Improving Mathematics Lecturers’ Content Knowledge through Lesson Study

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Abstract: This qualitative case study was conducted in a foundation center in a public university in Malaysia during the academic year 2018-2019. The purpose of this study was to investigate the impact of Lesson Study on mathematics lecturers’ content knowledge. The Lesson Study group comprises of seven mathematics lecturers, a physic lecturer, and the researcher. The group collaboratively discussed and designed five Research Lessons on the topic mathematics Function that was identified as one of the problematic topics that students face. Data gathered through interviews, mathematics tests, and observations on the lecturers’ activities in meetings and discussions. Data were analyzed descriptively. The results of this study show lecturers improved their content knowledge considerably through collaborative work and sharing of experiences on this topic. They learned to create new problems, discuss students’ misunderstanding of different subtopics, improve understanding of applications of subtopics, and design suitable Research Lessons based on problem-solving methods.

Keywords: Lesson Study, Mathematics Problem, Content Knowledge, Lecturers’ Knowledge

1. Introduction

A lecturer with a limited grasp of mathematics content knowledge has little room for progress or novelty in the classroom or the ability to fuel students’ interest in the subject. Mathematics content knowledge is a key component of teachers’ knowledge which aids them in improving the contents that they teach or to suit their teaching methods to the contents. It seems that mathematics educators with the low level of content knowledge have greater resistance in adopting a new...
method of teaching, such as problem solving approach because such approach requires the teachers to guide, assess and confirm different solutions given by students. Shulman (1986) categorized teacher knowledge and attended to the role of content in teaching as a special kind of technical knowledge, which is the key to the profession of teaching. Content knowledge shows teachers’ understanding of the subject matter taught. According to Shulman (1986), “the teacher need not only understand that something is so, but the teacher must also further understand why it is so?” (p. 9). Thus, the emphasis is on a deep understanding of the subject matter taught at school. There seems to be an agreement that teachers need to have a deep understanding and knowledge of mathematics they teach, and they need to be able to use this knowledge in their teaching. Teachers need to know how their teaching material is connected to other topics, both prior to and beyond their education (Conference Board of the Mathematical Sciences, 2012; NCTM, 2000).

Teachers’ content knowledge is related to the mathematical quality of their instructions and teaching styles (Baumert et al. 2010; Hill & Ball, 2009; Shulman, 1987). Similarly, Norton (2018) explained that knowledge of mathematics content to be taught is a key component in influencing the quality of teaching; furthermore, teachers who know more mathematics are likely to teach it better. The main goal of the current study was to investigate the impact of Lesson Study on mathematics lecturers’ content knowledge in foundation level. The researchers were studying the impact of Lesson Study on common content knowledge (CCK) and specialized content knowledge (SCK) of mathematics lecturers on the topic of mathematics function. The specific research questions of this study are as follows:

1. What is the impact of Lesson Study on CCK of lecturers?
2. What is the impact of Lesson Study on SCK of lecturers?

2. Literature Review

Lesson Study as a model of mathematics teachers’ professional development has been used in Japan since the 1950’s (Abiko, 2011). This method emphasizes on problem solving based on cognitivism learning theory. In this approach, teachers need a high level of content knowledge because they need to discuss, improve, create, and confirm the solutions given among teachers and students. Two Japanese words, “Jugyo” and “Kenkyu” means lesson and study, respectively thus the term “Jugyo Kenkyu” which was translated to Lesson Study by Yoshida in 1999 (Kazemi, Zamar, & Ghafar, 2014) in his doctoral dissertation (Doig & Groves, 2011). In 2011, Lesson Study was formally introduced in the Malaysian education system by the Ministry of Education through the Professional Learning Community (PLC) (Zanaton & Marziah, 2017). As a result, many others tried it out. One such example was the implementation of Lesson Study, which was carried out in 42 secondary schools in one of the Malaysian states to improve the quality of teaching (Matanluk, Johari, & Matanluk, 2013).

Teachers’ content knowledge is a fundamental component of their teaching effectiveness. Shulman (1986) defined content knowledge as the teacher’s knowledge of mathematics subjects matter taught. Ball et al. (2008) further extended the concept of content knowledge into two; CCK and SCK. They defined CCK “as the mathematical knowledge and skill used in settings other than
teaching” (p. 399). The CCK, the knowledge requires solving problems correctly, and it is employed in various situations, not only in teaching. Teachers apply their CCK using terms and notations in mathematics, and calculations. When teachers don’t have a sufficient amount of knowledge in CCK, as a result, both instructions and precious time are lost (Ball et al., 2008). This kind of knowledge assesses mathematical knowledge that is not essentially related to teaching. For example, teachers should be able to solve the problem “Prove that the number of subsets of the set $A = \{x_1, x_2, \ldots, x_n\}$ is $2^n$” through different ways. The second category of content knowledge is the special content knowledge (SCK), the content knowledge is unique to teaching. Teachers apply this domain of knowledge to analyze students’ errors, their misunderstandings, provide creative solutions, pertain to different subtopics, and they explain, and justify reasons underlying mathematical procedures and algorithms. Teachers also use SCK in explaining mathematics in such a way, not required in other situations, to facilitate and encourage students’ learning (Ball et al., 2008). In the other words, SCK evaluates the mathematical knowledge used in teaching, but students are not directly taught. For instance, for the problem “if $x_1 + x_2 = 1, x_1 + x_3 = -38$ and $x_2 + x_3 = -15$ then find the values of $x_1 + x_2 + x_3$ and $x_1 + 2x_2 + 3x_3$.” The non-routine solutions $x_1 + x_2 + x_3 = \frac{1}{2} (x_1 + x_2 + x_1 + x_3 + x_2 + x_3)$ = \frac{1}{2} (1 - 38 - 15) = -26 and $x_1 + 2x_2 + 3x_3 = x_1 + x_3 + 2(x_2 + x_3) = -38 + 2(-15) = -68$ by teachers considered as SCK of them.

According to NCTM (2000), a task is regarded as a problem if the learner is engaged with the given task for the first time and the task provides a challenge to the learner. Otherwise, the task is merely an exercise. So problem solving refers to engaging learners in a task that they have not learned to solve before. Furthermore, open-ended problems depend on the ability of the problem solvers (Asami-Johansson, 2015). Therefore, the distinction between mathematics problem and mathematics exercise depends on many factors such as the education system and time it is given. For instance, the following problem after being discussed in the Lesson Study meeting becomes just an exercise for the teachers.

Problem: Evaluate $A = \int_0^\pi \frac{\tan x}{\tan x + \cos x} \, dx$.

When this problem is changed slightly, we can consider another problem such as the following:

Problem: If $A = \int_0^\pi \frac{\sin x}{\sin x + \cos x} \, dx$ and $B = \int_0^\pi \frac{\cos x}{\sin x + \cos x} \, dx$ find the value of $2A - B$.

In this research, every problem posed is related to students’ everyday life and other subjects such as physics and chemistry, which is considered as practical problem. For example, the following problem is considered a practical problem.

Problem: A closed cylindrical can is to hold one liter (1000 cm$^3$) of liquid. How should we choose the height and radius to minimize the amount of material needed to manufacture the can?

2.1. Lesson Study

Lesson Study is a concept of teaching that requires a group of mathematics teachers to work collaboratively on a topic and spend a lot of time in planning a lesson, teaching or/and observing the lesson. The post-lesson requires the teachers to reflect and discuss on the lesson that was taught
in order to improve student’s achievement in mathematics learning and problem-solving through effective teaching (Matanuk et al., 2013), which helps improve their mathematics knowledge. These lessons with Japanese term *gakushushido-an*, are called Research Lessons (Hemmings, Groopenbore, & Kay, 2011). Lesson Study points to a cycle of pedagogical progress, of which the Research Lesson is the key part (Lewis 2002). Many types of research have shown the usefulness of Lesson Study for mathematics teachers especially in improving their content knowledge (Coenders & Vehoeft, 2019; Fernandez & Yoshida, 2004; Fuji, 2016; Harsono, 2016; Mon, Dali, & Sam, 2016; Takahashi, 2006). Fujii (2014) suggested five phases for Lesson Study as follows:

a. Goal Setting: In this phase, teachers focus on long-term goals in order to improve the students learning and problem solving.

b. Lesson Planning: Teachers collaboratively plan to find suitable mathematics materials to improve students’ abilities, such as students’ higher order thinking and problem-solving skills.

c. Research Lesson: They design a suitable Research Lesson, and one member of the Lesson Study group teaches the Research Lesson, and other members observe and collect data in order to improve the Research Lesson.

d. Post-lesson Discussion: Through post-lesson discussion, teachers consider students learning, students’ misunderstanding, unit design, and disciplinary content to enrich the Research Lesson.

e. Reflection: Teachers discuss the new questions about the Research Lesson, and they collaboratively plan to solve these problems in the next cycle of Lesson Study. Also, in this phase, they prepare a report about the Research Lesson.

The Lesson Study methodology is framed within the paradigm of Action Research that in contrast, focuses on a team-oriented instructional design and shared responsibility for the instructional work and outcome (Elliot, 2015, 2019). Lesson Study is based on constructivism. Constructivism from a social constructivist perspective helps create a framework that assists the use of Lesson Study as a potential method for enhancing the proficiency of teachers. The application of constructivist principles allows the lecturers to have better perspectives about mathematics lessons and helps improve their abilities in problem solving. Hoover (1996) explained that constructivism provides a guide to the teachers on how to help students construct knowledge instead of merely transferring knowledge from the teacher to the students. Hoover further explained that teachers who align their teaching based on constructivist principles make use of the learners’ prior knowledge in providing learning environments that minimize inconsistencies between the learners’ existing knowledge and the new knowledge to be constructed. Hoover had also emphasized the need for teachers to engage students in learning and to get them involved actively in the learning process.

Social constructivism contributes to the social form of knowledge and the notion that knowledge/expertise is formed via not only through individual experience but also through the interaction and sharing among people (Gergen, 1995). This puts an emphasis on the idea that
knowledge created in response to social interactions like negotiations, reflection, discourse, and explanations.

Thus, social constructivism underlies the interaction that takes place when teachers sit together in planning the Research Lesson. During the initial phase of the Lesson Study, the Lesson Study groups identify a goal statement that describes qualities they would like to develop in their students. Together with the researcher, the novice and newer lecturers work together to develop a Research Lesson that may benefit students the most. This process of improving the Research Lesson continues with post-lesson discussions.

Another principle of social constructivism, which is the adaptive function, highlights how learners organize their experiences (Fleury, 1998). In the context of Lesson Study, confrontation with problems or discrepant events motivates the teachers to seek, test, and assess answers within socially collaborative environments. The continuous process in developing Research Lessons provide vast opportunities for teachers to reflect, analyze, create action steps, evaluate, and share their experiences with other educators.

The principles of social constructivism lie beneath Lesson Study and validate why each stage of the process for Lesson Study is crucial in enhancing professional knowledge and skills of the teachers.

3. Methods

3.1. Participants

This case study was conducted in a foundation center of a Malaysian public university during the first semester of the academic year 2018-19. Unlike schools, lecturers in foundation centers have at least Master’s degrees. In terms of the lecturers’ knowledge and competencies, it is more or less the same in all the foundation centers of the Malaysian universities. This foundation center was selected because it provides a conducive and supportive place for the Lesson Study to be implemented and the wide choices of classes to choose from. In this center, there were nine mathematics lecturers (four males and five females). Seven mathematics lecturers and a physics lecturer voluntarily agreed to participate in this study thus, the Lesson Study group for this study involved seven mathematics lecturers, a physics lecturer, and the researcher. The Physics lecturer provided practical problems for the Research Lessons since she knew what mathematics understanding need to be applied in studying physics concepts. Meanwhile, the researcher played several roles such as the coordinator, discussion leader, and Lesson Study group member. Once, permission was obtained from the Director of the Foundation Center, all lecturers who were identified as participants of this study were asked to sign the consent letter. Table 1 shows the biodata of the study participants.

Table 1: Biodata of the Study Participants
3.2. Procedure
The researcher selected the topic on function since the lecturers identified it as a problematic topic for the students. The Lesson Study group then collaboratively planned, discussed, and designed five Research Lessons. Before the meeting on each research lesson, the researcher introduced the topic to the lecturers and asked them to provide materials, examples, experiences, applications and the suitable pedagogy to implement the contents. This was followed with a session for the Lesson Study group to produce the research lesson. During the second session, one of the lecturers taught the research lesson and other lecturers observed the teaching. Therefore, they further enhanced the quality of research lessons. During this study that spanned for seven weeks with 20 hours of discussions, data on lecturers’ content knowledge were collected before and after the implementation of the Lesson Study through interviews, mathematics tests, and observations of the lecturers’ activities in Lesson Study process. The constructivist grounded theory emphasizes pursuing qualitative research with an “open mind” and a close relationship with participants while the researcher is part of the study (Meyer & Ward, 2014). Therefore, before starting this study, the researcher discussed several times with the lecturers and students about the education system, teaching method, students’ abilities, and textbooks in order to better understand the situation of the foundation center and to create a good rapport with the participants of the study.

3.3. Instruments
For the instruments, the researcher prepared interview protocol which contains four questions related to the CCK and three questions related to the SCK and sent these questions to four mathematics education experts in Malaysia, Turkey, and North Cyprus to ascertain the validity and suitability of the questions. The researcher then pilot tested these interviews protocol with three other mathematics lecturers in order to ensure that the questions are clear. Some examples of the questions in the interviews are “Do you have any experience working collaboratively with your colleagues as a team to achieve a goal?” (to know the spirit of mathematics lecturers in collaboration work about mathematics education) and “How do you connect your lessons to “real world” experiences?” (to check the number of practical mathematics problems in their classes). The interviews were recorded using an audio recorder.
Tests were also developed to tap the content knowledge of the lecturers before and after the implementation and involvement in developing the Lesson Study. The tests contained 15 questions, of which seven questions measured the CCK and eight questions measured the SCK of the lecturers. These questions were confirmed by the four mathematics education experts in Malaysia, Turkey, and North Cyprus. Additionally, three mathematics experts of a public university in Malaysia verified the suitability of the tests and the equivalence of pre-test and post-test.

These mathematics tests were pilot tested on three mathematics lecturers of the same university. The questions include the following: “How many one-to-one functions can we define from \( A = \{4,2,3,5\} \) to \( B = \{6,8,1,7\} \)” and “If \( f(x) = x(x + 1)(x + 2) \ldots (x + 100) \), find \( f(0) \).” The final versions of these interview protocols and tests were confirmed by experts in the Ethical Committee of the Research Management Center (RMC) of the university.

3.4. Data Analysis
Through several sessions, lecturers planned, discussed and designed five Research Lessons collaboratively. During this period of time, they shared their content knowledge and experiences on the teaching of these five topics. Data gathered through observations were analyzed descriptively. The data gathered from interviews and tests were compared descriptively to observe differences between the onset and end of the study.

4. Findings
The findings of lecturers’ content knowledge are summarized through observations, tests and interviews. Different methods were applied to triangulate the findings on the lecturers’ content knowledge.

4.1. Lecturers’ Content Knowledge as Identified through Observations
Lecturers with suitable content knowledge can create appropriate problems in different topics. In a meeting, lecturers discussed about the range of the function \( f(x) = \frac{x^2}{1+x^2} \). They emphasized on the limitation of the inverse function method to find the domain \( f^{-1} \) as the range of the function \( f \) through the property \( f(f^{-1}(x)) = x \). The following are some of the remarks made by the lecturers:

“It’s not a one-to-one function thus this method is not practical” (Lecturer B).
By inequality \( 0 \leq x^2 < 1 + x^2 \) and each side divided by \( 1 + x^2 \) we have \( \frac{0}{1+x^2} \leq \frac{x^2}{1+x^2} < \frac{1+x^2}{1+x^2} \rightarrow 0 \leq f(x) < 1 \) thus \( R = [0, 1] \) (Lecturer A).
This is an ingeniously method for the solution. My question is what about the range of the function \( g(x) = \frac{x^2}{1-x^2} \)? (Lecturer D)
I found the new method for the range of the function \( f \) and explained that \( f(x) = \frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2} < 1 \). Also it is clear that \( \frac{x^2}{1+x^2} \geq 0 \) thus \( R = [0, 1] \) (Lecturer C).”

Upon listening to their thoughts, the researcher then suggested that the following:

“For the range of the function \( g \) suggested by Lecturer D, we can find \( x^2 \) and use this inequality \( x^2 \geq 0 \).

\[
y = \frac{x^2}{1-x^2} \rightarrow y - yx^2 = x^2 \rightarrow x^2 = \frac{y}{1+y}
\]

\[x^2 \geq 0 \rightarrow \frac{y}{1+y} \geq 0 \rightarrow R = ]-\infty, -1[ \cup [0, +\infty[\]

The researcher then asked the lecturers to create new problems related to this problem. Lecturer F mentioned the following:

“I teach exactly the same textbook’s materials in my classes, thus I cannot create new problem on my own because in foundation level, it is so difficult to create new problems for students.”

The researcher explained that every small change in the previous problem that is a challenge for students can be considered as a new problem. After this explanation, the lecturers suggested many problems related to this example such as the range of the functions

\[
h(x) = \frac{2x^2}{1+x^2}, \quad k(x) = \frac{3x^2+2}{1+x^2}, \quad m(x) = \frac{x^2}{x^4+1}, \quad n(x) = \frac{5x^2}{1+x^4}.
\]

In the textbook, even and odd functions are defined as follows:

A function \( f \) is said to be even if and only if \( f(-x) = f(x) \) for all \( x \).

A function \( f \) is said to be odd if and only if \( f(-x) = -f(x) \) for all \( x \).

The definitions of even and odd functions as provided in the textbook are not clear for lecturers. For example, in a meeting, with six lecturers the researcher asked them, “Is the function \( f(x) = x^2 \) with \(-3 \leq x \leq 2\) even or odd?” Three lecturers answered this is an even function because this function satisfies the condition \( f(-x) = f(x) \). Two lecturers answered it’s not even because the graph of this function is not symmetric with respect to the \( y \)-axis and a lecturer responded that the chapter only discusses about the functions with domain which are real numbers. After the discussion with the lecturers, the definitions of even and odd functions were changed as follows:

A function \( f \) is called an even (odd) function if this function satisfies in two conditions as the following:

a. Domain \( f \) is symmetric with respect to the point \((0, 0)\)

b. \( \forall x \in D_f, f(-x) = f(x) \) (\( \forall x \in D_f, f(-x) = -f(x) \))

For another example, some lecturers believed that the function \( f(x) = \sqrt{x} + \sqrt{-x} \) is even because this function satisfies the condition \( f(-x) = f(x) \) as \( f(-x) = \sqrt{-x} + \sqrt{x} = \sqrt{x} + \sqrt{-x} = f(x) \).
But for this function $D_f = \{0\}$ therefore, $f = \{(0,0)\}$ means that the function $f(x) = \sqrt{x} + \sqrt{-x}$ is both even and odd.

Sometimes lecturers see a mathematics problem from specific angle. For instance, during the discussion with lecturers about the question “How many functions which are both even and odd can we find?”, lecturers had different ideas and some lecturers argued:

"Since $f$ is even $f(-x) = f(x)$ also $f$ is odd $f(-x) = -f(x)$ therefore, 
$f(x) = -f(x) \rightarrow 2f(x) = 0 \rightarrow f(x) = 0$"

They explained that there is only one function that is both even and odd. Other lecturers considered different domains for the function $f(x) = 0$ and argued that many different functions can be found that are both even and odd such as the following function 
$f = \{(-4,0),(-2,0),(0,0),(2,0),(4,0)\}$.

In textbook, the following problem on even and odd functions is included.

Problem: Show that

a. The sum and difference between even functions are even.
b. The sum and difference between odd functions are odd.
c. The sum and difference between even and odd functions are neither even nor odd.
d. The product between even functions is even.
e. The product between odd functions is even.
f. The product between even and odd functions is odd.

Although it is a good problem, all parts are rejected by considering the example $f(x) = 0$ and $g(x) = 0$. In other words, all parts of this problem are incorrect for instance, for part (a), if we consider $f(x) = 0$ and $g(x) = 0$, then $(f + g)(x) = f(x) + g(x) = 0 + 0 = 0$, so the sum of two even functions is odd. Thus, during discussions with the lecturers, this problem was improved as follows:

Problem: Prove that

a. The sum and difference between non-zero even functions are even.
b. The sum and difference between non-zero odd functions are odd.
c. The sum and difference between non-zero even and non-zero odd functions are neither even nor odd.
d. The product between non-zero even functions is even.
e. The product between non-zero odd functions is even.
f. The product between non-zero even and non-zero odd functions are odd.

In textbook, the chapter on vector is discussed before the chapter on functions. The solution shown below is related to the application of vector properties in functions. Lecturers believed that the problem “Find the maximum and minimum values of the function $f(x) = \sin x + b\cos x$” is so
difficult to solve. They collaboratively discussed and learned a new technique to find the solution through linking between two different topics; vectors and trigonometric function. To get the solution, the lecturers found the dot product of two vectors \( v_1 = (a, b) \) and \( v_2 = (\sin x, \cos x) \) through two different ways as follows:

\[
v_{1}, v_{2} = (a, b), (\sin x, \cos x) = asin x + bcos x
\]

\[
v_{1}, v_{2} = |v_{1}| |v_{2}| \cos \theta = \sqrt{a^2 + b^2} \sqrt{\sin^2 x + \cos^2 x} \cos \theta = \sqrt{a^2 + b^2} \cos \theta -1 \leq \cos \theta \leq 1 \rightarrow -\sqrt{a^2 + b^2} \leq \sin x + bcos x \leq \sqrt{a^2 + b^2}.
\]

In the function \( f(x) = a \sin x + b \cos x \) if \( a = b = 1 \) they obtained the function \( g(x) = \sin x + \cos x \). The lecturers further discussed about the maximum and minimum of this function through different methods as shown below:

a. By using previous method the Max and Min values of the function \( g \) are \( \sqrt{2} \) and \( -\sqrt{2} \) respectively because

\[
-\sqrt{1^2 + 1^2} \leq \sin x + \cos x \leq \sqrt{1^2 + 1^2} \rightarrow -\sqrt{2} \leq \sin x + \cos x \leq \sqrt{2}.
\]

b. \( y = \sin x + \cos x \rightarrow y^2 = 1 + 2 \sin x \cos x = 1 + 2 \sin 2x = 0 \leq y^2 \leq 2 \rightarrow -\sqrt{2} \leq y \leq \sqrt{2} \)

c. \( y = \sin x + \cos x = \sqrt{2}(\frac{\sin^2 x + 1}{2} \cos x) = \sqrt{2}(\cos \frac{\pi}{4} \sin x + \sin \frac{\pi}{4} \cos x) \rightarrow y = \sqrt{2} \sin(x + \frac{\pi}{4}) \rightarrow -\sqrt{2} \leq y \leq \sqrt{2} \)

d. \( y = \sin x + \cos x \rightarrow y' = \cos x - \sin x = 0 \rightarrow \tan x = 1 = \tan \frac{\pi}{4} \)

\( x = k \pi + \frac{\pi}{4} \rightarrow x = \frac{5\pi}{4} \rightarrow \text{Max} = g \left( \frac{\pi}{4} \right) = \sqrt{2} \), \text{Min} = g \left( \frac{5\pi}{4} \right) = -\sqrt{2} \)

Solutions (b) and (d) are related to the CCK and solutions (a) and (c) are related to the SCK. The lecturers had also shared on some of the students’ misunderstanding which they had experienced during their teaching. The Lesson Study group further discussed to help each other understand the reasons for each of the students’ misunderstanding in order to help students understand better. For instance, some common misunderstandings that students encounter are as follows:

1. \( -1 \leq \sin x \leq 1 \) and \( -1 \leq \cos x \leq 1 \rightarrow -1 \leq \sin x + \cos x \leq 1 + 1 \rightarrow -2 \leq g(x) \leq 2 \).
2. \( g^2(x) = 1 + 2 \sin 2x = 0 \leq g^2(x) \leq 2 \rightarrow 0 \leq g(x) \leq \sqrt{2} \).
3. \( y = \sin x + \cos x = \sin x + \sin \left( \frac{\pi}{2} - x \right) = \sin \left( x + \frac{\pi}{2} - x \right) = \sin \left( \frac{\pi}{2} \right) = 1 \rightarrow \text{Max} = \text{Min} = 1 \).
4. \( y = \sin x + \cos x \rightarrow y - \sin x = \cos x \rightarrow -1 \leq y - \sin x \leq 1 \rightarrow -1 + \sin x \leq y \leq 1 + \sin x \rightarrow -2 \leq y \leq 2 \).
5. \( y^2 = \sin^2 x + \cos^2 x \rightarrow y^2 = 1 \rightarrow y = \pm 1 \rightarrow \text{Max} = -\text{Min} = 1 \).

The researcher also discussed some trigonometric formulas with all the lecturers informally. For example, the researcher asked them how to help students find the value of \( \sin \frac{23}{6} \). It was observed that some lecturers used the formula \( \sin(\frac{4\pi}{3} - \frac{\pi}{6}) = -\sin \left( \frac{\pi}{6} \right) = -\frac{1}{2} \) and they were not able to remember these groups of trigonometric formulas easily. When the researcher explained how to
transfer these groups of formulas such as \( \cos(-\theta) = \cos \theta, \tan(\pi - \theta) = -\tan \theta, \cot(2\pi - \theta) = -\cot \theta \) and many other formulas using the trigonometric circle easily, they found it very interesting. These examples indicate that the lecturers improved their content knowledge (CCK and SCK) considerably because they had a platform to share their content knowledge about different topics, methods, solutions and ideas.

4.2. Lecturers’ Content Knowledge as Identified through Mathematics Tests

The lecturers were given 15 questions for pre-test and post-test. The maximum time taken by the lecturers to complete the test was one and a half hours. They were assured that the marks that they obtained would not be reported individually. Only the overall performance of all lecturers (seven mathematics lecturers) who participated in the study will be reported.

The concepts covered and categorization of questions (CCK and SCK) in Test 1 (pre-test) and Test 2 (post-test) are the same but some questions were changed slightly in Test 2. For example, the question “How many functions can we define from \( A = \{4,2,3,5\} \) to \( B = \{6,8,1\} \)” in Test 1 was changed to “How many one-to-one functions can we define from \( A = \{4,2,3,5\} \) to \( B = \{6,8,1,7\} \)” in Test 2. The question “If \( f(x) = \frac{(x-1)(5x^2-4x+7)}{1-2x^3+5x} \) find \( f(1) \)” in Test 1 was changed to “If \( f(x) = x(x+1)(x+2) ... (x + 100) \) find \( f(0) \)” in Test 2. Table 2 shows the percentage of lecturers getting the right answers for Test 1 and Test 2.

Table 2: The Percentage of Lecturers Getting Right Answers for Each Item in Test 1 and Test 2

<table>
<thead>
<tr>
<th>Category</th>
<th>No.</th>
<th>Question</th>
<th>Test 1</th>
<th>Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCK</td>
<td>1</td>
<td>Why ( {(4,6),(2,8),(5,6),(5,8)} ) is not a function from ( A = {4,2,3,5} ) to ( B = {6,8,1} )?</td>
<td>57</td>
<td>100</td>
</tr>
<tr>
<td>SCK</td>
<td>2</td>
<td>How many functions can we define from ( A = {4,2,3,5} ) to ( B = {6,8,1} )?</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>SCK</td>
<td>3</td>
<td>What is the range of the following functions?</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>a. ( k(x) = \frac{1+x}{2x+1} )</td>
<td>57</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. ( k(x) = 3x - 3[x] + 1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CCK</td>
<td>4</td>
<td>What is the difference between a sequence and a function?</td>
<td>29</td>
<td>100</td>
</tr>
<tr>
<td>CCK</td>
<td>5</td>
<td>How many functions can we find which are both odd and even?</td>
<td>29</td>
<td>100</td>
</tr>
<tr>
<td>SCK</td>
<td>6</td>
<td>What is the difference between the inverse function and the inverse of a function?</td>
<td>14</td>
<td>71</td>
</tr>
<tr>
<td>SCK</td>
<td>7</td>
<td>Draw the graph of a function ( f ) that ( \lim_{x \to 2} f(x) = 3 ) and ( f(2) = 1 ).</td>
<td>43</td>
<td>57</td>
</tr>
<tr>
<td>CCK</td>
<td>8</td>
<td>Does ( \lim_{x \to 2} (\sqrt{x - 2} + \sqrt{2 - x}) ) exist? Why?</td>
<td>57</td>
<td>100</td>
</tr>
<tr>
<td>CCK</td>
<td>9</td>
<td>What is wrong in the following inequalities? ( \frac{1}{8} &lt; \frac{1}{4} \to \frac{(\frac{1}{2})^3}{(\frac{1}{2})^2} \to \log(\frac{1}{2})^3 &lt; \log(\frac{1}{2})^2 \to )</td>
<td>57</td>
<td>100</td>
</tr>
</tbody>
</table>
\[
\frac{1}{3\log_{2}3} < 2 \log_{2}3 \rightarrow 3 < 2.
\]

| CCK 10 | Consider suitable marks for this student's solution, ranging from 0 to 4. Question: Find the tangent line of the function \( f(x) = x^2 + 3x - 1 \) at the point \( A(1, 3) \). Solution: \( f'(x) = 2x + 3 \) and \( m = 2(1) + 1 = 3 \) so \( y - 1 = 3(x - 3) \rightarrow y = 3x - 8 \). | 86 | 86 |
| CCK 11 | Consider suitable marks for this student's solution, ranging from 0 to 4. Question: Investigate the continuity of the following function at the point \( x = -1 \). \[
f(x) = \begin{cases} 
2|x| - 1 & x < -1 \\
x - 4 & x = -1 \\
3\cos(-\pi x) - 2 & x > -1
\end{cases} 
\]
Solution: \[
\lim_{x \to -1^-} (2|x| - 1) = 2(-1) - 1 = -3 \\
\lim_{x \to -1^+} 2|x| - 1 = 2(-1) - 1 = -3 \\
\lim_{x \to -1} 3\cos(-\pi x) - 2 = 3\cos(-\pi) - 2 = 3(-1) - 2 = -5
\]
\( f(-1) = -1 - 4 = -5 \) therefore, this function is not continuous at the point \( x = -1 \). | 71 | 86 |
| SCK 12 | What is the suitable solution method for the following question? Question: Which function is the inverse of \( f(x) = 2x^3 - 6x^2 + 6x + 2 \)?
a. \( f^{-1}(x) = \sqrt[3]{x+6} + 1 \)
b. \( f^{-1}(x) = \sqrt[3]{x+4} + 1 \)
c. \( f^{-1}(x) = \sqrt[3]{x+6} - 1 \)
d. \( f^{-1}(x) = \sqrt[3]{x+4} - 1 \) | 86 | 100 |
| SCK 13 | If \( f(x) = \frac{(x-1)(5x^2+4x+7)}{1-2x^2+15x} \), find \( f'(1) \). | 86 | 100 |
| SCK 14 | How can we answer to the following students' question? Question: Why must we learn the logarithmic functions? When is this topic applied in our life? | 71 | 86 |
| SCK 15 | How can we answer to the following students' question? Question: What is the importance of derivative of functions in human life? | 71 | 86 |
| Overall Mean | | 54.3 | 91.5 |

As shown in Table 2, the overall mean of the respondents' performance was 54.3 and 91.5 in Test 1 and Test 2, respectively. In these tests, seven questions (47%) were related to the CCK, and eight questions (53%) were related to the SCK. The average of the lecturers' performance for CCK...
questions in Test 1 and Test 2 were 59.1 and 96.0, whereas, for SCK questions, they were 50.0 and 87.5, respectively. It means lecturers improved their content knowledge during this study considerably. Table 2 also shows that lecturers had better performance in SCK rather than CCK. So the Lesson Study helped lecturers to give more attention to the SCK. For instance, three lecturers answered question 12 using the routine method related to the CCK as follows:

\[
f(x) = 2x^3 - 6x^2 + 6x - 2 + 4 \
\Rightarrow f(x) = 2(x^3 - 3x^2 + 3x - 1) + 4 \
\Rightarrow f(x) = 2(x - 1)^3 + 4 \
\Rightarrow x = \frac{y - 1}{2} \
\Rightarrow (y - 1)^3 = \frac{x - 4}{2} \
\Rightarrow f^{-1}(x) = \frac{3x - 4}{2} + 1
\]

A lecturer suggested the following method.

\[
f(x) = x^3 + 3x^2 + 3x + 1 \
\Rightarrow y + 1 = (x + 1)^3 = x + 1 \
\Rightarrow y + 1 = 3\sqrt{x + 1} - 1
\]

Also three lecturers used the method \(f(f^{-1}(x)) = x\) as follows:

\[
f(f^{-1}(x)) = x \Rightarrow 2(f^{-1}(x))^3 - 6(f^{-1}(x))^2 + 6f^{-1}(x) + 2 = x \
\Rightarrow 2(f^{-1}(x) - 1)^3 + 4 = x \Rightarrow (f^{-1}(x) - 1)^3 = \frac{x - 4}{2} \Rightarrow f^{-1}(x) = \frac{3x - 4}{2} + 1
\]

But they had later learned to solve it easily using SCK only by considering the property \((a, b) \in f \Rightarrow (b, a) \in f^{-1}\) as \((1, 4) \in f \Rightarrow (4, 1) \in f^{-1}\) and thus, only part (b) can be the possible answer. Also two lecturers solved question 13 using the following routine method:

\[
y = \frac{1 - 2x^3 + 5x}{1 - 2x^3 + 5x} \
\ln y = \ln(x - 1) + \ln(5x^5 - 4x^4 + 7) - \ln(1 - 2x^3 + 5x) \
\frac{y}{y} = 1 - \frac{1}{25x^4 - 4} - \frac{5 - 6x^2}{1 - 2x^3 + 5x} \
y' = \frac{((x - 1)(5x^5 - 4x^4 + 7))(1 - 2x^3 + 5x)}{(x - 1)(25x^4 - 4)} \frac{5x^5 - 4x^4 + 7}{1 - 2x^3 + 5x} \
y' = \frac{5(1)^5 - 4(1)^4 + 7}{1 - 2(1)^3 + 5(1)} + 0 - 0 = 2 \Rightarrow y'(1) = 2
\]

Other lecturers suggested applying the derivative formulas for this problem:

\[
f(x) = \frac{((x - 1)(5x^5 - 4x^4 + 7))((1 - 2x^3 + 5x))(1 - 2x^3 + 5x)^2}{(1 - 2x^3 + 5x)^2}
\]

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$f'(x) = \frac{(5x^5 - 4x + 7)(1 - 2x^3 + 5x) + (25x^4 - 4)(x - 1)(1 - 2x^3 + 5x)}{(1 - 2x^3 + 5x)^2} - \frac{(-6x^2 + 5)(x - 1)(5x^5 - 4x + 7)}{(1 - 2x^3 + 5x)^2} \rightarrow f'(1) = 2$

Both methods described by the lecturers are related to the CCK. There is a simple solution for this problem related to the SCK as follows:

$y = \frac{(x - 1)(5x^5 - 4x + 7)}{1 - 2x^3 + 5x} \rightarrow y = (x - 1) \left( \frac{5x^5 - 4x + 7}{1 - 2x^3 + 5x} \right)$

$y' = (1) \left( \frac{5x^5 - 4x + 7}{1 - 2x^3 + 5x} \right) + (x - 1) \left( \frac{5x^5 - 4x + 7}{1 - 2x^3 + 5x} \right) \rightarrow$

$f'(1) = \frac{5(1)^5 - 4(1) + 7}{1 - 2(1)^3 + 5(1)} + 0 = 2 \rightarrow f'(1) = 2$

The question changed in Test 2 as follows:

If $f(x) = x(x + 1)(x + 2) ... (x + 100)$ find $f'(0)$.

The concept of both problems in Test 1 and Test 2 are the same. All the lecturers answered this question easily, by increasing their SCK through discussion with each other. Their argument is shown below.

$f(x) = x[(x + 1)(x + 2) ... (x + 100)]$

$f'(x) = (1) [(x + 1)(x + 2) ... (x + 100)] + x[(x + 1)(x + 2) ... (x + 100)]$

$f(0) = [(0 + 1)(0 + 2) ... (0 + 100)] + 0 \rightarrow f'(0) = 100!$

4.3. Lecturers' Content Knowledge as Identified through Interviews

The researcher conducted two interviews with the lecturers at the beginning and end of the study to determine their content knowledge. Each interview contains seven questions of which four questions (57%) were related to the CCK and three questions (43%) were related to the SCK. Meanwhile, only one lecturer (12.5%) was familiar with the Lesson Study. Table 3 shows the summary of lecturers' answers for Interview 1.

<table>
<thead>
<tr>
<th>Question</th>
<th>Lecturer's Response (n, %)</th>
<th>Purpose/Findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>If you could teach just one subject within your content area, what would you choose? (CCK)</td>
<td>Calculus (3, 43%) Statistics (2, 29%) Probability (1, 14%) Algebra (1, 14%)</td>
<td>To find the lecturers favorite subject in order to show different abilities among lecturers because usually educators have better content knowledge and performance in their favorite subjects.</td>
</tr>
</tbody>
</table>
| What kinds of materials have you used to assess student strengths and weaknesses? (CCK) | Short quizzes (3, 43%)  
Classroom activities (3, 43%)  
Exam (2, 29%)  
Online exams (3, 43%) | Assessment is one important part of education to encourage students for better understanding thus this question introduces the methods of assessments among lecturers. |
|---|---|---|
| What curricular changes do you hope to see over the next few years? (SCK) | Improve the use of technology (2, 29%)  
Prepare the situation to use new educational methods (1, 14%)  
Consider the application of mathematics in the real life (2, 29%)  
Improve the mathematics problem solving in the classes because it is so poor now (1, 14%)  
Mathematics education is very superficial because of the number of topics and limitation in time (2, 29%) | This question determines the main problems about mathematics education in foundation center that affect on mathematics education. |
| Do you have any experience working collaboratively with your colleagues as a team to achieve a goal? (SCK) | No discussion (2, 29%)  
Assignments (1, 14%)  
Special topics (4, 57%)  
Exam questions (2, 29%)  
Weak students in mathematics learning (1, 14%) | To know the spirit of mathematics lecturers in collaboration work about mathematics education. According to lecturers responses they work isolate. |
| How do you connect your lessons to “real world” experiences? (SCK) | Show students some videos related to the mathematics (2, 29%)  
Use some examples related to the real life (3, 43%)  
Do no refer to application of mathematics (2, 29%) | To check the number of practical mathematics problems in their classes. According to my observations practical problem is not common in the classrooms. |
| What is the difference between mathematics exercise and mathematics problem? (CCK) | Mathematics examples inside (outside) the class are called exercise (problem) (1, 14%)  
Mathematics exercise discussed to improve students’ learning but mathematics problem related to the real world life of students (4, 57%)  
Definitions exactly the same of NCTM (2000) (2, 29%) | The aim of Lesson Study is to improve students learning through rich mathematics problems. Only 2 lecturers (29%) knew the mathematics problem. After this interview the researcher explained the mathematics exercise and problem for lecturers. |
How do you plan to maintain and improve your content knowledge? (CCK)

<table>
<thead>
<tr>
<th></th>
<th>Collaboratively work with colleagues (2, 29%)</th>
<th>Although lecturers' CK improves through their experiences and reading, collaboration and discussion among lecturers improve their CK significantly.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Workshops (1, 14%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Internet (4, 57%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Books (5, 71%)</td>
<td></td>
</tr>
</tbody>
</table>

In terms of their preferred content to teach, none of the lecturers mentioned trigonometry and geometry. It can be inferred as difficult to teach and also their lack of content knowledge in these areas. For their practices on assessment, majority of the lecturers used mathematics tests, quizzes or online exams to assess their students' abilities. Somehow none of them regard classrooms activities such as problem solving as a way to assess student learning. The lecturers also feel that the course is overloaded with content and with the time limitation, teaching of mathematics seem superficial and they are not able to focus on problem-solving. There have also not been many opportunities for lecturers to discuss and improve their content knowledge about different topics through collaborative work.

There seems to be a lack of awareness in focusing on practical problems in their classes. The majority of lecturers believed that the tasks given to students can be considered as problem. Not many are able to differentiate between problem and exercise. Majority of the lecturers believed that they can improve their content knowledge by getting the information from the web and books. They may not realize that there are techniques, methods, solutions and shortcuts that lecturers can learn through discussions and by sharing their experiences with others. In summary, problem solving and practical problem solving is not a focus in this foundation centre. The lecturers were again interviewed at the end of the study. The summary of their responses is shown in Table 4.

<table>
<thead>
<tr>
<th>Question</th>
<th>Lecturer's answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Did you enjoy participating in the Lesson Study program? (CCK)</td>
<td>All lecturers said that this program was enjoyable for them because during this study they learned a lot of new things about the mathematics thus their CK improved considerably.</td>
</tr>
<tr>
<td>What is the impact of Lesson Study on your mathematical content knowledge? (CCK)</td>
<td>All lecturers confirmed that their content knowledge improved. Four lecturers C, D, E and F (57%) responded discussion about different mathematics problems with a lot of solutions were so useful to improve their content knowledge. Lecturer B (14%) explained that in this method instead superficial teaching emphasized on the theoretical knowledge behind the mathematics concepts thus her CK improved. Lecturer A (14%) said the teaching approach improved her CK. Lecturer G (14%) answered Lesson Study through discussion and studying improved his CK.</td>
</tr>
</tbody>
</table>

Table 4: Mathematics Lecturers' Content Knowledge at the End of the Study
### What curricular changes do you hope to see over the next few years? (SCK)

Three lecturers A, F and G (43%) explained that they improved the contents in textbooks with suitable mathematics problems and practical mathematics problems. Lecturer B (14%) said that majority of students just memorized the mathematics and she hopes that the method of teaching change to learn conceptually. Lecturer C (14%) said that he hopes he can use Lesson Study in this center in the future and share the methods of teaching. Lecturer D (14%) told that she hopes the technology, facilities and technological knowledge of lecturers will improve in the future. Lecturer E (14%) suggested improving the textbooks by reducing the number of topics, choose better students for this center and improve the quality of teaching in high schools in order for students learn conceptually instead of memorization.

### What are the important challenges among students to learn mathematics deeply? (CCK)

Five lecturers A, B, C, D and F (71%) said that in primary and secondary schools, students memorize mathematics materials thus in foundation they are faced with serious difficulties about problem solving. Two lecturers E and G (29%) answered that students basic knowledge are so poor.

### What kinds of materials have you used to assess student strengths and weaknesses? (SCK)

All lecturers answered that prior to the Lesson Study programme, they only use tests and exams but in Lesson Study, they evaluate the strengths and weaknesses of students through their activities on problem solving individually and team work in every session.

### Do you think that Lesson Study approach is a good approach that connects the lesson to real world experiences? (SCK)

Six lecturers (86%) described that Lesson Study is a good approach because they can find some applications in real world through collaborative work for all mathematics contents. Lecturer B (14%) explained although this is a good and suitable approach, it needs high level of teaching knowledge meanwhile it takes time and energy.

### How do you plan to maintain and improve your content knowledge? (CCK)

Three lecturers A, C and D (43%) answered that they used some books, articles and conferences in order to improve my abilities in problem solving and practical problem solving. Lecturer B (14%) explained that she needs to understand deeply about mathematics 1 and 2 thus she study the textbooks in degree level deeply and she plan to continue her study in PhD. Three lecturers E, F and G (43%) responded that they plan to work collaboratively with their colleagues because they learn many new things that they cannot learn in books, articles and internet.

All lecturers believed that Lesson Study increased their content knowledge through discussions about different methods of solving a given problem, techniques, and experiences about problem solving. For instance, lecturers learned “performance assessment” is another way to examine students’ abilities because problem solving methods in teaching stress not just the correct answer but, more importantly, the method and process. They found that problem solving is the heart of mathematics education, and students cannot learn it deeply and experience the beauty of it without problem solving activities. Also, during the Lesson Study process, they learned something new about the concepts of function that they cannot find in any books or web easily. For example, one of the lecturers explained that:
“When I was a student in high school and university level mostly, I memorized the mathematics formulas, theorems, methods and shortcuts and applied them in the exams but mathematics teaching helped me to learn many new things about the mathematics conceptually. In this Lesson Study approach, I learned a lot of methods, techniques and several solutions for some problems. In fact, this programme improved my teaching knowledge especially about content knowledge. During this study my colleagues shared their knowledge and experiences about different subtopics so I improved my content knowledge”.

5. Discussion

In this center, lecturers tend to teach exactly the same materials as in the textbooks provided because of the lecturers’ lack of content knowledge and the constraint of teaching time. Students, through their primary and secondary school education, may not have acquired the ability to solve problems well and moreover, they may not have understood the principles underlying procedures, concepts and theorems. Thus, lecturers in the foundation classes are not faced with challenging questions from students; therefore, there is no urge for them to improve their content knowledge. This can be detected by the explanation provided by lecturer B as follows:

“I conducted a research in my classes during this semester (two classes with 94 students) about the teaching of the circle equation. I considered two methods as follows:

First method: Firstly, I introduced the definition of the circle and its properties. After that, I explained that the distance of arbitrary point $A(x,y)$ on the perimeter of the circle to the centre of it $O(a,b)$ is equal to the radius of circle $(r)$. So we have $AO = r$ thus 

$$\sqrt{(x-a)^2 + (y-b)^2} = r \rightarrow (x-a)^2 + (y-b)^2 = r^2.$$ 

So the equation of circle is $(x-a)^2 + (y-b)^2 = r^2$.

Second method: In this method, I explained the definition of circle and introduced the equation of circle as $(x-a)^2 + (y-b)^2 = r^2$ without the theory behind it. I asked students which method is better for teaching the equation of the circle. About 90% of students answered the second method is better because we only memorize the circle equation in order to apply in solving exercises”.

Khalid (2017) explained that “teachers are so accustomed to teaching the traditional way as some of them could not envision how mathematics can be taught through problem solving and even if they had seen the technique, were not confident enough to try it in their own class. This lack of exposure and experience in teaching through problem solving may be solved through Lesson Study.” (p. 53). This program provided an experience to the lecturers on how to teach through problem solving confidently. The principles and reasoning behind the mathematics concepts or theorems are important to be shared with students to enhance their problem solving skills and to improve their higher order thinking. Lecturers need to be aware that the transfer of mathematics knowledge, which is done superficially, would not build students thinking in mathematics. The lecturers also need to see the opportunities of enhancing their own content knowledge through such engagement with other colleagues. They need to see that problem solving method is a very strong approach in improving their content knowledge because it helps in guiding, assessing, and

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discussing with students. It also helps in making decision to confirm or reject different methods of solving a given problem.

One of the important benefits of Lesson Study is related to improving the SCK among lecturers. Creative solutions not only help lecturers to have better views on mathematics concepts but they can create excitement to students because they can see the power of mathematics and thus improve their interest and motivation in learning mathematics. In international assessments such as Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA), most of the questions are related to the SCK and some questions require students to engage with multiple-choice questions. So educators teaching at any level of education should have suitable SCK to prepare their students for such assessments. For example, the following is a question related to 8th grade in TIMSS (2015).

Problem: A straight line on a graph passes through the points (3, 2) and (4, 4). Which of these points also lies on the line? A. (1, 1) B. (2, 4) C. (5, 6) D. (6, 3) E. (6, 5)
For this question, students do not need to use the routine method by applying the equation of straight line. They only need to apply the slope of the line \( m = \frac{4-2}{4-3} = 2 \), also for points (3, 2) and (5, 6), \( m = \frac{6-2}{5-3} = 2 \). Thus point (5, 6) is the answer.

Another example which is related to the TIMSS Advanced (2015) for 12th grade students is as follows:

Problem: Two mathematical models are proposed to predict the return \( y \), in dollars, from the sale of \( x \) thousand units of an article (where 0 < \( x \) < 5). Each of these models, \( P \) and \( Q \), is based on different marketing methods.

\[
\text{Model P: } y = 6x - x^2 \\
\text{Model Q: } y = 2x
\]
For what values of \( x \) does model \( Q \) predict a greater return than model \( P \)?
A. 0 < \( x \) < 4  B. 0 < \( x \) < 5  C. 3 < \( x \) < 5  D. 3 < \( x \) < 4  E. 4 < \( x \) < 5
Also for this question, students do not need to make an inequality and solve it by using sign-chart to find the answer. They can find the answer easily by testing two values \( x = 4 \) and \( x = 5 \) to find that 4 < \( x \) < 5 is the answer.

Also, during this study, the lecturers learned some creative solutions and techniques related to the SCK, which is a part of lecturers’ content knowledge that they cannot find in the books or web easily. It is of utmost importance that students are given the experience to learn mathematics conceptually and appreciate the beauty of mathematics. This can be enhanced if the lecturers continually improve their content knowledge. For instance, the lecturers solved the problem “If \( f(x) = \frac{(x-1)(5x^2-4x+7)}{1-2x^3+5x} \) find \( f'(1) \)” through routine way in pre-test that this method needs a lot of calculation but later they learned new technique and solved the problem “If \( f(x) = x(x+1)(x+2)...(x+100) \) find \( f'(0) \)” in post-test through the following creativity method.

\[
f(x) = x(x+1)(x+2)...(x+100) \rightarrow f(x) = x[(x+1)(x+2)...(x+100)]
\]
In current study, observations, tests and interviews showed lecturers improved their content knowledge about the mathematics function during Lesson Study program. For example, lecturers discussed about several methods to find the range of the function \( f(x) = \frac{x^2}{1+x^2} \) that explained in details in observation part. They improved their content knowledge and learned new methods to find the range of the functions. In post-test they had excellent performance to find the range of the function \( y = \frac{|x|}{2|x|+1} \). They solved this problem based on their learning in the discussion meeting on the range of the function \( f(x) = \frac{x^2}{1+x^2} \) as follows:

\[
y = \frac{|x|}{2|x|+1} \rightarrow |x| = 2y|x| + y \rightarrow |x| = \frac{y}{1-2y} \\
|x| \geq 0 \rightarrow \frac{y}{1-2y} \geq 0 \rightarrow y \in \left[ 0, \frac{1}{2} \right]
\]

Therefore, Lesson Study is a beneficial method in education to improve content knowledge of lecturers through discussion with their colleagues especially about SCK. In this approach they can share their content knowledge and experiences in different topics.

6. Conclusion

The results of this study, which was concluded through observations, mathematics tests and interviews, provided evidence that Lesson Study is an effective approach in improving content knowledge of lecturers significantly. The lecturers were given opportunities to discuss and share on problems that can be solved in different ways, develop problems, discuss students’ misunderstandings and shared their experiences about different topics so that they can help each other to improve their content knowledge. The findings of this study about the content knowledge of lecturers are in line with studies by Coenders and Verhoeof (2019), Harsono (2016), and Mon, Dali, and Sam (2016).

Lesson Study has many advantages compared to other professional development programs because it promotes the practice of innovative teaching (Suhaili & Khalid, 2011; Suhaili, Shahrill, & Khalid, 2014). They also stated that it provides educators with a platform to improve and reflect on their own teaching. It is also a form of the professional development program, but it is not confined to a specific time. Based on the finding of this study, Lesson Study is a suitable and effective professional development program for mathematics lecturers in the foundation center because lecturers usually teach one subject in each semester. So they have ample opportunity to coordinate sessions to discuss about different mathematics topics, in order to improve their content knowledge collaboratively. It is suggested that the Lesson Study approach be made as a required professional development program for lecturers in the conduct of a course.

References


