reflection. Assistance during preparation aimed to provide opportunities for teachers to adapt learning activities to the conditions of their students. Mentoring after implementation was intended to help teachers reflect on the learning done related to the principles of RME and aspects that need to be considered for further learning. Reflection was carried out through interactive discussions between teachers and researchers.

Teachers’ PCK data were obtained through observations, tests, and interviews based on the PCK framework proposed by Chick, Baker, and Cheng (2006). The PCK indicators are integrated with the RME principle, as shown in Table 2.

<table>
<thead>
<tr>
<th>PCK Category</th>
<th>Evidence when teachers...</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Learning strategies</td>
<td>Discussing or using strategies or approaches for learning certain mathematical concepts or skills based on RME principles.</td>
</tr>
<tr>
<td>2. Student's Way of Thinking</td>
<td>Identifying a certain level of understanding of students or students' ways of thinking about concepts.</td>
</tr>
<tr>
<td>3. Demonstrating a deep understanding of the foundations of mathematics (content knowledge)</td>
<td>Demonstrating understanding of concepts while identifying mathematical aspects in depth and detail.</td>
</tr>
</tbody>
</table>

Table 2: A Framework for Analyzing Teachers' PCK

The 1 and 2 categories data (learning strategies and identifying students' ways of thinking) was collected through observations when the teacher taught decimals in their classes based on the video lesson. Furthermore, the data was confirmed to the teacher’s reflection after the learning process. Both data were also reconfirmed through semi-structured interviews conducted after the learning. Data concerning category 3 of teacher content knowledge was obtained through tests conducted before the discussion about the scope of the decimals and during the teaching of decimals.

Data analysis was then undertaken through a data simplification process for more comprehensive research (Fraenkel & Wallen 2006). Also, the data of this study were analyzed through the stages of data reduction, data presentation, and conclusions (Milles & Huberman, 1994).

RESULTS

The following explanation describes the PCK of the two teachers (T1 and T2) for each PCK category, learning strategies, students' way of thinking, and content knowledge. The two teachers taught 70 minutes for the first meeting and 90 minutes for the second meeting.
Learning strategies

The observed aspect is how the teacher carries out learning in line with other RME principles: the activity principle, level principle, intertwinement principle, interaction principle, and guidance principle. Before teaching, the researchers conducted interviews with teachers regarding learning strategies.

The first meeting

The question asked during the interview was, "what is the planned teaching strategy for the class tomorrow?". The answers of the two teachers are as follows.

The first meeting aims to find the meaning of decimals. My strategy is to give students activities to weigh spices, such as garlic, ginger, and cinnamon. Students bring the spices per group before the learning. I have put it in a plastic bag that weighs 0.5 kg; then, it will be weighed using two scales that say 500 grams and 0.5 kg. Then, students weigh the objects around them, such as books, drink bottles, etc. From this activity, students see themselves where the decimals came from. Before weighing, I will also ask students to measure a string whose length is not a natural number, for example, 2 cm over or 2.4. ... I plan students to work in groups so that they interact with each other... I will guide them (T1)

Since this lesson has the theme of My Hero and is related to the ancient kingdom, I try to relate it to decimals. For example, my students and I will weigh rice to get decimals. I will tell the students to bring the rice later, or I will just bring it myself. I happen to have rice at home. Then, students will be asked to measure the sticks that I have prepared with a ruler that closes the original number then we write down the others. The original one is 1 to 100; later we will just make scale 1 to 10. The goal is that when we ask students to measure the sticks that I have prepared later, the size is not exactly integer.... . I plan students to work in groups so that they interact with each other... I will guide them (T2)

Next, the researcher asked the question, "why did you choose this strategy?" and the teacher's answer is as follows.

T1: So that students learn through experiences directly and real, in accordance with RME.
T2: So that students find the meaning of decimals according to the RME, students understand better if they see directly the number 0.5 on the scales; it is more real than just reading the writings in textbooks.

From the answers of the two teachers above, it is known that teachers T1 and T2 applied a strategy based on the video lesson and related it to the teaching principles according to RME. We conclude that the teacher only knew four of the RME principles: reality principle, activity principle, and interactivity principle, and guidance principle. They did not link their strategy to other RME principles: level principle and intertwinement principle.

Based on observations in class, it is known that T1 began the lesson by showing the story of the Muara Takus Temple taken from a textbook. In the story, there is an article about the size of the temple wall fence, which is 1.5 km. So the two teachers challenged the students to express their opinions regarding the number of 1.5. The following is the dialogue between T1 and the students.
T1: Who knows what 1.5 means?
S1: Half
S2: One plus half
S3: One fifth
T1: Thank you, many of you have expressed your opinion. How it looks if it is on a number line? Who can come to the front of the class?
S4: (Drawing the number line below)

![Number line diagram]

T1: There are one, one point five, and two. Aren’t there?
S: (chorus) Yes
T1: (The teacher draws a number line containing 4 and 5). Does anyone know if there is a number in the middle of this?
S: (chorus) Yes.
T1: Come on, who wants to come and write the number in the middle?
S5: (Drawing)

![Number line diagram with 1.5 marked]

From the script above, it is known that students can write numbers between two consecutive integers.

Next, T1 asked the students to measure the length of a rope, sketch the length in a number line, and write the length in their language. At that time, the teacher asked students directly to write in the symbol of decimals. Some students had difficulties, T1 guided them, as shown in the following dialogue.

T1: What is the length of your rope?
S: (Put her rope on the ruler)
T1: What is the length?
S: 8
T1: 8 point…?
S: (Silent)
T1: 8 point 9, isn’t it?
Based on the dialogue above, the teacher directly informed the length of the rope in decimals. It should be the student using her expressions, such as 8 cm over 9 small lines or almost 9 cm. Next, the teacher asked students to do the activity (iv): rewriting the object's length in the activity (ii) using the decimals symbol. Students' answer is seen in Figure 1.

<table>
<thead>
<tr>
<th>Name</th>
<th>Object</th>
<th>Measurement in decimals symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balca Meizira P</td>
<td>tali kuer</td>
<td>8,9</td>
</tr>
<tr>
<td>Nojuha thufoilih D</td>
<td>tali kuer</td>
<td>6,5</td>
</tr>
<tr>
<td>Ananda nur A.</td>
<td>tali kuer</td>
<td>3,2</td>
</tr>
<tr>
<td>Sybillia Doniswa</td>
<td>tali kuer</td>
<td>3,4</td>
</tr>
<tr>
<td>Inda Nico M.</td>
<td>tali kuer</td>
<td>7,7</td>
</tr>
<tr>
<td>Abidira Nur A.</td>
<td>tali kuer</td>
<td>5,2</td>
</tr>
</tbody>
</table>

Figure 1. The results of measurement of length by students

After activity (iv), T1 was back to activity (iii): measuring the weight of items. Students weighed the items and saw the number of 500 grams on the first scale and 0.5 kg on the second scale. Then, the teacher instructed students to write the display of the scales on the whiteboard, as shown in Figure 2.

Figure 2: Weighing Results (i) 500 gram, (ii) 0.5 kilograms, and (iii) representation

After the teacher introduced decimals to students using scales, T1 asked students to work on the activities on the worksheet in groups. One of the results of students' work is displayed in Figure 3.
Before ending the lesson, the teacher asked students, “what does 0.5 mean?”. Most students said ‘a half’, “one-second”. ‘What is decimals?’ one of the students answered that decimals are numbers with points. The dialogue of T2 with the students during the lesson is as follows.

T2: From the story, we know that the temple is surrounded by a wall of 1.5 x 1.5 m. Does anyone know the meaning of 1.5?

S1: One and a half

T2: Yes, one and a half kilometers. Does anyone else know?

S: (Silence)

T2: If you don't have one, I'll bring a ruler, we will measure the sticks. We will it measure from zero. Please, one student comes forward.

S2: (Holding the stick)

T2: How long is it?

S2: Six and a half

T2: Try writing it, how should it be written?

S2: (Writing 6.5)
Then the teacher asked other students to measure the stick, and the length was 3.5. The teacher drew a number line containing 7 and 8.

T2: What is the value between 7 and 8?
S: (Silence)
T2: Come on, go ahead, don't be shy.
S3: Between 7 and 8?
T2: Yes, let's write on the whiteboard, not just sitting down.
S3: (Writing)

![Number line with points at 7 and 8]

T2: Between 7 and 8. Between 3 and 4, the middle is 3.5. What is this between 7 and 8?
S3: (Silence)
T2: Between 7 and 8!
S3: (Silence)
T2: Can S3 answer?
S3: (Shaking his head)
T2: If you can't, you can sit down. Come on, who have the answer?
S4: (Writing)

![Number line with point at 7.5]

T2: Is the answer correct?
S: (Chorus) Yes...
The dialogue above indicates that the teacher only repeated the number in the middle but did not associate it with fractions so that some students had difficulty in making sense of it. The teacher also did not ask why S4 answered 7.5.

Next, the teacher asked students to measure the objects around them using a ruler, as shown in Figure 4.
The numbers written on the rulers are multiples of five, and the lines provided are only for integers and their halves. So, students’ answers in the group worksheets were only two kinds: half fractions and whole numbers. There was even a group writing the length of all items in the symbol of decimals when it should be in integers. For instance, the length of “pelok air” (tumbler) was 2.1 m but it should be 21 cm (see Figure 5). However, the teacher did not have time to respond and the students did not present their answers.

Another activity carried out by T2 to introduce decimal numbers is to carry out weighing activities. T2 used rice to weigh and compare its size as T1 in Figure 2.

The second meeting

T1 and T2 displayed a picture of the location pointer to start the lesson, as shown in Figure 6.
Teachers (T1 and T2) asked which one was the furthest and the closest, “who can sort from closest to furthest?” Next, the teacher shows the continuation of the temple story in the textbook, as shown in Figure 7. Then, the teacher asked one of the students to write down the decimals in the story on the board “How can we sort from the smallest to the largest? We will discuss it today”.

Figure 7: Text about Temples which Containing Decimal Numbers

T1 asks two students poor seed to two block glasses. The first student filled the seeds in the first block glass of 3.2; then the second student filled the seeds in the second block glass block of 3.7. After the two-block glass are filled, the teacher shows the difference between the two-block glasses. This activity is carried out several times with other numbers so that students can understand the location or sequence of decimal numbers. This activity can be seen in Figure 8 below.

Figure 8: Teacher T1 Demonstrating the Sequence of Decimal Numbers

Inside the temple complex, there are statues, a building called Bale Agung, inscriptions (slate) and several temples, including the Naga Temple, which is 4.83 meters wide, 6.57 meters long, and 4.70 meters high. In addition, there is a temple that is considered the most sacred, namely the main temple. It consists of three terraces with a total height of 7.19 meters.
After the demonstration, students were asked to order the decimal numbers on the worksheet. Student worksheet answers of T1s’ class and T2s’ class can be seen in Figure 9(i) and (ii).

Next, the teacher introduced the relationship between decimals and fractions. The teacher used cardboard media containing shaded boxes. The teacher explained about $\frac{5}{10}$ by showing the students ten squares with five squares shaded as 0.5. Next, for $\frac{6}{10}$, the teacher showed ten squares with six boxes shaded as 0.6 (Figure 10).

The answers of the worksheet about converting fractions into decimals and vice versa for the students of T1 and T2 are presented in Figures 11 (i) and (ii).
Students' way of thinking

Data about student thinking obtained through observations and quizzes given to students at the end of each lesson. The quiz results showed that four out of 30 students in the T1 class had misconceptions about the location of decimals. Figure 12 describes the example of students' misconceptions.

Figure 12 shows students' understanding of decimals between 3 and 4, namely 3.4, even though the specified line mark is located right in the middle between the numbers 3 and 4. Furthermore, in the second question, students wrote that the decimals between 5 and 6 was 5.6, even though the specified dash did not reach the middle between 5 and 6.

Based on the reflections made by researchers and teachers at the end of the meeting, T1 and T2 planned to overcome students' misconceptions using an image of a scale face. Here's a dialogue between T1 and her students.

T1: I will display an image of the scale face (using PowerPoint) so that all of you can see it. What is the maximum weight on the scale?
S: (Answering in unison) 2 kilograms.
T1: Let's read it together.
S: (reading in unison) 100 grams, 200 grams, .... 1 kg, 1.8 kg, 1.9 kg, 2 kg.

Then the teacher showed an imitation of the scale face and asked several students in turn. The teacher asked the students, "What is the number after 1.5?" The student answered 1.6 by pointing the arrow at the image of the scale face. Then the teacher asked the other students one by one, "What are other numbers between 2 and 3?" The students answered 2.1; 2.2; 2.3; 2.4; 2.5; 2.6; 2.7; 2.8 and 2.9. The teacher asked one of the students to write down the decimals on the scale face, as shown in Figure 13.

![Figure 13. Teacher Using Scales Face as a Model](image1)

The decimal numbers that students have written on the scale face were then released from the scale to form a number line (see Figure 14(i)). This activity was done by T1 so that students understood that the numbers on the scales could also be put into the number line. Then the teacher and students write the meaning of decimals using various ways, as in Figure 14(ii).

![Figure 14: (i) Students Writing Down Decimals (ii) The Meaning of Decimals](image2)
Content knowledge

Regarding the teacher's answers of the test about ordering decimals, the researchers interviewed T1 and T2.

The excerpt of the interview with T1 is as follows.

R: Which is the greater decimals, 4.7 or 4.18 and 2.007 or 2.0045. What is your answer and why?
T1: 4.7 is greater than 4.18 and 2.007 is greater than 2.0045. The reason is that we look at the first number after the point. If it has the same main number and so on, let's look at the numbers behind it. Like 2.007 and 2.0045, first, let's look at the number behind the point, which is 0, look again behind it, which is also 0. Then, look at the next numbers. For 2.007, it shows 7, and for 2.0045, it shows 4. Because 7 is greater than 4, then 2.007 > 2.0045

The excerpt of the interview with T2 is as follows.

R: Which number is greater, 0.75 or 0.8?
T2: 0.75 is less than 0.8. If it is sorted 0.75; 0.76; 0.77; 0.78 and so on, 0.75 does not reach 0.8.
R: Which number is greater, 3.92 or 3.480?
T2: 3.92 is greater than 3.480 because of the second number after the point. For example, for 3.92, the second number is 9 and for the number 3.480, the second number is 4. Because 9 is greater than 4. So, 3.92 is greater than 3.480.

Both teachers misunderstood decimals as point numbers. Even at the end of the first meeting, T1 concluded that decimals are numbers based on ten, namely 0,1,2,3,4,5,6,7,8,9. T1 wrote it down in Powerpoint and presented it to students. The two teachers also misread the decimals of two digits behind the point, for example 3.92 they read three point ninety-two; it should be three point nine two. Based on the reflection after teaching, the teacher realized their misunderstanding about decimals, that it is different to the decimal system.

DISCUSSION

Teachers’ knowledge regarding teaching strategy focused more on reality principle, activity principle, interactivity principle, and guidance principle of RME. Both teacher T1 and T2 asked the students to measure items and write the length using the decimals symbol before they knew the decimals symbol from the scale. Both teachers did not follow the sequence of activities in video lesson. In the video lesson, the sequence of activities was measuring the length without the decimals symbol, then measuring the weight of items and write them in decimals symbol. The last was measuring the length of items using decimals symbol. Referred to the level principle, teacher ask a student to solve a problem in an informal rather than formal way (Gravemeijer, 1994; Treffers, 1987; van den Heuvel-Panhuizen & Drijvers, 2014). Not implementing the level principle impacted the intertwinement principle. This research suggests teachers make connections between measuring the length and weight; therefore, teachers should implement the intertwinement principle.
During the lesson, the teacher seemed busy with demonstrating the media and sometimes ignored the students' incorrect answers. For instance, students wrote the length of all items in the symbol of decimals when it should be in integers. For instance, the length of “pelok air” (tumbler) was 2.1 m rather than 21 cm. Another example, some students wrote 3.4 between 3 and 4, and 5.6 between 5 and 6. Star and Strickland (in Amador, 2014) stated that teachers are rather weak in observing events in the classroom and interpreting students' understanding. Chick and Baker (2005) explained that teachers give varied responses to students' misconceptions. Their responses are influenced by the type or nature of the task given, and their PCK is related to the emphasis on procedural and conceptual aspects.

This research begins by providing training to teachers and providing video lessons, lesson plans, and student worksheets. Teachers were asked to understand all the materials provided and may revise them following the characteristics of students and the available facilities. However, the teachers did not fully follow the important activities. One of the activities in the video lessons that the teachers did not implement was weighing 10 packs of green beans, each weighing 100 grams or 0.1 kg. The teacher and students take 3 of 10 packages of green beans and then read the scales (0.3). Students are directed to find the relationship between \( \frac{3}{10} \) and 0.3. This is in line with the findings of Ahmad and Sultana (2013) reporting that some teachers teach according to what they believe to be true and even teachers fail to carry out activities based on what has been written in the lesson plan. Tanudjaya and Doorman (2020) added that the lack of information also caused this weakness.

Teachers' knowledge of decimals is limited to recognizing symbols, namely point and numbers, unrelated to various representations, such as measuring length and weight. This finding is in line with Widjaja (2008), revealing that the models for learning decimals presented in the Indonesian textbooks are more symbolic, such as emphasizing positions of points rather than lengths of lines. When sorting decimals, the teacher only provided reasons based on the digits after the point, so that students experience misconceptions in learning. Shulman (1986) asserted that "lack of content knowledge is likely to be as useless pedagogically as content-free skill". There is a correlation between PCK and CK (Krauss, 2008). Ball et al. (2009) also added that the mastery of the subject that will be delivered to students is a must for teachers. The very basic in teaching is teacher competency. The research results of Turnuklu and Yesilderes (2007) stated that as a teacher, it is impossible to teach mathematics without knowledge of how to convey the mathematical concept. This finding is important to the next teacher professional development program in developing teachers’ PCK through RME. The level principle and intertwinement principle need to be explained and practiced more.

CONCLUSIONS

Teachers’ PCK through PMRI workshops was measured on three aspects: learning strategies, identifying students’ ways of thinking, and understanding of content were unsatisfactory. Teachers only implemented only four of the six RME principles observed, namely: reality principle, activity principle, and interactivity principle, and guidance principle. They paid less attention to the other RME principle: level principle and intertwinement principle. In addition, teachers PCK in
designing teaching strategies were more likely to prepare learning resources, worksheets, and classroom management. Teachers were less experienced in making conjectures to students’ possible answers and preparing strategies to anticipate them. In addition, the teacher’s knowledge about the content, third aspect of PCK measured in this study, was also unsatisfactory.

It is suggested that to help students be engaged in learning mathematics, the teacher must design learning related to the Realistic Mathematics Education approach, using appropriate teaching aids that are easy to understand by students and preparing questions, to lead students to a higher understanding of mathematics. Teachers are expected to improve their content knowledge as it is the main requirement in designing activities and helping students learn mathematics. Future researchers are expected to design training to help teachers improve their content knowledge mastery and ask teachers to conduct peer teaching to enhance their PCK.

To improve teacher PCK, it is recommended that teacher professional development workshops be carried out by modeling based on lesson plans and worksheets. This means that the researcher as trainer in the workshop acted as teacher and teacher as participant acted as student. So that participants can follow the sequence of activities in the lesson plan and understand how to complete the activities on the students' worksheet. Another advantage of this modeling is that trainers can analyze participants' content knowledge. After the modeling process, researchers discussed the six RME principles one by one, and put more emphasis on the principal level and intertwinement principle.

Teacher content knowledge is the main requirement in teaching. Therefore, during the workshop, there is a special session focus on analysis of students' misconceptions. The results of the students' misconception analysis were discussed as well as adding to the teacher's content knowledge. Mentoring to improve teachers' PCK needs to be carried out continuously so that RME principles can be implemented by teachers during teaching.

REFERENCES


Android Application Development: Permutation of the Same Elements Based on Realistic Mathematics Education

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Abstract: The objective of this development research is to create an android application of probability theory of the same element permutation material based on RME as a learning medium, as well as to assess the quality of the apps created for utilize in learning mathematics. This research used research and development (R&D) method with the 4D development paradigm. There are four steps to the process: define, design, develop, and disseminate. The information analysis approach utilized is a media achievability test conducted by media specialist and material specialists, and a media quality test was conducted by students as responders. The feasibility test results show that the android application of the probability theory of the same element permutation material based on RME is feasible with good criteria, based on an assessment of 87.50 percent by media specialist and an assessment of 84.93 percent by two material specialists. The results of media quality test of 60 students from partner schools showed good criteria, based on an assessment of 80.20 percent by students as responders. The data from the tests suggest that the android application of the probability theory of the same element permutation material based on RME is possible to use as a source of mathematics learning.

INTRODUCTION

The fourth industrial revolution needs students to acquire 21st century skills and mathematical capabilities, such as communication, teamwork, critical thinking (Subekti & Prahmana, 2021), and problem-solving abilities (Widodo, et al. 2019), as well as creativity (Maryanto & Siswanto, 2021) and technological abilities (Siswanto, et al. 2019). Education helps students to achieve a goal in developing interests, talents and behavioral patterns that are useful for their lives (Siswanto & Ratiningsih, 2020). As a result, they encourage instructor to create or execute novel instructing strategies to support students’ creativity within the classroom, and the present school curriculum should stress the importance of growing students’ creativity (Chan & Yuen, 2014; Qian & Clark, 2016; Subekti & Prahmana, 2021).

The rapid development of science and technology today requires education in Indonesia to take a part in using technology as innovation in learning through the curriculum (Pramadana, et al. 2018).
In Indonesia, the 2013 Educational programs is presently being actualized, which stresses ICT literacy in learning and integrates all disciplines with the utilization of ICT (Siswanto et al., 2019). Because of the rapid advancement of technology, teachers must employ learning media that is up to date. A smartphone is one type of media that can help with learning.

Smartphones are mobile phones or mobile phones that are more practical than computers and can be used anywhere (Pramadana, et al. 2018). Smartphones have the potential to be developed into interactive media for students. Mobile innovation will have a noteworthy impact on understudy learning (Churchill, 2008; Churchill, et al. 2015). Smartphones have made it possible to offer a variety of multimedia, including graphics, video recording, and integrated media (Zhang & Wu, 2016). Students may study anywhere while using their cellphones for social media or leisure (García, Welford, & Smith, 2016). The base of the system utilized is one of the factors to consider when transforming cellphones into mobile learning (m-learning). According to the StatCounter Globalstats survey results from July 2020–July 2021, 91.80 percent smartphones in Indonesia use Android as operating system, followed by iOS 7.98 percent, Windows 0.05 percent, Tizen 0.01 percent, series 40 0.02 percent, and other 0.15 percent (Statcounter Global Stats, 2021). This implies that the Android mobile operating system is used by nearly all cellphones in Indonesia.

Android is a mobile operating system that is built on a customized version of Linux (Pramadana et al., 2018) which was developed by Andy Rubin and colleagues since October 2003 through the Android Company, Inc (Siswanto et al., 2019). Google purchased and took over Android as part of a strategy to integrate it into the mobile sector (Siswanto et al., 2019). Android is still the number one operating system on smartphones today, even though Android also has advantages such as user friendly and open source. User friendly means the android system is very easy to run. Open source means that users can freely develop their own version of the Android system.

The learning media created in this research is an android application that contains material on the probability theory of permutations of the same elements based on RME. This application is a follow-up study of a combinatorics application based on RME which has been made and published by Siswanto, Hilda, and Azhar (2019). In this application, it contains material, practice questions and evaluation of learning from the theory of probability permutation of the same elements with examples of cases of compiling card numbers that were developed based on RME. RME is a mathematics learning technique that provides pupils with real-world experiences. The RME approach’s fundamental introduction is that underestudies ought to be given the chance to rediscover mathematical concept and ideas with the direction of the instructor by exploring various circumstances and issues that are genuine to them. RME is based on three fundamental principles: guided reinvention through progressive mathematization, didactical phenomenology or phenomena in learning, and emergent models or generating models (Afriansyah, 2016).

In this study, an android application for the theory of probability permutation of the same elements based on RME was developed using a mobile device with the characteristics of using real
problems, using the results of thinking and model construction from students, mathematical modeling, the occurrence of interactions in classroom learning, and the relationship between the subject. So, in this study, we will pack an Android application based on RME with the same element permutation probability theory content in such a way on a mobile device to make it simpler for instructors, especially students, to learn material anywhere and at any time.

M-learning technology can make it easier for students to learn (Portelli & Eldred, 2016). M-learning facilities allow students to learn more freely since they are portable and can be carried place (Kennewell & Beauchamp, 2007), this is one strategy for achieving learning objectives. M-learning is the confluence of e-learning and mobile computing in which materials must be accessible from anywhere, rich in interactions, powerful support for successful learning, and performance-based evaluation (Riady, et al. 2016). M-learning systems take advantage of the mobility nature of mobile phones, to provide a learning function that can be done anywhere and anytime (Pramadana, et al. 2018). Recognizing the potential of mobile technology as a learning resource for students and as a tool to enhance educational activities (Chao, et al. 2011), teachers must be able to use and exploit technology in the learning process, as well as create chances for app creation utilizing smartphones (Demidowich, et al. 2012).

Based on the reasons for the investigate over, the authors want to improve mobile technology-based learning "Android Application Development: Permutations of the Same Elements Based on RME" at the high school level which is named "Permutation Applications" and continue research on RME-based android applications that have been made previously. The learning application developed in this study contains material, practice questions and evaluation of learning from the material permutation probability theory of the same element as the case example of compiling card numbers that were developed based on RME and will be packaged in such a way as to make it easier for teachers, especially students in learning the material anywhere and anytime.

**RESEARCH METHODS**

This study's Method is a research and development method that attempts to generate specific goods and assess their efficacy (Sugiyono, 2019). This research used research and development (R&D) method with the 4D development paradigm. There are four steps to the process: define, design, develop, and disseminate (Siswanto et al., 2019).

We employed a media validation questionnaire as the tool. The media validation questionnaire is made up of a achievability evaluation sheet and a media quality evaluation sheet that are both arranged employing a Likert scale. achievability evaluation sheets for material specialists and media specialist, and quality evaluation sheets for students' public tests (Widoyoko, 2012).

The data analysis method employed is both qualitative and quantitative. A feasibility test and a media quality test utilizing a questionnaire instrument were used to collect qualitative data. This
application will be verified by material and media specialists to verify the viability of the created application. Before being disseminated, a public test on students from partner schools is conducted to assess the quality of the application that has been created.

RESULTS AND DISCUSSION

Define

This research is a continuation of the research by Siswanto, Hilda, & Azhar (2019) which has been published previously and is also in the stage of conducting a preliminary study including literature study and field study. The literature study carried out is mobile-based learning, and a RME. While field studies on the use of RME in mathematics learning by teachers and restricted interviews with teachers and specialist.

Design

This stage’s goal is to create a learning material. We created a draft of the RME application in this step, which includes the processes of creating a narrative board design (manual) and an illustration design (graphic design). The android application that was created contains material, practice questions and learning evaluations from the material permutation probability theory of the same elements with examples of cases of compiling card numbers that were developed based on RME. Based on the design stages completed in each problem:

**Manual Storyboarding:** The Permutation Manual's narrative or plot is described in the Story Board Manual for the same elements that will be used. Table 1 depicts a sample of the Manual Story Board Composing Card Numbers Stages.

<table>
<thead>
<tr>
<th>Pictures / Videos</th>
<th>Sound / Dubbing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concept of Permutations for the same Element</td>
<td>Opening Music or Introduction</td>
</tr>
<tr>
<td>When Zainal will finish his task. Zainal is having a hard time and asks his brother for help. The older brother helps Zainal a card game.</td>
<td>&quot;Sis, there is something I still don't understand.&quot;</td>
</tr>
<tr>
<td>&quot;What material I will ask?&quot;</td>
<td></td>
</tr>
<tr>
<td>&quot;Material permutations if there are several elements in common&quot;</td>
<td></td>
</tr>
<tr>
<td>Brother shows the arrangement of cards by lining up on the table with the numbers 1 3 5 5. It shows that there are numbers that can be</td>
<td>&quot;Okay. Let me give you an example. Now we will arrange numbers with four number cards where two cards have the same number, namely five.&quot;</td>
</tr>
</tbody>
</table>
created from four number cards, two of which contain the same number.

Brother also shows many 4 factorial ways, because there are cards that have the same number written on them. That is number 5.

Zainal still doesn't understand why cards 1 and 2 are considered the same.

Brother again proved it by showing the arrangement of cards by paying attention to the color of the same number and the arrangement without distinguishing the color of the same number which resulted in 24 ways of permutation of 4 numbers with 2 the same numbers. The arrangement without distinguishing the same number, the number of numbers that can be formed is \(\frac{24}{4!} = \frac{4!}{4!}\).

"Are the numbers formed by Card 1 and Card 2 the same?"

“Why are Card 1 and Card 2 considering the same?”

"If all the numbers on the card are different, then the number of ways to arrange the numbers is 4 factorials, which is 24 ways"

"Look at method 1 and method 2. Are the numbers formed?"

"Look at method 3 and method 5. Are the numbers formed the same?"

"Look at method 4 with method 6. Are the numbers formed the same?"

"Look at the 7 methods with the 8 methods. Are the numbers formed?"

By paying attention to the Arrangement column without distinguishing the same numbers, the number of numbers that can be formed is 24 divided by what?

"2 answer Zainal"

Table 1: Storyboard Manual Draft Arrange the Numbering Cards

Graphic Storyboarding: Storyboard Graphics provides a sketch of the narrative storyline of compiling Card Numbers showing how to arrange cards by paying attention to the same color number and arrangement without distinguishing the color of the same number. Figure 1 shows a snippet of illustration design or graphic design compiling card numbers.
Develop

This step of development attempts to turn the design into a learning material. At this step, a draft of an android application material for the theory of probability permutations of the same components is carried out using an example of the case of assembling card numbers, which was produced based on RME includes combining animation phases or motion graphic animation (visual effects) and narrative dubbing.

In this process, you mix permuted frames with the same elements for how to arrange cards that have been stated in visual effects in this procedure, and then offer sound based on the manual dubbing narration. The next step is to examine the duration, dubbing, and transitions between frames in the completed video. The animation process (Visual Effect), Narrative Dubbing, and Motion Graphic Animation film are depicted in Table 2.

<table>
<thead>
<tr>
<th>Ways of working</th>
<th>Arrangement by paying attention to the same number</th>
<th>Arrangement without distinguishing the color of the same number</th>
<th>Ways of working</th>
<th>Arrangement by paying attention to the same number</th>
<th>Arrangement without distinguishing the color of the same number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1355</td>
<td>1355</td>
<td>13</td>
<td>5135</td>
<td>5135</td>
</tr>
<tr>
<td>2</td>
<td>1355</td>
<td>1355</td>
<td>14</td>
<td>5153</td>
<td>5153</td>
</tr>
<tr>
<td>3</td>
<td>1535</td>
<td>1535</td>
<td>15</td>
<td>5315</td>
<td>5315</td>
</tr>
<tr>
<td>4</td>
<td>1553</td>
<td>1553</td>
<td>16</td>
<td>5351</td>
<td>5351</td>
</tr>
</tbody>
</table>
Dissemination

Following the completion of the application, the product is validated by material and media specialist. This step involves expert validation and test. The result is criticism and ideas that may be utilized to revise the created material so that it can ended up indeed way better.

The objective of material validation is to look at the reasonability of the material given within the android application of the same element permutation probability theory as an example case of assembling card numbers delivered utilizing RME. Using a Likert scale, material specialists give ratings and recommendations on the material presented in the application. Validation was performed by two individuals: Rudi Dwi Pramono, S.Pd, a mathematics teacher at MA Kafila International Islamic School and Nurlaela Rahmawati, S.Pd, a mathematics teacher at South Tangerang 6 High School, which became a partner school in this project.

Table 2: Combining Animation Phases or Motion Graphic Animation (Visual Effects) and Narrative Dubbing on How to Arrange Cards Permutations.

<table>
<thead>
<tr>
<th></th>
<th>Learning Materials</th>
<th>Exercises</th>
<th>evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1535</td>
<td>1535</td>
<td>5513</td>
</tr>
<tr>
<td>6</td>
<td>1553</td>
<td>1553</td>
<td>5531</td>
</tr>
<tr>
<td>7</td>
<td>3155</td>
<td>3155</td>
<td>5135</td>
</tr>
<tr>
<td>8</td>
<td>3155</td>
<td>3155</td>
<td>5153</td>
</tr>
<tr>
<td>9</td>
<td>3515</td>
<td>3515</td>
<td>5315</td>
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<tr>
<td>10</td>
<td>3551</td>
<td>3551</td>
<td>5351</td>
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<tr>
<td>11</td>
<td>3515</td>
<td>3515</td>
<td>5513</td>
</tr>
<tr>
<td>12</td>
<td>3551</td>
<td>3551</td>
<td>5531</td>
</tr>
</tbody>
</table>

Diagram 2: Material Expert Assessment
The assessment result from Rudi Dwi Pramono, S.Pd and Nurlaela Rahmawati, S.Pd as material specialists are 82.29% in learning material aspects, 85.00% in exercise aspects, and 87.50% in evaluation aspects. Overall, the quality of the material is 84.93% with excellent standards judged by two material specialists.

Before being tried on the public, specially partner school students, the validation of the android application media material for the theory of probability permutation of the same elements as the case example of compiling card numbers developed based on RME was tested first. The aspects evaluated include software engineering, authoring, display, and dubbing by Endy Syaiful Alim, S.T., M.T., Ph.D., Head of the Information Technology Development Agency, validated the system (BPTI) Universitas Muhammadiyah University Prof. DR. HAMKA is Computer Science specialist.

The assessment result from Endy Syaiful Alim, ST, MT, Ph.D as a media specialist is 87.50% in display quality aspect, 87.50% in authoring aspect, 87.50% in software engineering aspects and 87.50% in dubbing aspect. Overall, the quality of learning media is 87.50% with excellent standards by a media specialist.

The application trial phase with testing and disseminating the android application of the same element permutation probability theory material as the case example of compiling card numbers that was developed based on RME to students in partner schools. This is done to evaluate the application's functionality on different Android devices and using Google form to determine the level of product achievability. Respondents were asked to install the program on their smartphone before using it. The application is deployed via bluetooth, SHAREit, and the researcher-prepared download URL from Google Drive.
Based on the results of a general trial to 60 students from partner schools, it was found that 81.73% in display quality aspect, 82.45% in authoring aspect, 81.00% in software engineering aspects, and 75.64% in dubbing aspect. Overall, the quality of learning applications is 80.20% with excellent standards rated by 60 students from partner schools. It is appropriate for use as a source of maths learning. Taking after the trial, which was constrained to students at accomplice schools employing a Google forms, the application was upgrade, and the next stage was the distribution stage. This stage deploys the application. The application developed in this study is released through the Google Play Store.

CONCLUSIONS

Development of learning media for android application based on RME for the theory of probability permutation of the same elements named "Permutation Application" using the 4D development paradigm consists of four phases. Define as a preparatory study including library investigate and field studies. Design is the form of drafting an RME application with storyboard design (manual), graphic design, combining animation phases or motion graphic animation (visual effects) and narrative dubbing. Disseminate, is the form of expert validation and test. The finished product is an android application material for the theory of probability permutation of the same elements as the case example of compiling card numbers that was developed based on RME named “Permutation Application”. This application was created as a learning medium. The android application that was created contains material, practice questions and learning evaluations from the material permutation probability theory of the same elements with examples of cases of compiling card numbers that were developed based on RME that can be gotten to through Android-based smartphones. This application is bundled in an attractive that it is expected to be a practical and enjoyable learning medium that can be gotten at any time and from any location, as well as to increase student interest in learning mathematics and to be utilized as a implies of free study and as a source of student references.
Overall, the quality of the material is 84.93% with excellent standards judged by two material specialists. While the quality of learning media is 87.50% with excellent standards by a media specialist and the quality of learning applications is 80.20% with excellent standards rated by 60 students from partner schools. Based on the information acquisition comes about, the Android application named "Permutation Application" for the theory of permutation of the same elements as the case example of compiling card numbers, which was developed based on RME, should be utilized as a source of learning math.

REFERENCES


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Developing Realistic Mathematics Problems Based on Sidoarjo Local Wisdom

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Abstract: Education currently only prioritizes mastery of scientific aspects and students' intelligence. Math problems are still related to fictitious general knowledge. For this reason, local wisdom-based learning is needed whose learning is packaged using objects, events, and various things that are close to students' lives to raise the local potential of regions in Indonesia. Therefore, the researchers developed Realistic Mathematics Problems Based on Local Wisdom (RMPBLW) Sidoarjo. The question is expected not only as a measuring tool for students but also as a step in character building by introducing local cultures to students. The purpose of this study was to develop mathematical problems based on Sidoarjo’s local wisdom on valid and reliable flat-shaped materials. This type of research is research and development or Research and Development (R&D). This study uses 3 stages of test tests, namely expert validation to test the feasibility of the questions, test the practicality in terms of readability of the questions with small group students, and test the validity of the items and the reliability of the questions with quantitative methods. This study produced 15 mathematical questions based on Sidoarjo's local wisdom on flat-shaped material that had been declared valid with an $r$-value > $r$-table (0.4438) and was declared reliable with a reliability value of 0.97.

INTRODUCTION

The success of the implementation of learning can be measured by the existence of a test of learning outcomes, especially in the cognitive aspect. Learning outcomes tests are generally carried out by giving a series of questions according to learning achievement indicators. This is done to obtain information related to understanding and mastery of student learning. However, the Mathematics questions that are studied and worked on are still about general knowledge and rarely involve the content and context that is often experienced around students. So there need to be questions that are developed based on the content and context that surrounds students.

Along with the development of the curriculum in Indonesia, in addition to making students proficient in science, learning carried out in schools should also instill the values of the nation's character. This is according to Law Number 20 of 2003, Article 3, which explains that the
development of capabilities and character formation, as well as forming a dignified nation's civilization, is a function of national education (National Educational Department, 2003). Character formation is closely related to character building and character education. People with character have a personality, a sense of character, and a pattern of behavior (Samrin, 2016).

But in reality, the formation of character and cultural values of the nation in students is still rarely applied in learning mathematics. This can be seen from the lack of students who know and recognize their own regional culture, especially in Sidoarjo. Whereas in essence, character education is an educational system that seeks to instill noble values to school members which include components of knowledge, awareness or willingness, and actions to implement these values.

One form of implementation of character education is to apply local wisdom-based learning. With local wisdom-based learning, local potential in Indonesia can be raised (Prasetyo, 2013). The implementation of local wisdom in learning can also make the nation's morality increase, the quality of education increases, and the quality of implementation and educational outcomes also increases. (Chairiyah, 2017)

So important and strategic is the value of local wisdom in nation-building, it is very natural that character education focuses its studies on extracting local wisdom values that live in Indonesian society and culture with Bhinneka Tunggal Ika. As it is known, that the traditions and culture contained in local wisdom play an important role in developing the personality of the younger generation in which each tradition has superior values. Especially in Sidoarjo, students who are living in Sidoarjo must know Sidoarjo's local wisdom itself, one of which is through education.

Local wisdom is a view of life and knowledge as well as various life strategies in the form of activities carried out by local communities in responding to various problems in meeting their needs. Sidoarjo local wisdom is the result of the Sidoarjo community through their experiences and traditions and is not necessarily experienced by other communities. Therefore, each region must have its local wisdom or culture. There are several of Sidoarjo’s local wisdom that can be related to mathematics, especially in junior high school material. Sidoarjo local wisdom such as Batik Jetis (traditional painting in a piece of cloth in Jetis village), Kirab Tumpeng Pitu (caravan of seven rice cones), Lelang Bandeng (milkfish auction tradition), Nyadran (the tradition of cleaning the tomb of elders), Wayang Silat (puppet silat), Ludruk (Java’s traditional theater), Reog Cemandi (the traditional art of Cemandi village), Tari Banjar Kemuning (traditional dance of Banjar Kemuning village), Ruwat Desa (the tradition of praying for the safety of the village). (Anggraeni et al., 2019).

Once the importance of local wisdom is known to students, so there is a need for learning that is tied to local wisdom, one of which is the development of local wisdom-based ones. If we know, Sidoarjo's local wisdom can be applied to various mathematical materials, but in this study it is more on flat-shaped materials, namely Batik Jetis, Kirab Tumpeng Pitu, Ruwat Desa, and Nyadran.
The application of local wisdom can be done through development based on the cultural context and local wisdom of Sidoarjo.

The questions based on the cultural context and local wisdom of Sidoarjo are closely related to real problems that are close to students' lives because apart from being a tool, mathematics is also a human activity. One approach that has characteristics that are close and relevant to everyday life is Realistic Mathematical Education (RME). Tønnes stated that RME is a learning theory that connects human activities with existing reality (Fadlila & Sagala, 2021). RME has characteristics that are close and relevant to the daily activities of the students themselves so that it enables students to see the mathematics that comes from everyday life. Treffers and Freudenthal explain that the process of rediscovering mathematical concepts is related to the search for patterns and relationships starting with realistic problems, trying to describe with language and symbols created by yourself, modeling, symbolizing, schematization, and defining which also starts with realistic problems and goes on. Overtime can find a way that can be used to solve similar problems without resorting to the help of realistic problems (Natalia, 2017).

It is known that learning using the RME approach can improve students' literacy skills (Sumirattana et al., 2017), students' mathematical communication skills (Habsah, 2017; Sa’id et al., 2021), higher-order thinking skills (Fadlila & Sagala, 2021), and also the problem-solving ability and mathematical confidence (Yuanita et al., 2018). The real world is the starting point for the development of mathematical concepts (Doorman et al., 2007). Muchlis’ research revealed that the mathematical problem-solving ability of students who studied with a Realistic Mathematics approach was significantly better than students who studied with a conventional approach (Efrida et al., 2012). In learning mathematics with the Realistic Mathematics Education approach, mathematical concepts are obtained through students' thinking processes, so this approach is a student-centered learning strategy (Danoebroto, 2013). The real world is the starting point for the development of mathematical concepts.

Based on the advantages of applying RME in mathematics learning, it is necessary to have an instrument to support the implementation of RME learning, namely the development of questions. However, to find out more about the importance of developing questions in RME, it is necessary to review the stages of the thinking process adopted by RME as shown below.

![Figure 1. Levels in Model Development (Bakker, 2004; Gravemeijer & Bakker, 2006)]
In Figure 1 it can be seen that situational is the initial level in the model development stage, where development and the model are still developing in the context of the problem situation using. Furthermore, at the second level, students begin to build a model to describe the context situation or known as the model of. Furthermore, at level three, the developed model has led to the search for solutions. Furthermore, at the last level, students have used symbols and mathematical representations, which is the stage of formulating and affirming mathematical concepts built by students. These stages are very appropriate when students are faced with situations that are very close to everyday life, one of which is problems related to local wisdom where students live. These stages are very appropriate when students are faced with situations that are very close to everyday life, one of which is problems related to local wisdom where students live. This study aims to describe the process and results of developing Realistic Mathematics questions based on valid and reliable Sidoarjo local wisdom. The problem development process in this study is based on the Plomp development model which consists of four phases, namely: initial investigation phase, design phase, realization phase, test phase, evaluation, and revision.

METHOD

According to the purpose of this study, which is to produce valid and reliable Realistic Mathematics Problems Based on Local Wisdom (RMPBLW), the development of questions in this research uses the Plomp model which consists of 1) initial investigation phase, 2) design phase, 3) realization phase/ construction, 4) test, evaluation, and revision phase, and 5) implementation phase. This research was only carried out until the test, evaluation, and revision phases because this research was carried out in a pandemic condition where not all schools carried out learning in schools.

In the initial investigation phase, an investigation was conducted on the local wisdom of Sidoarjo related to the content of flat shapes. The investigation was carried out by examining various sources in the form of relevant online articles discussing Sidoarjo local wisdom. In the design phase, the researcher designs questions by adjusting the context and content of the questions. At this stage, the researcher chose to develop multiple-choice questions to facilitate the analysis of the validity and reliability of the questions. Then in the realization/construction phase, the researcher made the questions according to the plan to produce the first question instrument (RMPBLW I).

At the test, evaluation, and revision stages, the researcher validated RMPBLW I to 3 validators. The three validators are experts in the fields of geometry and education, so all three were asked to validate the feasibility of the items based on content, context, and also language accuracy. Instrument validation was carried out through a qualitative questionnaire which contained a
validator's statement regarding the feasibility of the questions and suggestions for improvement in each question. The results of the expert validation were used to revise the first instrument of question and corrected it to become the second instrument of question (RMPBLW II).

Next, a test of the readability of RMPBLW II was conducted on 5 students who were selected based on the grade level corresponding to the context of the question. The five students were given a question readability questionnaire to determine whether students were able to read and understand the meaning of the questions or not. If there are questions that have not been able to be read by students, then revisions are made again to produce the third instrument of question (RMPBLW III).

RMPBLW III was then tested on 20 junior high school students to measure the validity and reliability of RMPBLW III. The selection of 20 students was adjusted to the suggestions of the research partner teachers, all of which represented the students' ability level. In addition, the condition of partner schools has not yet fully carried out learning activities so that researchers can only try out the third question instrument on 20 students. The problem is tested for students twice at different times or known as the test-retest method. Furthermore, the analysis of validity was carried out by the matter of using the Pearson Product Moment correlation with α = 0.05 and an analysis of reliability using Alpha Cronbach.

Broadly speaking, the implementation of this research can be seen in the following development flow chart.
Figure 2. Development Flow
RESULT AND DISCUSSION

1) Preliminary Investigation

At the initial investigation stage, data on local wisdom was obtained which was analyzed from various sources and as presented in the following table.

<table>
<thead>
<tr>
<th>Resources</th>
<th>Local Wisdom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antara Mempertahankan Batik Tulis Sebagai Produk Budaya Lokal dan</td>
<td></td>
</tr>
<tr>
<td>Kontribusi Ekonomi: Seminar Nasional &amp; Workshop: Peningkatan Inovasi</td>
<td></td>
</tr>
<tr>
<td>dalam Menanggulangi Kemiskinan. Surabaya Wardani, Kusuma. 2015.</td>
<td></td>
</tr>
<tr>
<td>Menggali Potensi Sentra Industri Kreatif Sidoarjo, Jawa Timur. Dalam</td>
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<td>Proceeding Seminar Nasional: Peran Strategi Seni &amp; Budaya dalam Membangun</td>
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</tr>
<tr>
<td>Komunikasi Ritus dalam Tradisi Nyadran di Sidoarjo. Kanal: Jurnal Ilmu</td>
<td>Nyadran and Ruwat Desa</td>
</tr>
<tr>
<td>Hutama. Mei 2018. Tindak Tutur Dalam Tradisi Nyadran (Nglarung Sesaji)</td>
<td></td>
</tr>
<tr>
<td>di Dusun Kepetingan Desa Sawoan Kecamatan Buduran Kabupaten Sidoarjo:</td>
<td></td>
</tr>
<tr>
<td>Dr. Soetomo Surabaya. Sangadji, dkk. Juni 2015. Kajian Ruang Budaya</td>
<td></td>
</tr>
<tr>
<td>Nyadran Sebagai Entitas Budaya Nelayan Kupang di Desa Balongdowo –</td>
<td></td>
</tr>
<tr>
<td>Indrassusiani, Renyta. Maret 2018. Partisipasi Masyarakat Dalam</td>
<td></td>
</tr>
<tr>
<td>Melestarikan Tradisi Kirab Tumpeng Pitu Sebagai Kearifan Lokal di Dusun</td>
<td></td>
</tr>
<tr>
<td>Njaretn Kelurahan Uragang Kecamatan Sidoarjo Kabupaten Sidoarjo.</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. List of Sidoarjo’s Local Wisdom Information

Table 1 is the data containing several local wisdom that can be used to develop questions on flat-shaped materials.

2) Fase Design

After identifying local wisdom that can be applied to the flat shape material, the next step is to design the content and form of the questions given. In this development, the questions developed are in the form of multiple-choice questions. The questions were developed in the form of multiple choice because multiple-choice questions have advantages, namely practicality in scoring and minimizing errors in scoring (Susongko, 2013). Azwar stated that multiple-choice questions have higher objectivity and generally have satisfactory reliability (Kadir, 2015). In addition, students
generally tend to like working on multiple-choice questions compared to essay questions (Tozoglu et al., 2004). The design can be seen in the Table 2.

<table>
<thead>
<tr>
<th>Local Wisdom</th>
<th>Context</th>
<th>Content</th>
<th>Number of Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nyadran</td>
<td>• Making a part of a boat</td>
<td>- Area measurement</td>
<td>1, 2</td>
</tr>
<tr>
<td></td>
<td>• Provision of Space</td>
<td>- Area measurement</td>
<td>3, 4, 6</td>
</tr>
<tr>
<td>Ruwah Desa</td>
<td>• Seat Position</td>
<td>- Distance</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>measurement</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Carnival Path</td>
<td>- Mileage</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Time</td>
<td>8</td>
</tr>
<tr>
<td>Kirab</td>
<td>• Making Tumpeng (Rice</td>
<td>- Area measurement</td>
<td>9, 10</td>
</tr>
<tr>
<td>Tumpeng Pitu</td>
<td>Cone)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Preparing the place</td>
<td>- Area measurement</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>of celebration</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Making a batik</td>
<td>- Number of patterns</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Area measurement</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 2: Problems Design According to Content/Material

The design of the questions is adjusted to the content/language, construction, and language as follows.

a) Content/Material
   - A clear scope of questions and answers
   - Conformity with the Core Competencies and Basic Competencies of Mathematics in 7th grade
   - Suitability competence (urgency, relevance, continuity, and accuracy of daily use)

b) Construction
   - Formulation of the sentence on the question using the word question.
   - Interrelated concepts.
   - There is clear instructions/information on how to do the questions.
   - If there are tables, pictures, graphs, or the like presented legibly, clearly, and functionally.
   - Suitability with the general level of understanding of students in 7th grade.

c) Structure of Language
   - Consistent with Improved Spelling.
   - The questions are not complicated and do not contain multiple interpretations.

An analysis of the local wisdom that exists around Sidoarjo students is carried out so that the questions developed can be felt real by students and are often encountered by students. So that students do not feel unfamiliar with the questions being worked on. In addition, with questions
that adapt local wisdom around students, students will enjoy and enjoy learning mathematics more (Sa’id et al., 2021).

3) Realization/Construction

At this stage, the resulting sheet produced RMPBLW Phase I is prepared at the design stage and adjust aspects of the content/materials, construction, and language. The results of the RMPBLW Phase I was 15 numbers multiple-choice.

4) Test, Evaluation, dan Revision

a) Expert Validation

At this stage, validation was carried out to 3 expert validators where all three assessed the feasibility of this realistic question in terms of content/material, construction, and language. The three validators provide qualitative assessments and provide suggestions for improving the RMPBLW phase I. The results of the feasibility test according to the validators and suggestions for improving the RMPBLW phase I.

<table>
<thead>
<tr>
<th>Validator Code</th>
<th>Number of Questions Without Revision</th>
<th>Number of Questions With Revision</th>
<th>Suggestion</th>
<th>Result</th>
</tr>
</thead>
</table>
| P-1            | 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15 | 1, 7                            | - No. 1: Revision of question context.  
- No. 7: Revision of question instruction | Valid with minor revision |
| P-2            | 5, 10, 13                                      | 1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 14, 15 | - No. 1, 2, & 12: Revision of question context.  
- No. 7: Revision of question instruction  
- No. 3, 4, 6, 8, 9, 11, 14, 15: Revision of question information content. | Valid with minor revision |
| P-3            | 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 | 1                                | - No. 1: Revision of question information content. | Valid with minor revision |

Table 3: Expert Validation Results and Suggestions for Improvement of RMPBLW

Based on the results of expert validation, most of all questions were feasible in terms of content/material, construction, and language. There are suggestions for improving content/materials, namely on questions number 1, 2, 6, 8, 9, 12, 14, and 15, and suggestions for improving the construction of questions on numbers 1, 2, 3, 4, 7, and 11. In general, improvements related to image changes, questions, and sentence structure.
The accuracy of the content and the construction of the questions need to be considered so that the results of the RMPBLW become more realistic and can be easily understood by students. The validity of the content is needed, especially on things that have just been developed and the validity of the constructs also needs to be considered to see the feasibility of transforming ideas and concepts according to the existing reality. (Taherdoost, 2016)

b) 1st Revision

At this stage, improvements were made to RMPBLW phase I adjusting the suggestions from the validator and the suitability of the questions with the level of mathematical ability of 7th-grade students in Sidoarjo in general. The following are the questions before and after revision that were included in the RMPBLW phase II.

<table>
<thead>
<tr>
<th>Before Revision</th>
<th>After Revision</th>
</tr>
</thead>
<tbody>
<tr>
<td>To commemorate the Nyadran tradition which is held every month before the fasting month, people in Balongdowo Village, Buduran District, Sidoarjo make boats that are used to sail to perform Nglarung Sesaji Rituals (tradition of drowning offerings). The lid of the boat is in the shape of a curved rectangle. What is the area of the lid of the boat if it is 6 meters long and 2 meters wide?</td>
<td></td>
</tr>
</tbody>
</table>
| a. 10 m²  
| b. 11 m²  |
| c. 12 m²  
| d. 13 m²  |
| e. 14 m²  |
| To commemorate the Nyadran tradition which is held every month before the fasting month, people in Balongdowo Village, Buduran District, Sidoarjo make boats that are used to sail to perform Nglarung Sesaji Rituals (tradition of drowning offerings). The residents will make boats with lids. The lid of the boat is in the shape of a curved rectangle. What is the area of the lid of the boat if it is 6 meters long and 2 meters wide?  |
| a. 10 m²  
| b. 11 m²  |
| c. 12 m²  
| d. 13 m²  |
| e. 14 m²  |

Table 4: Revision of Problem Number 1

In question number 1, improvements were made by adding the sentence “Local residents will build a boat with a lid”. Improvements were made to the construction section of the question to better provide students with an understanding of the need for caps on boats.
Before Revision

Following is the shape of the boat sail that will be used by the people of Sawohan Village, Buduran District, Sidoarjo to commemorate the Nyadran tradition (Nglarung Sesaji). The screen is made of cloth. If people want to make the screen themselves, then how many square meters of fabric is needed?

| a. 25 m² | b. 30 m² | c. 35 m² | d. 40 m² | e. 45 m² |

After Revision

Following is the shape of the boat sail that will be used by the people of Sawohan Village, Buduran District, Sidoarjo to commemorate the Nyadran tradition (Nglarung Sesaji). The screen is made of cloth. If people want to make the screen themselves, then how many square meters of minimum fabric area is needed?

| a. 25 m² | b. 30 m² | c. 35 m² | d. 40 m² | e. 45 m² |

Table 5: Revision of Problem Number 2

In question number 2, improvements were made to the questions and provided more real picture illustrations so that students could understand the meaning of the questions better.

Before Revision

The Ruwat Desa tradition in Sidoarjo is held every year. One of them by holding a thanksgiving in a large place. Below is one of the places used for the Ruwat Desa event. If the shaded part is a thanksgiving place, then how wide is it?

| a. 56 m² | b. 57 m² | c. 58 m² | d. 59 m² | e. 60 m² |

After Revision

The Ruwat Desa tradition in Sidoarjo is held every year. One of them by holding a thanksgiving in the village field. Below is one of the places used for the Ruwat Desa event. If the shaded part is a thanksgiving place, then how wide is it?

| a. 56 m² | b. 57 m² | c. 58 m² | d. 59 m² | e. 60 m² |

Table 6: Revision of Problem Number 3

In question number 3, improvements were made by replacing the phrase “in a wide area” with the phrase “in the village field” and adding a more realistic picture.
One of the neighborhoods in Bluru Kidul Village will hold a Ruwah Desa celebration in an empty rectangular field measuring 15 m × 20 m. The field will be covered with a carpet measuring 3 m × 2 m. How much carpet (shaded area) is needed to cover the entire field?

<table>
<thead>
<tr>
<th>Option</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>50 carpets</td>
</tr>
<tr>
<td>b.</td>
<td>51 carpets</td>
</tr>
<tr>
<td>c.</td>
<td>52 carpets</td>
</tr>
<tr>
<td>d.</td>
<td>53 carpets</td>
</tr>
<tr>
<td>e.</td>
<td>54 carpets</td>
</tr>
</tbody>
</table>

Table 7: Revision of Problem Number 4

In question number 4, improvements were made by deleting the image according to the suggestions from the validator. Then, there are improvements to the question instructions, namely “Look at the picture below to answer questions number 5 and 6!” be “Look at the picture below to answer questions number 7 and 8!”.

If it takes me 1 minute to walk 50 m, then how long will it take to circle the road?

<table>
<thead>
<tr>
<th>Option</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>75 minutes</td>
</tr>
<tr>
<td>b.</td>
<td>80 minutes</td>
</tr>
<tr>
<td>c.</td>
<td>85 minutes</td>
</tr>
<tr>
<td>d.</td>
<td>90 minutes</td>
</tr>
<tr>
<td>e.</td>
<td>95 minutes</td>
</tr>
</tbody>
</table>

Table 8: Revision of Problem Number 8

In question number 8, improvements were made by adding the sentence “The activity committee wants to estimate the time needed”. The addition of this sentence is so that students can understand the need to estimate the time so that the event can go according to plan.
Every hamlet in Urangagung sub-district, Sidoarjo made a cone for the celebration of the Tumpeng Pitu Kirab tradition. They are competing to make & decorate the cone as beautiful as possible. One of them is like the picture on the side. The picture is a picture of a cone when viewed from above. The circle has the smallest radius of 8 cm. If each circle has a different radius of 2 cm for each level, then calculate the largest area of the cone!

a. 615 cm²  c. 617 cm²  e. 619 cm²  
b. 616 cm²  d. 618 cm²

Every hamlet in Urangagung sub-district, Sidoarjo made a cone for the celebration of the Tumpeng Pitu Kirab tradition. They are competing to make & decorate the cone as beautiful as possible. One of them is like the picture on the side. The picture is a picture of a cone when viewed from above. The circle has the smallest radius of 8 cm. If each circle has a different radius of 2 cm for each level, then what is the area of the largest tumpeng rice circle?

a. 615 cm²  c. 617 cm²  e. 619 cm²  
b. 616 cm²  d. 618 cm²

Table 9: Revision of Problem Number 9

In question number 9, improvements were made to the construction of the question, namely from the sentence “then calculate the largest area of the cone!” be “then what is the area of the largest tumpeng rice circle?”.

See the picture below to answer questions number 11 – 13!

In Njaretan hamlet, Urangagung sub-district, Sidoarjo, every month, the local people carry out the tradition Tumpeng Pitu Kirab, which is to make 7 cones which will be paraded around the village. The existence of traditions these due to the discovery of “Situs Sendang Agung” (the Great Spring Trip) by one of the residents, and the residents believe that water is efficacious. If the land surrounding the well is to be tiled, what is the area of the land?

a. 31 m²  c. 33 m²  e. 35 m²  
b. 32 m²  d. 34 m²

See the picture below to answer questions number 11 – 13!

every month in Suro, people in Njaretan hamlet, Urangagung sub-district, Sidoarjo, carry out the tradition Kirab Tumpeng Pitu, which is to make 7 cones which will be paraded around the village. The existence of traditions these due to the discovery of “Situs Sendang Agung” (the Great Spring Trip) by one of the residents, and the residents believe that water is efficacious. If the land surrounding the well is to be tiled, what is the area of the land?

a. 31 m²  c. 33 m²  e. 35 m²  
b. 32 m²  d. 34 m²

Table 10: Revision of Problem Number 11
In question number 11, improvements were made by changing the sentence “In the hamlet of Njaretan, Urangagung, Sidoarjo every month of Suro” to “Every month of Suro, the people in Hamlet Njaretan, Kelurahan Urangagung, Sidoarjo carry out the tradition of Kirab tumpeng Pitu”. This improvement is done so that students do not get confused when reading the narrative questions.

<table>
<thead>
<tr>
<th>Before Revision</th>
<th>After Revision</th>
</tr>
</thead>
<tbody>
<tr>
<td>How many tiles are needed to cover the entire soil around the well if 1 tile is 50 cm × 50 cm?</td>
<td>How many boxes of tiles are needed to cover the entire soil around the well if 1 tile is 50 cm × 50 cm and 1 box contains 4 tiles?</td>
</tr>
<tr>
<td>a. 110 tiles</td>
<td>a. 30 boxes</td>
</tr>
<tr>
<td>b. 120 tiles</td>
<td>b. 35 boxes</td>
</tr>
<tr>
<td>c. 130 tiles</td>
<td>c. 40 boxes</td>
</tr>
<tr>
<td>d. 140 tiles</td>
<td>d. 45 boxes</td>
</tr>
<tr>
<td>e. 150 tiles</td>
<td>e. 50 boxes</td>
</tr>
</tbody>
</table>

Table 11: Revision of Problem Number 12

Improvements to question number 12 are done by changing the question that originally asked for “how many tiles are needed” by adding a description of the contents of 1 box of tiles. This is done so that students can know that when buying tiles it is not possible to buy units but several boxes.

<table>
<thead>
<tr>
<th>Before Revision</th>
<th>After Revision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batik Jetis Village is one of the traditional batik production sites in Sidoarjo. One of them is Batik Jetis which is a typical batik of Sidoarjo. In addition to written batik, there is also a stamped batik. The image above is an example of its creation. If 1 batik motif measures 10 cm × 20 cm, how many batik patterns can fill 1 piece of cloth with the same motif?</td>
<td>Batik Jetis Village is one of the traditional batik production sites in Sidoarjo. One of them is Batik Jetis which is a typical batik of Sidoarjo. In addition to written batik, there is also a stamped batik. The image above is an example of its creation. If 1 batik motif measures 10 cm × 20 cm, how many batik patterns can fill 1 piece of cloth with the same motif?</td>
</tr>
<tr>
<td>a. 300 patterns</td>
<td>a. 300 patterns</td>
</tr>
<tr>
<td>b. 350 patterns</td>
<td>b. 350 patterns</td>
</tr>
<tr>
<td>c. 400 patterns</td>
<td>c. 400 patterns</td>
</tr>
<tr>
<td>d. 450 patterns</td>
<td>d. 450 patterns</td>
</tr>
<tr>
<td>e. 500 patterns</td>
<td>e. 500 patterns</td>
</tr>
</tbody>
</table>

Table 12: Revision of Problem Number 14

In question number 14, changes are made to the size of the cloth from 4m × 2m to 2m × 1m.

c) **Readability Test Phase and 2nd Revision**

This legibility test stage was carried out on July 20, 2021, by visiting the homes of 5 readability test subjects. The five subjects came from 5 different schools in the Sidoarjo area. The activity was...
carried out by providing RMPBLW phase II and a questionnaire sheet to get comments from students regarding students' understanding of the questions and the clarity of pictures. At this stage, there are only responses to question number 4 while the other 14 questions have been considered suitable for use because they did not receive responses from the 5 students regarding the readability and understanding of the questions. The suggestion for question number 4 is that there is a hint of "shaded part" while there is no picture in question number 4. So it is necessary to make improvements to question number 4. Following are the results of the improvement of the questions after the readability test.

<table>
<thead>
<tr>
<th>Before Revision</th>
<th>After Revision</th>
</tr>
</thead>
<tbody>
<tr>
<td>One of the neighborhoods in Bluru Kidul Village will hold a Ruwah Desa celebration in an empty rectangular field measuring 15 m × 20 m. The field will be covered with a carpet measuring 3 m × 2 m. How much carpet (shaded area) is needed to cover the entire field?</td>
<td>One of the neighborhoods in Bluru Kidul Village will hold a Ruwah Desa celebration in an empty rectangular field measuring 15 m × 20 m. The field will be covered with a carpet measuring 3 m × 2 m. How much carpet is needed to cover the entire field?</td>
</tr>
<tr>
<td>a. 50 carpets  c. 52 carpets  e. 54 carpets</td>
<td>a. 50 carpets  c. 52 carpets  e. 54 carpets</td>
</tr>
<tr>
<td>b. 51 carpets  d. 53 carpets</td>
<td>b. 51 carpets  d. 53 carpets</td>
</tr>
</tbody>
</table>

Table 13: Revision of Problem Number 4

The results of the improvement of the questions from the readability test were then compiled into RMPBLW phase III which was then used to test the validity and reliability of the items.

d) Validity and Reliability Test

Test the validity and reliability of the items carried out thorough tests given to 20 subjects for two tests. This method is known as the retest method. The tests were conducted respectively on 23 and 25 July 2021. The test was carried out for 45 minutes. The following is a table recapitulation of student answers.
From table 17, it can be seen, there were 3 questions, that can be solved well by the students, namely questions number 1, 4, and 10 (in the first test stage), and questions numbered 1, 4, 9, 10, 11, and 12 (in the second test stage). This can be seen from the number of students who answered the questions correctly, reaching more than 50% of students who took the test. In table 17 also, it can be seen that there were questions that did not change, many students answered correctly and there were questions that increased the number of students answered correctly after the retest was held. Like question number 9, from 9 students who answered correctly, 10 students answered correctly. Then questions number 2, 7, 8, 11, and 12 also experienced an increase in the number of students who answered correctly even though questions numbers 2, 7, and 8 had not reached 50% of students who answered the questions correctly. There is also question number 10 which experienced a decrease in the number of students who answered correctly, from 10 students to 9 students.

The following are examples of student answers who answered correctly on the number of questions that could be answered correctly by more than 50% of students and the answers of students who
answered incorrectly on the number of questions that could be answered correctly were less than 50% of students.

Figure 4. Example of Student Answers Who Answered Correctly

Figure 5. Example of Student Answers Who Answered Wrong

From the students' answers shown in Figure 18 and Figure 19, students were able to solve problems related to the perimeter and area of simple flat shapes. However, students will answer incorrectly on a flat shape question whose context is expanded. Like question number 4, it asks about the size of the prayer room page but there are still many students who answer wrongly. Then from the students' answers to question number 7, it can be seen that students have not been able to solve the
problem if the unit of measure is made differently. Then students also have difficulty answering questions about flat shapes with the context of area and circumference associated with time.

Although there are still students who have not correctly completed the questions given, the validity and reliability of the items must be considered. The results of the item validity test for both the first and second tests are presented as follows.

<table>
<thead>
<tr>
<th>Number of Question</th>
<th>r-value 1st Test</th>
<th>r-value 2nd Test</th>
<th>Description</th>
<th>Reliability of 15 Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4870</td>
<td>0.5696</td>
<td>Valid</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.5252</td>
<td>0.4820</td>
<td>Valid</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.5651</td>
<td>0.5196</td>
<td>Valid</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.5876</td>
<td>0.5213</td>
<td>Valid</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.5079</td>
<td>0.5495</td>
<td>Valid</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.6038</td>
<td>0.4820</td>
<td>Valid</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.4770</td>
<td>0.4634</td>
<td>Valid</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.4565</td>
<td>0.6481</td>
<td>Valid</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.5174</td>
<td>0.4790</td>
<td>Valid</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.4988</td>
<td>0.5516</td>
<td>Valid</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.6619</td>
<td>0.5791</td>
<td>Valid</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.4853</td>
<td>0.4965</td>
<td>Valid</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>0.5495</td>
<td>0.5883</td>
<td>Valid</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>0.5642</td>
<td>0.5934</td>
<td>Valid</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.5016</td>
<td>0.5136</td>
<td>Valid</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Table 15: Results of Validity and Reliability Test Items ($\alpha = 0.05$).

Based on table 4 above, 15 questions that have been developed and tested on students can be used because all questions have reached valid and reliable values. This is indicated by the $r$-value of each question being greater than the $r_{table}$, where the $r_{table}$ of 20 samples = 0.4438 and the reliability value of 0.97, which indicates that all the questions developed are in the very high category. In this study there were no questions whose results were invalid. If there is a possibility that the question is invalid, then the question will not be analyzed further (Santoso et al., 2017). The validity and reliability of the questions or all the instruments used in learning are indispensable in learning (Md Ghazali, 2016; Yue Li, 2016) so that it can be applied according to the purpose (Sireci & Faulkner-Bond, 2014).

**CONCLUSIONS**

Good question development is needed in measuring students' abilities. Content and context need to be considered in preparing questions. The development of questions related to local wisdom can be studied by studying relevant literature regarding the implementation of local wisdom in each region, in this study precisely the local wisdom of Sidoarjo. The results of the assessment are used
to determine the context that will be applied in the development of questions and adapted to the competencies to be assessed, in this case, the content of the questions. In addition, the selection of the form of questions developed must also adjust to the accuracy of measurement and students' readiness in working on the questions. In this study, the type of multiple-choice questions was chosen because the scoring measurement was more objective and more attractive to students in the process.

Validation testing by experts is needed in the question development process. In this study, the development of this RMPBLW has reached the conditions suitable for use based on the results of expert validation and readability tests. However, improvements based on expert advice are needed to produce better questions. From the results of the analysis of the validity and reliability of the RMPBLW, it has also been found that it meets the valid and reliable criteria for each item that has been developed. This is indicated by the value of the validity of each item that exceeds the value of \( r_{table} = 0.4438 \) and the reliability value reaches 0.97. With this condition, the developed RMPBLW can be used in the field as a form of implementation of questions based on local wisdom and supports the implementation of mathematics learning that puts forward realistic questions.

Based on the results of the last trial, it was found that students tend to have not shown their abilities in-depth, especially in terms of processes, so researchers need to do further research. For further research, the form of the question instrument given can be in the form of a combination of multiple-choice questions accompanied by a place to write down the problem-solving procedures for each question. So that the form of multiple-choice questions can be used to get an objective score, and the addition of a place to write down the problem-solving procedures is expected to measure students' cognitive abilities more deeply. In addition, the questions based on local wisdom which are applied in learning make students more familiar with local wisdom, especially local wisdom in Sidoarjo, so that students are easier to understand and solve problems because they are related to students' daily lives. It is known that the application of local wisdom based on mathematics problems can improve students' understanding of mathematics problems. (Kaunang et al., 2018)

This RMPBLW also still requires further research, namely on the matter of discriminating power and the level of difficulty of the questions that have not been seen. In addition, further action is needed, especially in terms of the effectiveness of applying questions based on local wisdom in learning and assessing students' cognitive abilities. However, the feasibility of using and the validity and reliability of the items provide a discourse that realistic questions based on local wisdom can be applied well in learning mathematics.

REFERENCES


Development of Instrument Test Computational Thinking Skills
IJHS/JHS Based RME Approach

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Abstract: The increase in the need for critical and analytical thinking among students to boost their confidence in dealing with complex and difficult problems has led to the development of computational skills. Therefore, this study aims to develop an instrument test for computational thinking (CT) skills in the mathematics-based RME (Realistic Mathematics Education) class of the Grade VIII students of JHS/IJHS. This is a Research and Development research carried out using the Plomp model. Data were collected from 30, 27, 22, and 23 Grade VIII students of JHS Negeri 7 Watampone, JHS Negeri 1 Patampanua Pinrang, IJHS Negeri 1 Makassar, and JHS IT Ulul Al-Baab all in South Sulawesi, Indonesia. The data collected were qualitatively analyzed, and the findings showed that more than 50% of the students gave a positive response. Therefore, the students' responses to the questionnaires met the criteria "achieved" without needing instrument improvement. Furthermore, the reliability level of the RME-based computational capability test instrument for multiple-choice issues tested in 4 schools obtained reliability scores of 0.709 and 0.781, which indicated that the developed questions were reliable. Based on data analysis used to measure computational skills in mathematics, out of the 102 test subjects of the assessment instrument, 2 students (1.96%) have a high level of computational skill-based RME approach. Of the remaining 11 (10.78%), 57 (55.8%), 20 (19.6%), and 12 (11.7%) students are in the good, moderate, poor, and very poor categories.
INTRODUCTION

The rapid development of science and technology is marked by the era of industry 4.0, which triggers the need for the renewal of skills and knowledge. According to Hussin (2018) and Lase (2019), this development significantly influences the academic sector. It is also called education 4.0, especially at the formal level, including primary (ES/PS) and secondary educational systems (JHS/IJHS and SHS). This has led to several challenges at the elementary, junior, and senior high schools and the radical conversion of traditional learning practices to modern approaches. In curbing these threats, the government implemented a new orientation involving revising primary and secondary educational curriculum based on literacy and relevant competencies that students need to possess in this era. Some of the skills constitute complex problem solving, creative, innovative, critical thinking skills, communicative and collaborative abilities, emotional intelligence, assessment, and decision making (Mougenot, 2016). These skills are also included in the mathematics curriculum, especially critical thinking and complex problem-solving.

Critical and computational thinking (CT) abilities are related. In recent years, CT has become an emerging global trend in the educational sector (Bustillo & Garaizar, 2014) and is relevant for the future (Adler & Kim, 2018; Maharani, Kholid, Pradana, & Nusantara, 2019; OECD, 2018; Phillips, Yu, Hameed, & Akhdary, 2017). NRC (2011) further stated that its essence is to solve complex problems by breaking them down, making it easier to realize more efficient and automated solutions. Computational thinking improves the ability to solve daily issues (Haragus & Katai, 2020; Kalelioglu, Gulbahar, & Kukul, 2016). Besides, an approach to problem-solving refers to the basic concepts of computing, which is a way to realize abstract concepts into something concrete (Wing, 2008; Romero, Lepage, and Lille, 2017). Furthermore, Thinker and NRC (2011) proposed the importance of advancing computational thinking skills in various sciences. However, CT is not only associated with the thinking process (J. Lockwood & Mooney, 2017), problem-solving (Grover & Pea, 2013; Haseski, Ilic, & Tuğtekin, 2018; Namukasa, Patel, & Miller, 2017), and determining new questions (Barr & Stephenson, 2011), rather it is also centered on individual abilities, and perspectives as well as cognitive factors (Deschryver & Yadav, 2015). Wing (2016) stated that it is a fundamental skill that needs to be possessed by everyone (Kafai, Burke, & O’Byrne, 2016; Kim, Kwon, & Lee, 2014), and not only computer scientists (Williamson, 2015). Some studies stated that CT (Gadanidis, 2017; Rambally, 2017; Son, 2016) plays an important role in solving mathematical problems, which is a constructive process (Benakli, Kostadinov, Satyanarayana, & Singh, 2016; Junsay, 2016; E. Lockwood, DeJarnette, Asay, & Thomas, 2016). Based on these opinions, CT skills need to be developed, especially in mathematics classes, which means that it is necessary to carry out a new orientation in respect to this subject, such as designing an instrument for assessment, thereby promoting CT development (Alfansuri, Rusilowati, & Ridlo, 2018; Deschryver & Yadav, 2015; Wing, 2006). This is expected to help students develop decision-making and problem-solving skills in mathematics (Bower, Wood, Lai, Howe, & Lister, 2017). Therefore, the computational process is
a means to understand natural and social phenomena (Denning, 2019). CT is also developed during solving daily problems associated with computing (Sung, Ahn, & Black, 2017). Meanwhile, students adopt suitable techniques to find solutions through fun activities (Tim Olimpiade Komputer Indonesia, 2017).

Based on the results of observations made in some schools, it was discovered that the process of learning mathematics has not maximally facilitated the development of students’ computational thinking skills. Teachers rarely apply content related to daily problems to develop CT abilities. As a result, the students’ computational thinking skills during math lessons are still low. In accordance with the data acquired from TIMSS, Indonesia is at a low level. Meanwhile, in 2015, it was ranked 44th out of 49 countries, while the results of the 2018 PISA (Program for International Student Assessment) showed that the nation was categorized under the low-performance quadrant with an average and OECD math score of 379 and 487, respectively (OECD, 2019). The outcome of the PISA in the reading, science, and math categories stated that it was ranked 74th out of 79 countries (KumparanSAINS, 2019). According to Harususilo (2019), math and science scores were below average, with the least and highest mathematics scores of 379 and 591 achieved by Indonesian and Chinese students. Therefore, the students’ computational thinking skills in mathematics are still low according to TIMSS and PISA. Another deliberation is centered on the fact that the learning process does not promote the development of children’s thinking skills (Sanjaya, 2016). Kemp (1994) stated that the basic knowledge that needs to be possessed in terms of understanding computational algorithms is mathematical reasoning because the subject matter presented is not a routine issue, therefore, the students first need to be equipped. This led to the suspicion that their poor CT ability in mathematics subjects is due to the implementation of minimal learning and assessment activities. It is also commonly found that the test instruments used by teachers are only derived from package books recommended by the school authority (Sutriani, Sukmawati, & Rukli, 2021). Meanwhile, as a facilitator, teachers are expected to optimize all activities during learning until the assessment stage. Assessment is described as collecting and processing information to determine the students’ learning outcomes (Hanifah, 2019). It is undeniable that presently, the assessment of mathematics education relies more on tests (Sumaryanta, 2014), although teachers have evaluated students’ knowledge (cognitive) and skills through assignments (Irmayati, 2017). However, it simply means something is wrong with the applied scoring system (Sumaryanta, 2014). Therefore, it is necessary to revise the assessment model to hone students' computational thinking skills in mathematics lessons. Teachers are one of the causes of the inadequate development of this attribute in learners, especially during math lessons. Preliminary studies found that apart from incapability to improve computational thinking skills, teachers as facilitators have never developed RME-based test instruments or Realistic Mathematics Education. These are considered relevant to the development of computational thinking skills because their approach emphasizes daily problems or real contexts. This is in line with Anasrudin et al.’s research (2014), stating that the RME approach emphasizes the importance of real contexts and the process of constructing mathematical knowledge by the students. A realistic approach is described as a
method that associates the subject matter with practical problems, especially those experienced (activity) daily through horizontal and vertical mathematical processes (Wahyudi, 2016).

The selection of an appropriate assessment model that tends to hone the students' computational thinking skills in mathematics lessons is then essential to be conducted. One solution to renew this developmental process is authentic assessment. According to the 2013 curriculum, this model emphasizes areas that need to be evaluated. Furthermore, both processes consist of various assessment instruments that are adapted to the demands of SK (Competency Standards), KI (Core Competencies), and KD (Basic Competencies) (Kunandar, 2013). In addition, various studies, such as Malik & Wara (2018), Hilda Nuruslimah (2019), Mufidah (2018), and Fajri & Yurniwati (2019), have been performed on CT to improve and describe its skills and profiles. This means that no research has been carried out on developing the test instruments based on the RME approach. However, it is necessary to pay attention to assessment constructing tools, including integrating computational thinking and subject matter (Tang, Yin, Lin, Hadad, & Zhai, 2020). Therefore, this is related to the development of computational thinking test instruments based on the RME approach.

RESEARCH METHODOLOGY

This Research and Development (R & D) analysis is described as the evaluation of basic and active study (Hasyim, 2016). It adopted the Plomp model, consisting of 4 phases, expressed in the following sections.

Preliminary Investigation Phase

The preliminary investigation phase is the initial stage, carried out to evaluate certain needs to discover the basic challenges required in developing the instrument through an analysis of the curriculum, students, and school materials. Meanwhile, various references related to R & D, research instruments, and computational skills in mathematics were collected. The information obtained was then analyzed based on observation and interviews with mathematics teachers organized in schools where the research was conducted. Furthermore, (a) an analysis of the 2013 curriculum was also performed, based on the acquired data. This aims to improve the students' computational thinking skills. (b) Student analysis, in this phase, those categorized under the 3 abilities, including high, medium, and poor, was further observed in respect to their computational thinking skills based on indicators and (c) material analysis, it was discovered that the 2013 curriculum for junior high school mathematics was used to develop the assessment instrument. This comprises a system of linear equations with 2 variables, circles, and plane shapes.
Design Phase

The design phase is the solution to challenges encountered in the previous stage, besides the resulting instrument is the answer. The developed tools were instrument validation sheet, test blueprint, students’, and teachers’ responses to the questionnaires. This stage aims to design an instrument for assessing computational thinking skills based on the RME approach identified due to the results obtained from the preliminary investigation phase. It consisted of a blueprint, test questions in essay format, answer sheets, and criteria, including scoring guidelines. The test questions were designed in accordance with the analyzed materials and indicators of computational thinking in mathematics, also known as preliminary design. Meanwhile, 40 questions in the form of multiple-choice and essays were designed. A test blueprint, answer keys, assessment guidelines, and considerations to check the validity of computational thinking skills in mathematics questions were also constructed.

Realization Phase

This is the creation phase of the design comprising instrument validation sheets, test blueprint, students’, and teachers’ responses to the questionnaires. It is also called prototype I because it is validated at the evaluation phase after designing a prototype from the previously outlined factors. The results were re-examined by referring to 3 characteristics, namely content, construct, and appropriate language tested for validity by experts in accordance with the theoretical rationale and consistency in construction, therefore, all assessment instruments were analyzed.

Evaluation Phase

The evaluation phase consists of 3 parts, the first aspect is the question validation. This is performed by selected validators regarded as experts in the fields of instrument development and mathematics. The second part carries out a limited trial on revised questions validated by experts. Furthermore, the data obtained from this phase were analyzed for reliability, difficulty level, and discriminating power. However, supposing the test criteria is met, then it proceeds to the subsequent stage, and assuming otherwise, it is revised and re-tested, thereby enabling the product to meet the stipulated yardstick. The third part was considered to have met the criteria after carrying out a limited trial analysis and prototype. Furthermore, the research subjects were tested during the case field trials using 30, 27, 22, and 23 grade VIII students at JHS Negeri 7 Watampone, JHS Negeri 1 Patampanua Pinrang, IJHS Negeri 1 Makassar, and JHS IT Ulul Al-Baab. The following is the development flow chart according to the Plomp model.
Validation activities

Instrument validation was carried out with a test blueprint validation sheet, test questions, and answer criteria, consisting of 2 lecturers and a teacher. In this phase, the validators assessed 10 aspects related to the designed instrument (Prototype I). They stated that the prototype is also used without revision, some or all components of the question need to be revised. The following suggestions were made by the validators, presented in Table 1.

<table>
<thead>
<tr>
<th>Number</th>
<th>Validator</th>
<th>Instrument</th>
<th>Revision</th>
</tr>
</thead>
</table>
| 1      | Validator 1 | Test blueprint | a. Adjust the indicator items on the blueprint with the questions  
b. Correction of sentences described in numbers 2, 7, and 9. Provide images and link them with real contexts familiar to students |
Table 1: Suggestions for Revision from Validators

Table 1 describes the suggestions made by the validators regarding the developed instrument, which was further revised to produce a viable product. Some questions led to significant changes after the expert review process, namely numbers 2, 7, and 9, summarized in Table 2.
<table>
<thead>
<tr>
<th>Questions Number</th>
<th>Before Being Validated</th>
<th>Once Validated</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>A Garden is twice as long as its width. If its 24 cm, then the area of the park</td>
<td>Kgarebosi Field has 3 equally large football fields. Each football field has a ratio between length and width is 3:2. On the edge of the football field there is a road with a width of 3 m around the football field. If the perimeter of each football field is 3 km, the area of the Karebosi field…</td>
</tr>
<tr>
<td>7</td>
<td>In a parking lot there are 100 vehicles consisting of 4-wheeled cars and 2-wheeled motors. The total number of wheels in the parking lot is 300 pieces. The parking fee of a car is Rp5,000.00 while the motorcycle is Rp2,000.00, then the parking money</td>
<td>In the parking lot of Phinisi Point Mall there are 300 vehicles consisting of 4-wheeled cars and 2-wheeled motorcycles. The total number of wheels in the parking lot is 1000 pieces. Parking fee of a cars Rp5,000.00 while a motorcycle Rp3,000.00. The vehicle’s parking money is…</td>
</tr>
</tbody>
</table>
income from the vehicle is…

9. In a school there are several classrooms. If the number of seats per class is 40, then are 84 seats left. If the number of seats in each class is plus 6 then there will be a shortage of 12 seats. The number of classes at the school is…

In JHS 2 Makassar there are several classrooms. If the number of seats per class is 40, then there are 40 seats left. If the number of seats is added 2 then it will be short of 18 seats. The number of classes at JHS 2 Makassar is…

Table 2: Question Changes After the Expert Review Process

Limited Trial

In this phase, the revised questions were tested on 6 students in 4 JHS/IJHS. They answered 9 of them on the available answer sheets. After the trial process was completed, they were asked to comment on the questions.

Further Trial

The validated and subsequently revised prototype was tested on the research trial subject, namely grade VIII students of JHS/IJHS in South Sulawesi, consisting of 4 schools, JHS 7 Watampone, JHS 1 Patampanua Pinrang, IJHS 1 Makassar, and JHS IT Ulul Al-Baab. The trial was carried out in 1 meeting during class hours. Students were asked to take a computational thinking skill test containing 30 multiple choice and 15 essay questions.

The data collection techniques used were tests, students, teachers, and validators’ responses to the questionnaires. The research instrument used consists of test, students’ responses to the questionnaires, and a validation sheet. The test instrument is in accordance with multiple choice and essay questions indicator of computational thinking skills, namely decomposition, data representation, pattern recognition, algorithmic reasoning, generalization, and evaluation, based on the RME approach, presented in Table 3.

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Item Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Multiple choice</td>
</tr>
<tr>
<td>Decomposition</td>
<td>4, 7, 9, 11, 18, 21, 28</td>
</tr>
<tr>
<td>Data representation</td>
<td>1, 2, 7, 17, 18, 29, 30</td>
</tr>
</tbody>
</table>
Pattern recognition 3, 5, 6, 8, 10, 19, 27, 30 3
Algorithmic reasoning 1, 2, 3, 5, 7, 8, 11, 15, 20, 22, 27 4, 6, 7
Generalization 1, 2, 6, 8, 14, 16, 19, 23, 24, 25, 26 4, 10
Evaluation 4, 11, 12, 13, 20, 21, 25 5

Table 3: Item Number Indicator of Computation Thinking

The data analysis techniques consist of self-evaluation, prototyping, small group, and field research. The test instrument in the form of an essay was analyzed in the previous phase. The instrument step was performed in stages, including 1) evaluation of the test instrument in the form of an essay by obtaining the Content Validity Ratio (CVR). Afterward, a CVI (content validity index) analysis was carried out to determine the average reliability of the accepted questions. The results obtained from the CVR and CVI calculations are a ratio of numbers from 0 to 1.

RESULTS AND DISCUSSION

Results of Development of Computational Thinking Skills Based Assessment Instruments

The validity test was established with the Content Validity Ratio (CVR) to determine the suitability of the item with the material or topic to be measured based on the experts’ judgment. The results obtained from CVR and CVI showed that of the 40 items reviewed by 2 validators (experts) these items support the validity of the test, therefore, the prototype was reported to be valid. Apart from the judgment process, some students’ responses were based on the completed questionnaire. The responses were given to 3 (one-to-one) and 6 students (small group) outside the test subject. The questionnaires were distributed after students answered the questions concerning the given measuring instrument. The analysis of their opinion on the computational skills-based assessment instrument in the one-to-one and small group trials both obtained an average positive and negative responses of 79.16% and 20.83%, respectively. Therefore, the students’ responses to the questionnaires met the criteria of "achieved," and there was no improvement or revision of the developed instrument.

The reliability test was conducted based on the results of a field test involving grade VIII students of JHS 7 Watampone, JHS 1 Patampuan Pinrang, IJHS 1 Makassar, and JHS IT Ulul Al-Baab. Meanwhile, 30 grade VIII students were observed at the JHS 7 Watampone, 27 at JHS 1 Patampuan Pinrang, 22 at IJHS 1 Makassar, and 23 at JHS IT Ulul Al-Baab. Based on the results of the students’ work, the level of test reliability was calculated. The results of the reliability test are shown in Table 4.
Table 4: Data Reliability of Assessment Instruments

Table 4 shows that the assessment instruments are reliable, therefore, based on this analysis, the computational skills-based instrument was not revised. The difficulty level in accordance with the developed instrument was also obtained from the results of students’ work in the field test, as shown in Table 5.

Table 5: Analysis of the Difficulty Level of the Multiple Choice as an Assessment Instrument

Table 5 shows that the categories of the difficulty levels at the trial phase are divided into 3, including questions that are classified as easy with difficulty levels within the range of 0.7 to 1.00, medium with a difficulty level of 0.30 to 0.69, and difficult with a difficulty level of 0.00 to 0.29. For multiple-choice questions, 2 items were categorized as easy, 23 were classified as medium, and 6 as difficult, as shown in Table 6.

Table 6: Analysis of the Difficulty Level of the Essay as an Assessment Instrument
Table 6: Analysis of the Difficulty Level of Essay Question as an Assessment Instrument

For essay questions, 3, 3, and 5 items were classified as easy, medium, and difficult. The test result used to measure the students' computational skills is based on the final score obtained when working on the instrument. The analysis results of students' computational skills test are shown in Table 7.

<table>
<thead>
<tr>
<th>Number of Questions</th>
<th>Student Score</th>
<th>Frequency</th>
<th>Percentage (%)</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 questions</td>
<td>80 &lt; value ≤ 100</td>
<td>2</td>
<td>1.96</td>
<td>Excellent</td>
</tr>
<tr>
<td></td>
<td>60 &lt; value ≤ 80</td>
<td>11</td>
<td>10.78</td>
<td>Good</td>
</tr>
<tr>
<td></td>
<td>40 &lt; value ≤ 60</td>
<td>57</td>
<td>55.8</td>
<td>Moderate</td>
</tr>
<tr>
<td></td>
<td>20 &lt; value ≤ 40</td>
<td>20</td>
<td>19.6</td>
<td>Fair</td>
</tr>
<tr>
<td></td>
<td>0 ≤ value ≤ 20</td>
<td>12</td>
<td>11.7</td>
<td>Poor</td>
</tr>
<tr>
<td>Number of Subjects</td>
<td>39</td>
<td>100</td>
<td></td>
<td>Moderate</td>
</tr>
<tr>
<td>Average Score</td>
<td>47.35</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Analysis of Computational Skills Test Results at JHS 7 Watampone, JHS 1 Patampanua Pinrang, IJHS 1 Makassar, dan JHS IT Ulul Al-Baab

Based on data analysis to measure the computational skills of students at JHS 7 Watampone, JHS 1 Patampanua Pinrang, IJHS 1 Makassar, and JHS IT Ulul Al-Baab in mathematics, it was discovered that of the 102 test subjects of the assessment instrument, 2 trial students (1.96%) possessed excellent computing skills, while 11 of them (10.78%) were included in the good category, 57 (55.8%) in the moderate classification, 20 (19.6%) in the fair division, and 12 (11.7%) in the poor group.

After carrying out research on computational thinking in mathematics, it is obvious that this skill is still low in Indonesia. This ability is relevant to the reason mathematical abilities in TIMSS and
PISA are low. Saxena, Lo, Hew, and Wong (2020), stated that this was caused by the students’ failure to find solutions at the algorithm stage. The basic knowledge that needs to be possessed in terms of learning computational algorithms is mathematical reasoning. However, because the subject matter presented in the computational algorithm is not a routine issue, the students have to be equipped with creative mathematical reasoning. The results of the 2018 PISA (Program for International Student Assessment) showed that Indonesia was in the poor performance quadrant with an average and OECD math score of 379 and 487, respectively (Kemdikbud, 2019). The results of PISA in the reading, science, and math skills categories stated that Indonesia was still ranked 74th out of 79 countries (KumparanSAINS, 2019).

Meanwhile, based on the research findings, it was obvious that majority of the students were only able to work from the first to third stages. Figure 2 shows one of the incorrect results used by students to determine the answers to the questions.

![Susi gave 2 pencils and 3 books at a price of Rp 21,000,000 and Adam gave 5 pencils and 1 book at a price of Rp 20,000,000. Is a pencil more expensive than a book? Prove it!](image)

**Figure 2: Students’ Work Results for Problem 3**

Based on figure 2, it is evident that students are able to solve and represent certain problems related to the pattern recognition stage. This is because they do not completely understand the material explained by their teacher. Based on findings, it was concluded that some of the factors that cause poor mathematical computation were the adopted learning methods and models. These include students' experiences, less challenging statements from the teacher, ability to formulate questions, unequal treatment in each development, and lack of discipline during classes. Subsequently, CT assessments are still lacking in relation to the subject matter (Tang et al., 2020), even though it affects students' computational skills (Djambong & Freiman, 2016).

The poor computing skill is caused by several factors, including the current formal education, which tends to be trapped only in honing aspects of knowledge and understanding perceived as lower-order thinking abilities (Cansu & Cansu, 2019). Moreover, the students are asked to absorb anything the teacher says. Learning activities with the casting system leads to controlling the children’s capability, even though everyone is born with amazing potentials. The difficulty in understanding abstract concepts with teacher-centered learning methods is based on the fact that
students often succeed in solving certain problems and fail when their context is slightly changed. This is one of the reasons they are not accustomed to high-level thinking, namely understanding, application, and reasoning abilities. In learning, meaningful experiences need to be encountered by the students to develop optimal computational thinking skills (Fajri, Yurniwati, & Utomo, 2017). To determine the students' abilities, evaluations in tests are applied (Ling, Saibin, Naharu, Labadin, & Aziz, 2018). Based on the results of the previous discussion, several weaknesses were obtained during the development of this instrument, namely a) students are less able to solve computational skills questions, b) development of assessment tools are only intended for the grade VIII level, therefore computing abilities of those in other classes are immeasurable, c) students are not used to solving questions on such tools. In addition to the weaknesses of the research, this study also has certain advantages, including a) the developed instrument promotes students to improve their computational skills, b) it is used as an exercise to develop and optimize the computational abilities of those in grade VIII JHS/IJHS in South Sulawesi, c) teachers also use the developed instrument to improve their computing skills.

CONCLUSIONS

The process of developing a computational skill test instrument in mathematics for grade VII JHS/IJHS students in South Sulawesi was determined through 4 stages, namely (a) preliminary investigation, (b) design, (c) realization or construction, and (d) test, evaluation, and revision phases. The results of this study indicate that of the 40 items reviewed by 2 validators, it was reported that these support the validity of the test, meaning that the prototype is valid. Therefore, over 50% of the students gave a positive response. This implies that their responses to the questionnaires met the criteria of "achieved" without revision. The reliability level of the assessment instrument based on computational skill for multiple-choice and essay questions tested in 4 schools was 0.709 and 0.781, which indicates that the formulated problems are reliable. Based on data analysis carried out to measure computational skill in mathematics, it found that of the 102 test subjects, 2 (1.96%), 11 (10.78%), 57 (55.8%), 20 (19.6%), and 12 (11.7%) students are in the excellent, good, moderate, and poor category. Therefore, based on the assessment results, it was discovered that the average category is sufficient, less, and very less large. This proves that there is still no maximum effort to improve these skills because learning activities carried out with the casting system lead to controlling the children’s capability, even though everyone is born with amazing potentials. The difficulty encountered in understanding abstract concepts with teacher-centered learning methods is focused on the known fact that students often succeed in solving certain problems and fail when the context is slightly changed. This is one of the reasons they are not accustomed to high-level thinking, namely understanding, application, and reasoning abilities. Therefore, to overcome these problems, the results of this study tend to help teachers in the teaching and learning process, including the use of tests to measure the students' computational thinking skills. This developed product contains details about the scores obtained based on solved problems. The assessment guideline is a useful guide for teachers to assess the results of the
students' work in answering test questions, especially those related to computational thinking skills.

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Beliefs on Realism of Word Problems: A Case of Indonesian Prospective Mathematics Teachers

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Abstract: This study aims to investigate prospective teachers' beliefs toward the realism of mathematics word problems. The study employed both quantitative and qualitative analysis. A 36-item survey was distributed to 28 prospective mathematics teachers in one of the private universities in Indonesia. The survey was developed using the framework proposed by Palm (2006). The survey data was analyzed quantitatively by giving a score to each item's scale responded by participants. Then, interviews were conducted with six participants who have realistic, non-realistic, and neutral beliefs toward word problems. The interviews excerpts were analyzed qualitatively through excerpt coding. It was found that 20 participants possessed realistic beliefs toward word problems, while the rest possessed non-realistic and neutral beliefs (7 and 1 participant consecutively). Prospective teachers with realistic beliefs emphasized that any information presented in the word problem should simulate real life as accurately as possible. In contrast, those who have non-realistic beliefs stated that it was acceptable if it can be imagined. Neutral prospective teachers believe that word problems' realism is relative to cultural setting and students’ background.

INTRODUCTION

In today’s Mathematics teaching and learning context, it is never denied that one of the essential learning goals is to apply the knowledge obtained in the class into real life. The demand to contextualize teaching into students' lives increases as the need for real-life problem-solving skills increases. One of the ways used by teachers to introduce this contextual problem is by giving word problems.
A word problem is a written text containing a real-life situation completed by quantitative information and question. The answer can be obtained from the provided or inferred information given (Leder, et al. 2002). Despite the role as a tool to apply mathematical knowledge into a real-life situation, word problems in the classroom are sometimes considered "superficial" as they do not depict the realistic view of life. It hinders students from making sense of their learning back to a real-life context. For example, in the study by Palm (2008) on 161 fifth graders, the use of superficial solution strategies was found. Interestingly, this study revealed that the main reason for this was students’ beliefs in word problem-solving. In this study, students believed that they did not have to consider what was not written in the word problem (i.e., considering how real the information is in the real situation). Although some students found the unrealistic information given in the problem, they preferred to ignore it as long as the problem was answered.

A similar case was also found in the study of undergraduate students by Inoue (2005). From the worked tests given, most students' answers were unrealistic. However, after being interviewed, the unrealistic view emerged from a realistic effort to adjust to schooling norm during Mathematics learning; this realistic effort leads to their beliefs on what a "realistic" situation is in word problem-solving. Further, the classroom practices showed how teachers supported students to make sense of the problem realistically, from what the teacher believed as "realistic", which in other words, restricted the ways students see the problems from their point of view of life.

Students' unrealistic answers may come from teachers' typical word problems. Greer (1997) investigated this issue in the study when he claimed that the word problems' structures are mostly unrealistic. The word problems were found to be unproblematic, i.e., they can always be solved, and all information will always be used. On the other hand, unproblematic word problems do not always happen in reality. A study by Krawitz (2018) revealed similar findings. Students' exposure to unproblematic word problems made them overlook irrelevant information and thus gave unrealistic answers. This exposure came from the learning experience provided by teachers possessing such beliefs.

Some studies on beliefs toward word problems had been conducted in Indonesia. Wijaya, et al. (2015) showed that teachers mainly offer plain word problems without irrelevant or redundant information linked to their beliefs. Another study by Siswono, et al. (2019) found teachers' beliefs that the problem solving should be done instantly, not inferring a realistic view. Some other studies discussed prospective teachers' beliefs, although not focusing on word problem-solving. For instance, the study by Muhtarom (2019) revealed that prospective teachers, who believed that mathematics is dynamic and a way to solve real-life problems, gave a problem close to students' life. In another study (Muhtarom et al., 2017), most prospective teachers' had a Platonist view, which saw mathematics as a static and structured knowledge rather than a dynamic one. Despite many studies discussing beliefs toward word problem solving, studies of prospective teachers, or even the specific beliefs toward word problem realism, are still limited in Indonesia. Taking the facts into account, studying prospective teachers' beliefs related to the realism of word problems.
and, further, how the beliefs affect the decision to give it to students is necessary. Therefore, this study aims to investigate prospective teachers’ beliefs toward the realism of mathematics word problems by answering a research question: "How are the beliefs possessed by prospective Mathematics teachers toward the realism of word problem?"

THEORETICAL FRAMEWORK

Word Problem and Reality

Word problem, in general, is a problem being put in a real-life situation (Verschaffel, et al. 2010). The existence of real-life situations embedded in the word problem makes it both unique and challenging. Rather than directly offer a question demanding an answer, the word problem is presented by three components: a set-up component, an information component, and a question (Gerofsky, 2014). Those components are formulated around a real-life situation while at the same time bringing a mathematical purpose. However, the situation in which word problem is formulated may not be that realistic.

There had been numerous studies exploring the notion of “reality” in mathematics education, and indirectly in the case of word problems. One of the most referred one was that of Freudenthal who regarded reality as rich source to be utilized in teaching and that mathematizing, a process of organizing reality with mathematical meaning, should be learnt by students (Freudenthal, 1973). He explained the reality to be experientially real to students, or in other words directly correlated to students’ daily life (Gravemeijer & Terwel, 2000). However, he also admitted that mathematicians seemed to weigh logical connection more than reality as soon as the former brings faster progress. In this case, the practice of incorporating reality into the classroom may not be that apparent or dropped once the logical connection of mathematical matter seemed more promising. In the case of word problems, Boaler (1993) questioned explicitly, “how real is real”. She stated that for some word problems, the reality presented in the word problem sometimes should be perceived as real although some aspects of that reality were ignored in the “real life version”. This again showed that what is considered real in the word problems may not be students’ reality and allowed for some variations of what is considered as realistic (or unrealistic) word problems.

Galbraith and Stillman (2001) distinguish four types of word problems based on how the real world is integrated into the word problem. An "injudicious problem" does not use the real world into the word problem or even violate it. A "context-separable problem" uses a real-world context, but the answer is context-independent. A "standard application problem" expects realistic consideration for the situation, and the procedure cannot be directly presented. The last type, a "genuine modeling problem", requires personal real-life knowledge and considers it to solve problems. In another study by Greer, Verschaffel, and De Corte (2002), the first two characteristics are considered that of non-problematic problems.
Types of word problems showed to what extent the real world is incorporated into the problems. Interestingly, various studies promoted a more realistic word problem, which considered the context embedded in it, to support students' mathematical learning (Greer et al., 2002; Krawitz et al., 2018; Tarim & Öktem, 2014; Van Dooren et al., 2019). The thinking process in considering the reality presented in the problems will benefit students in exercising transformation from real-world information into the mathematical model. This process imposes a more meaningful role of mathematics toward students. However, this is not always the case. For instance, a study by Sumarwati et al. (2014) showed more non-problematic and simple word problems in elementary school textbooks. The word problems presented there are the ones whose context is not considered for solving them.

Beliefs Towards Word Problem

In the context of mathematics teaching and learning, belief is defined as subjective conceptions that are held to be true, which may be shown explicitly or implicitly, and affect mathematical learning and problem solving (Op’ T Eynde, et al. 2002). The practices in utilizing realistic word problems in the classroom are highlighted by several studies to be affected by teachers' beliefs (Beswick, 2005; Luz & Antoni, 2019). Despite being useful in promoting students' reasoning and meaningfulness of mathematics in their life (e.g., Boaler, 1993), there is a spectrum of beliefs on the realism of word problems.

In discussing the spectrum of beliefs on the realism of word problems, studies generally explain it in two complementary modes: realistic and unrealistic (Depaepe, et al. 2010; Luz & Antoni, 2019; Tarim & Öktem, 2014; Verschaffel, et al. 1997). When one believes that the real world can be totally or partly excluded from the word problem solution or modeling process, it can be inferred that an unrealistic mode is held. While realistic mode portraits beliefs of putting all real-world consideration into the solution and modeling process of the word problem.

In the case of word problems and reality, Palm (2006) offered a comprehensive framework on essential aspects in real life to be considered in word problems (Table 1). An individual’s conception of how each aspect should be considered determines his/her beliefs about word problems (WP). Palm mentioned an example of the ‘event’ aspect. An event of some people wants to go up in a lift in the morning (as shown in the Lift Problem in Figure 1) can be considered to have a fair chance of happening. On the other hand, picking marbles from an urn and noting their colors is not something people do and therefore cannot be considered a realistic event. When a person agrees that a word problem should contain a realistic event, it shows his/her realistic beliefs on word problems; on the other hand, if he/she thinks it is okay to include non-realistic events, he/she is considered to have non-realistic beliefs.

Another example is a consideration for the question aspect. In the question aspect, there needs to be a consideration of whether the word problem's question is in accordance with the possible question posed by a real-life situation. There may be no real-life question like the question given
by the Lift problem in Figure 2. People may want to know when it may be their turn to use the lift instead. So, when a person believes that a word problem should pose a real-life question, he/she has realistic beliefs. To summarize, the closer each aspect presented in the word problem to the real situation, the more realistic it becomes. Thus, a person who believes that each aspect should be as realistic as possible is considered to have realistic beliefs toward WP, while on contrary, the one who does not, is considered to have non-realistic beliefs.

<table>
<thead>
<tr>
<th>Aspect</th>
<th>Sub-aspects</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Event</td>
<td>A1. Event</td>
<td>The event described in the school task has taken place or has a fair chance of taking place</td>
</tr>
<tr>
<td>B. Question</td>
<td>B1. Question</td>
<td>The question being one that might be posed in the real-life event described is a pre-requisite for a corresponding real-life situation to exist. (The question would be asked in the described event</td>
</tr>
<tr>
<td>C. Information/data</td>
<td>C1. Existence</td>
<td>The information given should be available in the real-life context.</td>
</tr>
<tr>
<td></td>
<td>C2. Realism</td>
<td>The value presented in the WP should be reasonable or should be very close to the value in a real-life context</td>
</tr>
<tr>
<td></td>
<td>C3. Specificity</td>
<td>The context given in the WP should be specific with regards to students' life</td>
</tr>
<tr>
<td>D. Presentation</td>
<td>D1. Mode</td>
<td>The WP presented written, visually, or orally should be considered</td>
</tr>
<tr>
<td></td>
<td>D2. Language</td>
<td>Difficult terms, sentence structure, and amount of text should be considered</td>
</tr>
<tr>
<td>E. Solution Strategies</td>
<td>E1. Availability</td>
<td>The role of students in the story presented in WP should be clear</td>
</tr>
<tr>
<td></td>
<td>E2. Experienced Plausibility</td>
<td>Strategy experienced is plausible for both school situation and real-life situation</td>
</tr>
<tr>
<td>F. Circumstances</td>
<td>F1. Availability of external tools</td>
<td>Availability of external tools (for example, calculator or software) should be considered</td>
</tr>
<tr>
<td></td>
<td>F2. Guidance</td>
<td>Implicit or explicit hints in solving WP should be considered</td>
</tr>
</tbody>
</table>
F3. Consultation and collaboration
Whether WP should be solved individually or collaboratively should be considered

F4. Discussion opportunities
Opportunity to discuss the meaning of WP should be considered

F5. Time
Time restrictions should be made such that there is no significant difference between solving the WP in school and real life context.

F6. Consequences
Pressure and motivation created by how close the real-life context presented in the WP to the solver should be considered.

G. Solution requirements
The validity of the student's answer should be interpreted broadly as close as possible to the real-life context

H. Purpose

H1. Purpose in the figurative context
The purpose of finding the answer in WP should be made as clear as in the real-life context.

H2. Purpose in the social context
Whether the WP serves as a real social purpose in a real-life context should be considered (not merely math problem 'dressed up' in real-life context)

Table 1: The framework of word problem reality aspects by Palm (2006)

METHOD
The study employed both quantitative and qualitative analysis. The quantitative analysis was done descriptively, while qualitative analysis was done through excerpt coding. The following sections explain the instruments, data collection, and data analysis of the study.

Instruments
The study used two instruments for data collection, i.e., a survey of beliefs toward WP realism (English translation is provided in Annex 1) and interview guidelines. The survey was developed to measure prospective teachers' beliefs. On the survey, participants were asked to complete each item on a four-point-scale: strongly agree, agree, disagree, and strongly disagree. The 36-item-survey was constructed based on Palm's framework, as shown in Table 1.

Each sub-aspect was manifested into two survey items, a positive statement, and a negative statement. Each sub-aspect's positive item contained a statement showing positive beliefs toward
the sub-aspect while the negative item contained negative beliefs. For example, the positive item for the sub-aspect 'Event' is "The event described in the word problem should have taken place or have a fair chance of taking place in real life." In contrast, the corresponding negative item is "The event described in the word problem does not necessarily happen in real life." On positive items, the greater scale shows greater beliefs in word problems' realism, while on negative items, the greater scale shows greater beliefs in non-realism. For instance, participant responding 'strongly agree' in positive items is most likely having realistic beliefs toward word problems.

**Lift Problem**

This is the sign in a lift at an office block:

This lift can carry up to 14 people.

In the morning rush, 269 people want to go up in this lift. How many times must it go up?

**Bakery Problem**

In a bakery, you see a 20 cm long cylinder-shaped Swiss roll. A dissection straight through the cake produces a circular shape with a diameter of 7 cm. The points of time in a day when the Swiss rolls are all sold are normally distributed with mean of 5.30 p.m. and a standard deviation of 15 minutes.

What is the volume of the Swiss roll?

What is the probability that the Swiss rolls are all sold before 6.00 p.m. when the bakery closes?

**Figure 1**: Problems for the interview, adapted from Palm (2006)

The content validity of the survey was checked and revised by two mathematics education experts. The construct validity was checked by conducting a statistical test correlating each item score to the total score (all items are valid with value < .05). The reliability of the survey revealed the Alpha-Cronbach coefficient of 0.773, which is considered reliable.

The interview guideline was divided into two parts. The first one is the two-word problems adapted from Palm (2006), as given in Figure 1. The second one is the list of questions about those problems and were constructed for the study needs. The guiding questions were (but not limited to): (1) describe your thinking about the problem, (2) have you made realistic considerations about the problem? Explain your reason, (3) if you were to modify the problem, how would that be? (4) what are the things you consider in giving or creating a word problem, and why do you believe so?
Data Collection

An online survey was distributed to 28 Mathematics prospective teachers of the first to the fourth year in one university in Jakarta, Indonesia. The survey was created using an online survey platform, and its link was distributed through university email. The survey link was made available for two months and only received one response for each participant. Participants' responses were recorded automatically by the platform feature.

To further clarify participants' stance toward word problems, a semi-structured interview was conducted. Six prospective teachers were being interviewed, two from each beliefs category, i.e., realistic, neutral, and non-realistic. Each participant was neither informed about the word problems (as shown in Figure 2) nor about the list of questions prior to the interview to allow for original responses. Each participant was also notified separately upon the interview’s schedule and was interviewed individually to minimize the possibility of sharing information. The interview process was started by giving the participants time to read the first problem and solve it if they wanted to. After that, participants were asked the guideline questions related to their beliefs in the problem. This process continued similarly for the second word problem. All interview processes were recorded and transcribed for analysis.

Data Analysis

Prospective teachers' beliefs about the realism of word problems in Mathematics teaching was analyzed from Mathematics education students' responses to the survey and interviews. Participants' responses to the survey were converted to numerical values. For positive items, respond of "strongly disagree", "disagree", "agree" and "strongly agree" were converted to a score of 1, 2, 3, and 4 consecutively. On the other hand, for negative items, respond of those were converted to a score of 4, 3, 2, and 1 consecutively. The score of 3 and 4 indicated a tendency to realistic beliefs toward word problems. For each participant, the number of responses scoring 3 or 4 was counted. Then, the proportion of responses to all items was calculated by dividing the number of responses scoring 3 or 4 by 36. The illustration of this process was given in Figure 2.

A proportion greater than 0.5 inferred realistic beliefs and a proportion smaller than 0.5 inferred non-realistic beliefs. The exact proportion of 0.5 was also considered, which later was inferred as neutral beliefs. The same calculation process was done to responses in each aspect of the framework in Table 1.

<table>
<thead>
<tr>
<th>Participant A</th>
<th>Response</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 1 (positive item)</td>
<td>Agree</td>
<td>3</td>
</tr>
</tbody>
</table>
Interview transcripts were coded based on the aspect being identified and analyzed further to explain each type of prospective teachers' beliefs. Statements indicating opinions on a certain aspect were coded as given in Table 2. The opinion was coded referring to the description of each aspect (Table 1) inferred and then written into memos. For instance, an agreement or disagreement to numerical value presented in the word problem would be included to “information” aspect (code C), especially in “realism” sub-aspect; while an opinion to modify the question due to its impossible event to happen would be included to “event” aspect (code A). The code-based statements were collected and then categorized based on similarity of beliefs presented within them. The analysis from the interviews was to enrich the findings obtained from survey.

<table>
<thead>
<tr>
<th>Aspect</th>
<th>Code</th>
<th>Aspect</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event</td>
<td>A</td>
<td>Solution Strategies</td>
<td>E</td>
</tr>
<tr>
<td>Question</td>
<td>B</td>
<td>Circumstances</td>
<td>F</td>
</tr>
<tr>
<td>Information/ data</td>
<td>C</td>
<td>Solution requirements</td>
<td>G</td>
</tr>
<tr>
<td>Presentation</td>
<td>D</td>
<td>Purpose</td>
<td>H</td>
</tr>
</tbody>
</table>

Table 1: Coding for analyzing excerpts

**RESULTS**

Indicated by the proportion of realistic answers given by all prospective teachers answers on the survey, there was an overall trend of realistic beliefs toward word problems \((n = 19)\), while the rest of them showed non-realistic \((n = 7)\) and neutral beliefs \((n = 2)\). For example, 78.5% of prospective teachers responded "agree" or "strongly agree" to the item "The event described in the word problem should possibly take place or have a fair chance of taking place in real life." Similar
responses also emerged from the item "The value presented in the WP should be reasonable or should be very close to the value in real-life context," which yielded 96% of prospective teachers.

| Aspect               | % of participants |  |
|----------------------|-------------------|--|   |
|                      | Realism           | Non-Realism   | Neutral |
| Event                | 60.7              | 14.3          | 25.0    |
| Question             | 32.1              | 32.1          | 35.7    |
| Information          | 46.4              | 25.0          | 28.6    |
| Presentation         | 25.0              | 25.0          | 50.0    |
| Solution Strategies  | 64.3              | 14.3          | 21.4    |
| Circumstances        | 21.4              | 53.6          | 25.0    |
| Solution Requirements| 57.1              | 10.7          | 32.1    |
| Purpose              | 21.4              | 39.3          | 39.3    |

Table 2: Beliefs in each aspect of WP realism

Prospective teachers' responses were broken down into each aspect, and it was revealed that realism beliefs were not that prevalent (as shown in Table 2). More prospective teachers responded with realism beliefs in 'event', 'information', 'solution strategies', and 'solution requirements' aspects, while more of them held non-realism beliefs in the 'circumstances' aspect. More prospective teachers gave neutral beliefs in the 'question' aspect, but with more even distribution with other categories of beliefs. In the 'purpose' and 'presentation' aspect, non-realism and neutral beliefs were held most. To further understand prospective teachers' beliefs, an interview was conducted with two prospective teachers in each category of realistic, non-realistic, and neutral beliefs inferred from the survey.

Realistic Beliefs

Realistic beliefs revealed from prospective teachers' responses during interviews related to their views on the two-word problems. Their responses in the interview confirmed their stance inferred from the survey. Two prospective teachers with realistic beliefs tended to see word problems as close as possible to reality. One of the notable responses came from participant A. Her response related to the realism aspect of value presented in the lift problem was shown below.

*I think it is not realistic in the real world for two hundred sixty-nine people for one lift. You know, like there are, in the (our campus), there are six lifts for not more than 100 people, I think in one time.*
This response showed that she believed the value presented in the WP should be as close as possible to the value that existed in real life. After being asked her opinion regarding the possibility that the "real value" may make students struggle with the numbers instead of the problem itself, she emphasized that the case would actually be a good thing as it would lead to a fruitful discussion involving analytical thinking.

Another prospective teacher, participant B, stated an interesting view on how close a value should be to the real value with realistic beliefs. Participant B stated that she would choose the number of people for a struggling student that will lead to an integer number for the answer as it will have a more straightforward calculation. However, she stated the following related to a more advanced student.

_For students who have achieved the standard or more, I will give this problem so they will be more challenged. Yes, the activity in it is the reflection of daily life, and students can use their learning to do a real calculation that is more complex._

Participant B acknowledged that the real value might lead to a more complex calculation, making students feel more challenged. Regarding the mode and language used in presenting the problem, the two prospective teachers with realistic beliefs showed a preference for a more familiar word than a technical one for the bakery problem. The following is the excerpt of participant A's opinion on the word "dissection" in the bakery problem.

_The "dissection straight." I think "dissection straight" is not a familiar word. I think just that._

Participant A found the word to be confusing as it was not a familiar word for students. She further stated that the word might hamper students from imagining what was being asked in the problem. In this case, participant A showed a tendency to consider students' preference for words. Although not mentioning the same word, Participant B suggested removing the word "cylinder-shaped" and preferred giving students the picture of Swiss roll instead, as written in the excerpt below.

_But I will delete the word 'cylinder-shaped. The reason is that this word makes the problem wordy, and maybe students who actually can solve the problem but not familiar with the word may be distracted. Besides, if the picture of Swiss roll is provided, there is no need for the additional word._

Participant B, like participant A, consider the efficiency of words presented in the problem. The highlight of both participants' opinions is their reason for making the word portrays the real situation and help students 'imagine' the situation and 'not distracted' by the word instead.

The prospective teacher also stated their opinions on the question of the bakery problem. Participant A thought that there is no real intention of asking the volume of bread. Her suggestion to modify the question is given in the following excerpt.
What about giving information on how many productions made each day? Without giving diameter information. So, how much flour will be needed for one Swiss roll? After this, give the information of diameter, then it can be connected to asking the weight of Swiss roll.

Her suggestion showed a preference for putting the question as close as possible to real context. She wondered about the purpose of asking the volume of bread and thus asking the amount of flour needed will be more relevant. Participant B gave an opinion that it is not common to use bread as the context of finding volume. She stated as follows.

Interesting. It is my first time seeing a question like this. Usually, the concept of volume is introduced using water, sand, or other objects whose shape can change, adjusting its container.

Both prospective teachers did not agree to give the original questions to students. Participant B still deliberately allowed the question to be given to upper-level school but preferred to modify them. Participant A emphasized this case quite often, as illustrated in the interview record below.

Interviewer: if this one is an unrealistic problem, do you think it is okay to give students unrealistic problems?

Participant A: I think no

Interviewer: Why give students an unrealistic problem is not okay?

Participant A: Because I have been there as a student, and I was really confused about the questions because I just like this question, I mean like who give one lift for 269 people, I was like Oh My God, I think it would be beneficial for the students if they see the real world and of course, okay, real-world is there, but mathematics is also there. So, for example, NASA makes a rocket plane to outer space, so we can imagine the real making of a rocket plane is like this, so everything is not only an imagination.

Participant A shows her beliefs by stating her personal reflection as a student, and she emphasized the importance of seeing Mathematics not separated from the world at all. In another part of the interview, she repeated her stance as follows.

In my opinion, if we can give the realistic (problem), why not giving it?

Both prospective teachers with realistic beliefs showed their realistic view toward word problems by suggesting a more realistic version of the given word problems by modifying the information, question, or language and mode of presenting the problem. Their further stance also includes avoiding giving an unrealistic problem to students whenever possible.

Non-realistic Beliefs

Non-realistic beliefs were shown through two prospective teachers’ opinions on the realism of value presented in the Lift problem. Participant C preferred to give a value resulting in an integer
answer after being divided by 14. Her consideration on whether a problem with a non-integer answer could be given was due to students' level. Her opinion is given in the following interview record.

**Participant C**: After calculating, 269 divided by 14 is 18 point something, while 266/14 is 19. But again, I think where this problem context is located? If it is in senior high school, I think the first one is okay because they (students) will know decimal, but for junior high school, I think I will choose the second value.

**Interviewer**: Do you think the value of 266 is realistic?

**Participant C**: I think it is not. We don't really know how many people will be in the lift, right? I think 269 is more realistic, but 266 is more comfortable to solve.

The interview noted that Participant C chose a word problem based on its relevance to the students' grade level. Her consideration was heavier on calculation skills needed to solve the problem rather than how realistic the value presented there. Related to the Bakery problem, Participant A chose to have the volume question modified into the weight question in which the weight in each volume unit is known. However, her reason was not that the weight was more reasonable to be asked, but due to its relevance with high school students' skills. She also mentioned her view on the picture given in the question, as shown below.

*If needed, we can give images based on the context of the problem. I have once found an image in a problem, but they are not related. The image is just there but not related to the problem.*

Further, her general view on whether a realistic problem should be given to students was written below.

**Interviewer**: So, do you agree that word problems should be realistic?

**Participant C**: I don't really agree that word problems should always be realistic.

**Interviewer**: If the word problem is not realistic, do you think you will allow it to be given to students?

**Participant C**: Yes, as long as appropriate with the grade and learning objective.

Participant C believed that word problem does not have to be realistic as long as it is appropriate with students' level and the learning objective. She further emphasized that her priority in choosing word problems is their alignment with the learning objective. A similar belief was presented by participant D, who stated that some things cannot always be realistic. Her interesting thought is written below.

**Interviewer**: When we create a word problem, should it be realistic?
Participant D: Hmm, but don’t you think some things cannot be made realistic? It is good if it is realistic, but some things cannot be made realistic. For example, in geometry, the perfect square is the idea in our mind. In real life, we rarely find it.

She said that a realistic word problem is good, but she acknowledged that some things could not be made realistic. She thought that it would be helpful if the word problem were realistic, but due to the reason she mentioned before, she stated that as long as the problem can be imagined, that will be okay. Regarding the standard deviation information in the Bakery problem, participant D thought there is a rare possibility of its availability. However, despite thinking that, she stated as follows.

I think it is okay, but the teacher must explain it first, maybe if the students ask. For example, students read (the question) maybe they understand some words, but not understand the others. Maybe if the students get confused, the teacher can give clues.

Participant D showed affirmation of giving a word problem despite it being unrealistic in terms of its information. She further stated that the teacher might give students clues if they did not understand the information.

Neutral Beliefs

Some exciting opinions came from participants with neutral beliefs (with an answer proportion of 0.5). The interesting opinions differ by what they believe as 'acceptable' real-life situations. Participant E stated that what is considered real by some students may not be real for others. His opinion is stated below.

First, because this is a problem, I will see the students' grades. Is this for primary or secondary school students? This is about how realistic, right? The environments surrounding primary and secondary students must be different.

His thought revealed that what is considered realistic by a certain level of students might be unrealistic to others, because they might have different environments. He further explained that some WPs might be realistic for scientists but not for students.

Word problem is used to make students understand mathematics, but we know that many word problems are outside our daily lives, and I think closer to scientists' or academicians' lives. We may understand the context, but we don’t find it in our real-life.

Participant E thought that the concept of realism in life might be different from it in the context of science, including mathematics. On the other hand, participant F had an opinion on what is important in deciding if WP is okay to be given to students or not. She highlighted that if the problem could help students learn doing the procedure, it was fine. She exemplified this by telling her experience below.
Like a problem given to my sister yesterday, 'there is a kid who buys three buckets of fish, and there are 35 fish in each bucket. Then, it was asked how many fish the kid had. That is not realistic, right? If we think about it, it is not realistic for a kid to buy three buckets of fish, and moreover, there are 35 fish in each. But it is okay; I think because the purpose is to learn multiplication."

In her example, she mentioned that because the problem helped students learn multiplication, it was acceptable to be given, even with unrealistic values. Like Participant E, she showed beliefs on two versions of "realistic", realistic in life and mathematics. She explained that what is not realistic in life may be acceptable in the context of mathematics learning.

Because if we bring realism to real-life, for example, we go to a bakery and are asked to determine the cake’s weight, that is not realistic. But, in the context of mathematics, I think that is (short break) that is acceptable. Yes, that is calculating.

The notion of neutral beliefs shows the relativity of what participants perceived as "realistic". They may not have the exact definition of a realistic word problem, but they would consider the learning's cultural and developmental setting.

**DISCUSSION**

The findings of this research revealed some distinctions among the three types of beliefs possessed by prospective teachers. The main points are mainly about their definition of realistic WP and whether they will give realistic WP to students. Participants with realistic beliefs show a strong emphasis on making the word problem as closely as possible to students' lives. They believe that a realistic word problem should be manifested into the WP elements: the information, questions, and mode of presentation. They believe that the information should possess data, value, or facts available in real life with an accurate value as possible.

The question given in the WP should also be a question possible to be asked in real life. The mode of presentation, they believe, should avoid any ambiguity, and portray the real-life situation as straightforward as possible. This finding is similar to the beliefs found in the study by Chen et al. (2011). In the study, participants thought it is not realistic to measure the ability to run 1 kilometer far using an ability to run 100 meters. The participants thought that the problem should be formulated in an exact, unambiguous way.

A strong objection in giving unrealistic problems came from the participants with realistic beliefs. They believe that the consequence of making calculation more complicated by having close-to-real-life information will positively contribute to the discussion and thinking process during the learning process. This view has been inferred in a study stating that word problems should support students to acknowledge that mathematics learning is closely related to their life (Verschaffel, et al. 2001).
Participants with non-realistic beliefs have more practical consideration on whether a particular WP can be given to students. They mainly consider students' level of skill to do the needed calculation or procedure. Participants with these beliefs are found to prioritize less, if not disregard, the relevance of WP questions to real life. Most importantly, their beliefs show the stance of a good WP is the one that can be imagined; it does not have to fully simulate real life, if it is mathematically "acceptable". The beliefs are pretty interesting, as it resonates with what Galbraith and Stillman (2001) stated as a 'context-separable problem', in which a real-world context exists but is not used to solve the problem. Participants with these beliefs see the context-separable problem as an acceptable problem to be given to students.

Participants with neutral beliefs tend to question to whom the reality matters. Their opinion lies on a relative degree of what is called real. They consider several aspects of WP's realism, such as students' level of understanding, age, experience, and cultural setting. This "grey" beliefs have been noted by De Lange (1995), who stated that there might not be an exact way of determining the proximity of real-life to WP. This may be due to students' varied experiences and backgrounds, so, understandably, some prospective teachers believe it that way.

CONCLUSIONS

This study has described several points differentiating three types of beliefs possessed by prospective mathematics teachers toward WP: realistic, non-realistic, and neutral. By understanding the characteristics of each beliefs type, it is hoped that more effort can be made in promoting meaningful use of word problems to prospective teachers. Each beliefs’ characteristics can also contribute to the broader understanding of teachers’ beliefs, not only in word problem posing but also in the teaching practices in general. Besides, beliefs have been one of the essential factors in driving teachers' practices; thus, this study will provide ample insights into addressing the issue from an affective factor.

Due to the small sample used in this study, the findings cannot be generalized. Still, it provides valuable descriptions of prospective teachers' s toward WP's realism and its types of distinction. Further research can be done by investigating whether prospective teachers' beliefs are affected by their lecturers or whether the possessed beliefs are put into practice or compromised instead.

REFERENCES


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Annex 1. The items of questionnaire of beliefs towards word problem
(This is the English translation. The original questionnaire was given in Indonesian language.)

1. The event described in the word problem has taken place or has a fair chance of taking place.
2. Value presented in the word problem should make sense or close to the actual value in real life.
3. Difficult terms, sentence structure, and amount of text should not be avoided for the word problem creator.
4. The use of external tools (such as calculator and software) in solving word problems should be allowed.
5. Teachers should consider whether solving a particular WP will need a discussion based on the real-life context.
6. Teachers should accept answers that use real assumptions of the context presented in WP (for example, not impossible for someone to carry 80 watermelons, etc.).
7. The events described in WP don't have to be really possible in the real world.
8. The values used in WP do not necessarily correspond to real-world values.
9. WP makers as much as possible simplify or reduce terms, complex sentence structures, and sentence length in WP.
10. Cannot complete WP with tools (such as calculators or software).
11. Each WP should be able to be done without any opportunity for discussion about the different meanings.
12. When there is a question with the sentence 'Ani can make 100 statues in 1 hour', we can ignore the assumption that this sentence cannot happen in real life.
13. Questions like, “What is the volume of the loaf?” should not be given to students because in real life no one really needs to know the volume of a loaf.
14. The context presented in the WP should be specific based on the real life of the students who were given question.
15. Students should be able to position their roles in the context of the WP, not merely solving problems as people outside the story.
16. WP solution instructions (such as 'start by calculating the total cost') should not be provided as such guidance would not be available in real-life problems.
17. The WP completion deadline should consider the time to solve similar real-life problems.
18. The purpose of finding answers to a WP should be made as clear as possible like finding answers to everyday problems.
19. It is okay for a WP to ask things that you don't really need to know in the real world, for example, questions about the volume of a cylindrical cake.
20. The context in WP does not have to be based on the real life of students who are given questions.
21. Solving story problems with a point of view outside the context of the problem is fine.
22. It's okay to give hints (hints, clues) on how to work on WP.
23. The WP completion time limit can be determined without considering whether the same problem in the real world can be solved within that time limit.
24. The purpose of finding WP answers is okay only to solve math problems, even though it has little relevance to real life.
25. The information given to WP should actually exist in real life. Information about the standard deviation, for example, should not be given but calculated first.
26. The word problem, as the name suggests, must be given in the form of words.
27. Strategies for completing WP should be possible for students both in school situations and in their daily lives.
28. Whether a WP can be done individually or in groups based on its real-life context should be considered.
29. Consequences of completing WP should be made as close as possible to the real-life of students (eg by presenting WP answers about a policy to local officials, etc.).
30. WP is a daily life problem that requires mathematics to solve.
31. In WP, it's okay to give information about a value that doesn't actually exist in the real world, for example, information about the average deviation.
32. WP does not have to be given in words.
33. The strategies needed to solve story problems may only be completed at school, not in everyday life, because at school students receive the support of assistive devices.
34. Each WP should be done individually by students, regardless of the actual context.
35. WP's answer is quite related to the real context by discussing in class.
36. WP is a math problem wrapped in the context of everyday life.