Editorial from Bronislaw Czarnocha

Mathematics Teaching-Research in the time of Covid19:
Difficulties and Possibilities

COVID 19 is constantly on our minds to larger or smaller degree and investigations into student performance as well as learning mathematics during the pandemic are important to determine the best course of action. It is becoming clear that changes in pedagogy required from us by the pandemic will be substantial if we want to engage all possible available routes of e-learning. A bisociative frame is created between the past pedagogy and new circumstances, the structural frame, which is a prerequisite to creativity of Aha! Moment. Thus it is our own classroom creativity that is called here for engagement.

We start the issue with the work of Ariyanti and Santoso from Indonesia who inform about a simple yet statistically rigorous comparison of student work before and after Pandemic. Their results definitely demonstrate lowering of student achievement during the Covid19 distance learning.

We follow that analysis with the work of Baker and his colleagues in the Bronx who investigate the impact of COVID 19 restrictions upon the facilitation of creativity of Aha!Moment. Baker et al are using this occasion to lay the short background of the methodology of facilitation and assessment of the depth of creativity. They show that the characterization of Aha!Moment by three criteria of: search, connection and resulting novel process do a good job in the analysis of the depth of creativity within Aha!Moment.

They report that the main impact of the distance learning on the facilitation process is in the constraints upon interaction between students as well as upon student/teacher interaction created by the online approach. Since such an interaction is essential for the creativity facilitation process, one can expect lowering of the level of creativity in the mathematics classrooms.

The third article in the Covid series by Fuchs and Tsaganea provides multidimensional analysis of the COVID impact upon teaching in NYC as well as in the whole country. They discover quite a few advantages of online teaching in relation to the limitations of face-to-face teaching, which nonetheless has been seen as the best pedagogical method. However, they point out that the
societal changes due to COVID will stay with us much longer and they urge educators and students to develop mastery of online teaching and learning.

We supplement these three COVID related papers by the interesting paper of Stokes and Sanfratello, which although written before the pandemic struck, nonetheless offers an interesting pedagogy of “learning through doing”, that eliminates math anxiety. As the long term effects of the pandemic upon learning are still unknown the experiential approach based on patterns and deliberate practice while grounded in the problem solving model of creativity/innovation offers the pathway of success for those students who have experienced higher levels of math anxiety in new online learning circumstances.

We complete this issue with two papers analyzing classroom effectiveness of two geometrical software, Geogebra and Geometer Sketchpad. At present, geometrical oriented mathematical software might be very useful in contemporary online mathematical classrooms. It can provide mediating visual pathway between the student and the teacher while increasing and deepening their interaction.

Raj Joshi and Singh from Nepal demonstrate high effectiveness of Geogebra for learning linear equation through the simple experimental group/control group investigation. They point out to the versality of Geogebra to teaching variety of mathematical domains from arithmetic to calculus; it could be especially useful for distance learning.

The paper of Hartono from Indonesia investigates effectiveness of Geometer Sketchpad (GSP) in guiding student understanding of two dimensional objects. The author describes three months long teaching experiment comparing student (7th grade) achievement between two cohorts, experimental with GSP software, and control with traditional learning. The positive result of the teaching experiment needs to be repeated during pandemic, however it’s clear that both geometrical software can positively impact mathematics learning and teaching during that critical time.

Bronislaw Czarnocha

Chief Editor
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The Effects of Online Mathematics Learning in the Covid-19 Pandemic Period: A Case study of Senior High School Students at Madiun City, Indonesia

Gregoria Ariyanti, Fransiskus Gatot Iman Santoso

Mathematics Education Study Program, Widya Mandala Catholic University Surabaya, Indonesia

ariyanti_gregoria@yahoo.com; fransimansantoso@yahoo.com

Abstract: This study aims to determine the differences in students' mathematics learning outcomes before and after online learning and their positive responses to this method's use. Data were quantitatively and non-randomly collected from Senior High School students in Madiun city, Indonesia, in May 2020, by using the documentation and questionnaire methods. There are two sources of data, namely mathematics learning outcome and student's response using an online questionnaire. Data were used to determine the effect of learning during the COVID-19 pandemic. Meanwhile, the questionnaire was used to determine their responses. The results showed that the average of mathematics learning outcomes before online learning is greater than the average after online learning and students’ average positive response towards mathematics before online learning is greater than the average after online learning.

Keywords: Student mathematics learning outcomes, Online learning, COVID-19 pandemic

1. INTRODUCTION

Mathematics is one of the subjects learned in High Schools and Universities. It is one of the basic educational components that require students to be skilled and understand for the various methods used to structure lives. For instance at the end of 2019, the world was shocked by the inception of the coronavirus in Wuhan, one of the cities in China, which subsequently spread to several countries worldwide. Therefore on January 30, 2020, the World Health Organization (WHO) declared this outbreak as a Public Health Emergency. In a letter written by WHO's Director-General, Tedros Adhanom, to President Jokowi on March 10, 2020, the organization advised Indonesia to undertake several steps to prevent the spread of the virus declaring a national coronavirus emergency. The letter was also forwarded to the Ministries of Health and Foreign Affairs (Sari, 2020). Currently, COVID-19 has infected more than 4.7 million people, with 1.8 million recoveries, and more than 300 thousand deaths, based on updated across the world on May 17, 2020.
Therefore, President Joko Widodo has declared the virus a public health disaster, and it is recommended to work, study, and worship from home. The government's appeal to the community was also conveyed through the Minister of Education and Culture, Nadiem Makarim. Due to these new rules, learning activities that are usually carried out at school eventually take place online (Sari, 2020). Therefore, learning activities that are usually routinely carried out in schools finally take place online. Regarding learning from home, the Minister of Education and Culture emphasizes that online or distance learning is carried out to provide meaningful learning experiences for students, without being burdened with the demands of completing all curriculum achievements for grade promotion or graduation. The Minister of Education and Culture also recommends regions that have learned from home to ensure that teachers also teach from home to maintain teacher safety (Sari, 2020).

Online learning is education that takes place over the Internet. Online learning is just one type of distance learning for any learning that takes place across distance and not in a traditional classroom. One of the main reasons for this is it gives students' greater access to education in comparison to traditional methods of teaching as students can undertake their study from anywhere and at any time as well as being given the option to study part-time or full-time (El-Seoud et.al., 2014). Through online learning, interactive activities such as teacher-student interaction, student-student interaction, student–content interaction, and student–technology interaction are considered. Students participated in the blended learning course in which formative assessment was used to evaluate student learning outcomes by the combination of different learning activities through a learning management system (Nguyen, 2017). According to Salamat et.al. (2018) “Online learning can refer to the situation where the interaction between the students and the teacher is done through online system. Students are received training and taught through online system and teacher may also in the same building with them”.

Various school efforts have been made to continue learning even though the school is on vacation. Based on the description above, the following problem formulation is obtained:

a. Are there differences in students' mathematics learning outcomes before and after online learning?

b. Are there differences in students' positive responses to mathematics before and after online learning?

2 RESEARCH METHOD

The quantitative research method was used to determine the effect of the online learning outcome of mathematics in Senior High School students at Madiun. Furthermore, this is a quantitative research due to its ability to use samples to solve problems related to online learning. This research
used the one-group pretest-posttest design to compare the conditions before and after online learning (Wijayanto et al., 2017).

The research population consists of two Senior High School students in Madiun City with the Non-Random Sampling method. It is used to obtain data from teachers that are alumni of the Mathematics Education Study Program at Widya Mandala Catholic University Surabaya, Indonesia. Furthermore, this technique was chosen several cities implemented the Large-Scale Social Restrictions (LSSR) to stop the spread of COVID-19, with questionnaire research samples collected through telephone interviews.

The data collection techniques in this study are the documentation and questionnaire methods. The documentation is in the form of learning outcomes before and after online learning. Meanwhile, questionnaires need to be answered or responded to by students.

In addition, the Paired Sample Test was used because the secondary data are processed in intervals. “Paired samples have the same subject with different treatments” (Rea and Parker, 2014). The population normality test is also conducted as a prefix to statistical tests that were carried out using the SPSS program.

The population normality test uses the Kolmogorov Smirnov Goodness of Fit Test statistic and SPSS program with menu procedures of Analyze, Descriptive Statistics, and Explore. In Display, select Plot and check the Normality Plot to determine the two conditions. When both data are normally distributed, then the paired sample t-tests are used. Furthermore, data processing is carried out by using the SPSS program with the menu procedure of Analyze, Compare Means, and Paired Sample t-Test. The t-value is recorded as t-count (t-count) and df value as degrees of freedom to determine t table (t-table) based on the output results in the Paired Samples Test table. Assuming one or both of the data are not normally distributed, the non-parametric statistical testing, which is the Wilcoxon Signed Ranking Test, is used to test the two paired samples. Subsequently, data processing is carried out using the SPSS program with the menu procedure of Analyze, Compare Means, and 2 Related Sample, with the Test Type selected by Wilcoxon. Based on the output results in the Test Statistics table, the Asymp Sig. (2-tailed) is recorded as the Asymp count (Asymp_count).

3 DISCUSSION OF RESEARCH RESULTS

3.1 Discussion of Student Mathematics Learning Outcomes

Data analysis was conducted after obtaining data on student mathematics learning outcomes. The samples in this study are 96 students from two Senior High Schools. This is based on data processing on learning mathematics using the SPSS program, as shown in Table 1.
Table 1 - Statistics Description of Mathematics Learning Outcomes

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Minimun</th>
<th>Maximu m</th>
<th>Mean</th>
<th>Std. Deviatio n</th>
<th>Varian ce</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics Learning Outcomes Before Online Learning</td>
<td>96</td>
<td>60</td>
<td>100</td>
<td>83.60</td>
<td>9.153</td>
<td>83.779</td>
</tr>
<tr>
<td>Mathematics Learning Outcomes Before Online Learning</td>
<td>96</td>
<td>53</td>
<td>98</td>
<td>76.82</td>
<td>9.072</td>
<td>82.295</td>
</tr>
<tr>
<td>Valid N (listwise)</td>
<td>96</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A normality test is carried out to determine whether the data from the population is normally distributed. The result showed that outcomes before and after online learning do not come from normally distributed populations.

Table 2- Calculation Results for Normality Test with Kolmogorov-Smirnov Test before and after the online learning

<table>
<thead>
<tr>
<th></th>
<th>Kolmogorov-Smirnova</th>
<th>Shapiro-Wilk</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Statistic</td>
<td>df</td>
</tr>
<tr>
<td>Mathematics Learning Outcomes Before Online Learning</td>
<td>.172</td>
<td>96</td>
</tr>
<tr>
<td>Mathematics Learning Outcomes Before Online Learning</td>
<td>.170</td>
<td>96</td>
</tr>
</tbody>
</table>

a. Lilliefors Significance Correction

In Table 2, output in the Tests of Normality table obtained a significance value (Sig.) Of 0.00 for mathematics learning outcomes before and after online learning. This value is less than the significant level (α) of 0.05. This means that data before and after online learning does not come from the normally distributed populations.

These two paired samples test uses the non-parametric statistical testing method, namely the Wilcoxon Signed Ranking Test. Furthermore, the data processing is carried out using SPSS with
the menu procedure of Analyze □ □ Compare Means □ □ and 2 Related Sample, with the Test Type selected by Wilcoxon.

Table 3 - The Results of Wilcoxon Signed Ranks Test

<table>
<thead>
<tr>
<th>Mathematics Learning Outcomes Before Online Learning</th>
<th>N</th>
<th>Mean Rank</th>
<th>Sum of Ranks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative Ranks</td>
<td>63^a</td>
<td>48.02</td>
<td>3025.00</td>
</tr>
<tr>
<td>Positive Ranks</td>
<td>21^b</td>
<td>25.95</td>
<td>545.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mathematics Learning Outcomes Before Online Learning</th>
<th>Ties</th>
<th>12^c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>96</td>
<td></td>
</tr>
</tbody>
</table>

a. Mathematics Learning Outcomes After Online Learning < Mathematics Learning Outcomes Before Online Learning

b. Mathematics Learning Outcomes After Online Learning > Mathematics Learning Outcomes Before Online Learning

c. Mathematics Learning Outcomes After Online Learning = Mathematics Learning Outcomes Before Online Learning

Table 4 - Wilcoxon Test Statistics Table

<table>
<thead>
<tr>
<th></th>
<th>Mathematics Learning Outcomes After Online Learning - Mathematics Learning Outcomes Before Online Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z</td>
<td>-5.538^a</td>
</tr>
<tr>
<td>Asymp. Sig. (2-tailed)</td>
<td>.000</td>
</tr>
</tbody>
</table>

a. Based on positive ranks

b. Wilcoxon Signed Ranks Test

In the Test Statistics table, the Asymp Sig. (2-tailed) values obtained an Asymp Count of 0.000, which is less than the significant level (α) of 0.05. Therefore, there are differences in the average of
student mathematics learning outcomes before and after online learning. The average of the mathematics learning outcomes, before online learning is greater than after online learning.

3.2 Discussion of Student Responses towards Mathematics

The response questionnaire data were obtained from 75 Senior High Schools students in Madiun City. The responses using the SPSS program with normality test analysis showed that before and after online learning outcomes do not come from the normally distributed populations. The processing data obtained using the SPSS program with the Wilcoxon Signed Ranking Test is shown in Table 5.

**Table 5 - The Results of Wilcoxon Signed Ranks Test**

<table>
<thead>
<tr>
<th>Student Responses After Online Learning - Student Responses Before Online Learning</th>
<th>N</th>
<th>Mean Rank</th>
<th>Sum of Ranks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative Ranks</td>
<td>29^a</td>
<td>17.81</td>
<td>516.50</td>
</tr>
<tr>
<td>Positive Ranks</td>
<td>6^b</td>
<td>18.92</td>
<td>113.50</td>
</tr>
<tr>
<td>Ties</td>
<td>40^c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>75</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Student Responses After Online Learning < Student Responses Before Online Learning
b. Student Responses After Online Learning > Student Responses Before Online Learning
c. Student Responses After Online Learning = Student Responses Before Online Learning

**Table 6 - Wilcoxon Test Statistics Table**

<table>
<thead>
<tr>
<th>Z</th>
<th>Asymp. Sig. (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.615^b</td>
<td>.000</td>
</tr>
</tbody>
</table>

a. Based on positive ranks
b. Wilcoxon Signed Ranks Test
In the Test Statistics table, the \textit{Asymp Sig. (2-tailed)} values of $\text{Asymp}_{0.000}$ are obtained and less than the significant level ($\alpha$) of 0.05. Therefore, there are differences in the average results of student questionnaire responses before and after online learning. Table 5 shows that the average positive response of students’ towards mathematics before online learning is greater than after online learning.

4. CONCLUSIONS

The following conclusions were made based on the research:

a. The average of student mathematics learning outcomes before online learning is greater than after online learning.

b. The average student's positive response towards mathematics before online learning is greater than after online learning.

Positive responses to mathematics after students take online learning were obtained from questionnaires. The following are the results obtained from filling out the questionnaire as many as 75 students, namely:

a. Obstacles faced by students when participating in online learning, namely non-smooth signals, limited quota, the teacher directly gives questions without any material explaining how to solve problems so that students do not understand the material, and feel disturbed by noise in the home environment;

b. The efforts that have been made by students in overcoming these obstacles, namely: look for a smooth internet/wifi network to neighbors or other places outside the home; search for material on the internet (browsing) and access YouTube; asking friends and doing it together; studying in the room to avoid noise;

c. Media used in online mathematics learning, namely Whatsapp (WA), Google Classroom (GC), Zoom, E-learning schools, and e-mail;

d. Suggestion from students for online mathematics learning, that is, teachers should make videos or explain material through videos so that students can better understand completion steps and formulas that can be used.

References


Creating Meaning by Students within Class Discourse in Mathematics: Pre and Post the Transition to Online Learning due to Pandemic

William Baker

Hostos Community College, CUNY

Abstract: This Teaching-Research article contains a Pre and Post observation of class discourse from two instructors applying constructivist principles in their classroom to support student insight and shared construction of knowledge before and after the Covid-19 pandemic necessitated on-line learning. One goal of this article is to present the difficulties with translating moments of shared construction of knowledge from in-class to on-line learning in an urban community college within the City University of New York (CUNY) system. The moments of student insight and learning within classroom discourse takes place within a theoretical framework that integrates constructivist’s theory and research on creativity. This paper proposes three criteria for analysis of student insights based upon the work of Arthur Koestler (1964) to understand moments of creative insight. These three criteria are then applied to teacher led discourse and student insight, responses, and reasoning at different levels of development. Thus, a second goal of this work is to bring creativity research closer to the everyday mathematics classroom through analysis of class discourse using these three criteria.

Keywords: Accommodation, Bisociation, Discovery of Hidden Analogy, Matrix, Piaget-Garcia Triad, Schemes, Shared Construction of Knowledge, Teaching Research Team

THEORETICAL FRAMEWORK

Creativity within Accommodation

This work is founded on the premise, noted by Prabhu (2016) that all students, even those struggling to accommodate simple mathematical structures must experience moments of insight to appreciate and learn mathematics as anything other than a collection of meaningless rules. In this view creativity takes place within a continuum of difficult to observe moments of recognition, recall, and internalization of new content to more dramatic ‘Aha Moments.’ These moments of insight, especially at initial levels of learning, are often subtle as student abstract meaning through interaction with the instructor, or follow peer-peer and teacher-peer discourse e.g. what Mason (1989) refers to as a “delicate shift of attention.” More dramatic ‘Aha Moments’ occur as one develops a domain of processes related to a situation and can search through an increasingly wider collection or toolbox of such processes. The discovery of a connections to previously unrelated process in a new situation is a foundational component in accommodation, understood as the modification of one’s schemes.
Accommodation as described may be viewed as taking place within a problem-solving environment, in which a person has a goal but the solution to that goal does not readily fit into any of his routine frames of reference (i.e., it cannot be readily assimilated). Note, the term goal is used broadly here and can perhaps better be interpreted as motive. Thus, the goal may include the motive to follow social discourse or math presentations (textbook, video etc.) or inquire into how new content relates to one’s existing knowledge structures.

Schemes

The constructivist term “scheme” or “action scheme” is understood as essentially two connections between three steps within a problem-solving environment, as set forth by Glasersfeld (1995). The first connection is between features of a new situation (step 1) and appropriate solution activity (step 2). This connection is often the result of a moment of insight—what Koestler refers to as the discovery of a hidden analogy. The second connection is between the activity and progress towards the goal (step 3).

Matrices and Hierarchies

The term matrix is used by Koestler (1964) to denote any frame of reference to interpret, and respond to a situation, if it follows a set of rules (the code). To analyze moments of insight within class discourse and whether such insight resulted in successful learning or accommodation, the terminology (“schemes”) of constructivists is integrated with the terminology (“matrices”) of Koestler (1964). We note, that the term matrix is used more broadly (outside mathematics and science) than the constructivist notion of an action scheme. Koestler (1964, pp. 610-620) describes a hierarchy of matrices as the result of abstraction based upon two characteristics, first relevancy to any given situation (similar to an action schemes), and second, a conceptual network derived from the abstracted essence of the different situations it interacts with.¹

Three Components of Moments of Insight

The three components of creativity leading to accommodation are as follows: first, the search process to resolve the goal in a new situation; second, a resulting connection (previously hidden) between the situation and a relevant scheme; and third, the novel process that results. The creation of a novel process or creative product is a defining characteristic of creativity research e.g. Leikin & Pitta-Pantazi (2013). In this work, novel or original is taken as subjective (it is new for the learner), and a novel process is understood as a process born through reflection upon solution activity, e.g. process-object duality theories Dubinsky et. al. (1999). The notion of a connection between a new problem situation, and a previously overlooked scheme is central to Koestler’s notion of a hidden analogy and bisociation, and it is inherent in the previous definition of s’ scheme Glasersfeld (1995). The understanding that the search process within a new situation varies depending upon the level of development is a foundational characteristic of the Piaget-Garcia Triad (1983). We note that the degree of originality and depth of the resulting connection will also vary depending upon the level of development.
Piaget-Garcia Triad Level 1

Students in level 1 view mathematical procedures as externally directed rules that must be memorized. Steffe (1991) present the first level as a transition from empirical reasoning (perceptual real-life objects) to internal reasoning (mental reference to perceptual objects required) and, finally, through interiorization (birth of abstract conception and process) to the ability to engage in activity independent of perceptual objects. In a classroom setting outside the use of ‘manipulatives’ to understand procedures, internalization often involves the solver’s creation of meaning not from manipulating perceptual objects, but from reproducing solution activity modelled by a mentor or peer. Norton and D’Ambrose (2008) suggest that social constructivist pedagogy based upon Vygotsky’s notion of internalization involves reconstruction of modeled activity through conscious reflection and not simply imitation, yet they acknowledge that this explanation poses a dilemma for constructivist pedagogy, particularly with students at the first level of development. When does the teacher’s action promote consciousness as opposed to imitation? Vygotsky (1997) notes that “intelligent conscious imitation comes instantly in the form of insight, not requiring repetition.” (p.221).

Piaget and Garcia (1983) characterize the first level as one in which the search process is localized, that is on an existing scheme that is not sufficiently understood i.e., intuitive, semi-conscious or externally driven. Moments of insight are thus, often subtle, and undetected. In the first level, the connection and resulting activity are typically internal processes that becomes interior when one can perform them independently i.e., anticipate the results of an activity without the need to engage in such activity, Dubinsky et. al. (1999).

Piaget-Garcia Triad Level 2: Toolbox of Schemes

In the second level the search process extends beyond an existing scheme that one does not fully understand or exterior modeling (conscious imitation), and towards features in a new situations that require one to modify, adapt or generalize an emerging process. As the student transitions beyond the need to connect processes to a specific situation they simultaneously develop the ability to apply it to related situations, and thus develop a sense of ‘problem types.’ The collection of processes that apply to a situation develop into domain of processes for a problem type, what we have referred to as toolbox. The connections often integrate schemes appropriate for a given problem type or domain, including those learned in a previous math class. These connections may involve a synthesis of the reasoning of one process with another to form a new reasoning or code i.e. the bisociative synthesis of unrelated matrices Koestler (1964). At this level, the novel processes that result are typically new not only for the individual, but also for the other students in the class. The resulting novel connections are new not only to the individual but typically to the entire class, and to some extent the teacher as well.

As students transition between the second to third level, moments of insight are often distinct examples of ‘Aha Moments.’ Thus, they occur during the transition from incubation to illumination within the four stages of creativity developed by psychologist Graham Wallas (1926): preparation, incubation, illumination (insight), and verification.
Piaget-García Triad level 3 Connections that Transcend Domain

In the third level, the solver can increasingly transcend their toolbox of schemes, crossing domains when necessary. In this level, hierarchies of schemes (matrices) are being developed as connections are between problem types that cross domains even different fields. The novel processes that result are often new to the instructor as well as of interest to the mathematics community. Koestler (1964) provides copious examples of creative discoveries by research scientists and mathematicians. In mathematics education the third level is typically the domain of the study of ‘gifted students’ and it is at this level that the creative process is often characterized by notions such ‘flexibility’ and ‘fluency’ or ‘divergent thinking’ e.g. (Leikin & Pitta-Pantazi, 2013), (Singer et al., 2017).

The Role of the Teacher: Didactic Contract

Norton and D’Ambrosio (2008) characterize pedagogy, based upon Vygotsky’s notion of (social) internalization, as a cycle of “assistance” or guidance by the teacher modeling or explaining mathematical content and reasoning followed by “internalization” viewed as “…the internal reconstruction of an external process…” (Vygotsky, 1978, p.56). These authors suggest that pedagogy based upon Vygotsky’s work should present content within the upper reaches of a student’s ability, at which he or she can function only with assistance by a mentor—in other words, his or her zone of proximal development or ZPD.

The Didactic contract (Sarrazy and Novotná 2013). between the teacher and students is built upon a ‘less-is-more’ principle which underlies the tenets of constructivism and guided discovery methodology used within constructivist teaching experiments. This principle simply put states that, the less direct instruction, the more opportunity for the student to actively internalize content. As students begin to form interior constructions, instructional methods to assess whether the process have become interior include leaving time delays e.g. ‘the next-day-effect, introducing new problem features that require generalization, or coordination with related processes in the student’s toolbox. This approach begs the question of how to teach, assist struggling students, promote student insight, and share creation of knowledge in social discourse.

The Teaching Research Team

In this work, both the observer and instructors are members of a teaching research team at a community college in the CUNY system. The classroom instruction methodology of two instructors on the team were observed to analyze how they supported student insight, and shared construction of meaning and knowledge within social discourse both before (in-class) and after (on-line) instruction was necessitated by the Covid-19 pandemic.

Methodological and Theoretical Research Questions

1. What was the methodology of the instructors to motivate students and support the shared construction of knowledge in a flexible and creative learning environment, and how did the shift to online learning necessitated by the Covid-19 pandemic effect-limit this shared learning experience?
2. How useful are the three criteria developed within the Piaget-Garcia Triad in classifying and analyzing the continuum of student insight and responses from subtle shifts of attention to noticeable ‘Aha Moments’ within class discourse?

**EMPIRICAL INVESTIGATIONS: TEACHING RESEARCH**

In an earlier article Baker et. al. (2019) the teaching methodology of Professors Stachelek (College Algebra) and Wolf (Pre-Calculus) to promote student participation based upon their insights in the classroom learning process was reviewed.

**Professor Stachelek, In-Class Lesson:**

The first technique Stachelek employed was board work by volunteer students using their notes from the previous day. The students had to: name, sketch and find the domain/range of common ‘parent’ functions i.e. the linear function ‘f(x) =x, the quadratic f(x) = x^2 , the rational function f(x) = 1/x and the radical function. Scaffolding assistance included peer-assists from colleagues although these were limited to short directives e.g. do this, draw that… and hints from the instructor whose role The students readily and correctly completed the graph and found the domain-range of the linear and quadratic functions yet had much more difficulty with the rational and radical functions. The instructor seized on this opportunity to (re)introduce interval notation involving the concept of infinity.

This instructional methodology can be classified as level one (Piaget & Garcia Triad) because it was designed to give hands on practice of review of notes i.e. ‘conscious imitation’ and is a good exercise to promote internalization. In lower algebra courses these students clearly experienced many examples of linear and quadratic functions, and these are less cognitively demanding, especially the linear function. However, they struggled with the sketch of a radical, (it looks more like a sequence of line segments connecting points than a curve) finding the domain and range is especially challenging. Thus, the students consciously imitated the previous day work of instructor Stachelek from their notes, they began within their comfort zone (linear and quadratic) and moved to the upper level of their ZPD (radical and rational).

**Inducing a Gestalt Experience**

Then Professor Stachelek introduced a very creative methodology he titled “guess the parent function,” he asked the students for small reasonable domain values of x and then gave them corresponding values of y in an x-y table format and asked them to guess the function. He presented two examples the first involved a linear function with an additive constant (y = x + k) and the students readily grasped that it was both linear and the constant. Then he introduced a quadratic function with an additive constant (y = x^2 + k) and the students struggled to understand it. This methodology was designed for the second level, as it required students to transition from an imitation of what was previously done (linear scheme) to a new less intuitive situation (quadratic scheme). The instructor scaffolded this challenge with clear and repeated instructions (e.g. “what parent function is this?”). The question-statement itself suggests to students it may not be the same as the previous linear scheme. However, for a while, it would appear most were trying...
to interpret the new information using their linear schemes, finally in response to the instructors repeated questioning one student realized it was not linear as the previous one and responded that it was quadratic.

**Aha Moment**

Immediately after the first student suggested that it was a quadratic parent, a second student had an “Aha” moment, exclaiming “I got it” as she explained what the function was. It appeared that she had a Gestalt-like “Eureka” moment in which the correct solution came about only after she abandoned her linear scheme and the hidden “quadratic scheme” was presented by her peer. When Professor Stacheleak asked the class to break into groups and present their results on the board, she was both a leader in her group and willingly assisted other groups when they had trouble presenting their results. Although she was clearly an advanced student, it was also clear that her realization resulted in positive affect and motivation to participate and assist other students. In summary, the technique of presenting a situation that readily assimilated into an existing scheme and immediately following this occurrence with a situation that superficially appears equivalent but requires a much less commonly used scheme was very effective in producing an “Aha Moment.”

According to Salamat et. al. (2018) “E-learning us utilizing electronic technologies to get access to educational curriculum” (p.2) These authors characterize e-learning as having potential to make learners independent and to bring about more equality in access to learning, it may motivate students and provide them with more flexibility and instructor access. However, Ariyanti (2020) reports statistical evidence that student performance on learning objectives decreased because of online learning during the pandemic. These results are not mutually exclusive e-learning may have much to offer, but not for unprepared instructors and students.

**Professor Stacheleak, On-Line Lesson:**

The on-line lesson by Professor Stacheleak was a creative format using the game show *Jeopardy* through “screen share”; the lesson objective was to review topics in elementary statistics for the final exam. The *Jeopardy* game board contained 4-5 columns-topics throughout the course and different levels of difficulty (i.e., rows-money). The topics included normal distribution, probability, linear regression, the binomial probability distribution, confidence intervals, and hypothesis testing. The game format was clearly exciting for the students and they enjoyed choosing topics at different ranges of money, yet they were often silent when presented with the *Jeopardy* question.

This review lecture would be classified as requiring second level development, in that the situations presented required the students to search for, recall, and connect schemes that had been learned weeks even months ago to appropriate solution activity. We review one example that highlighted the teacher-led discourse observed: the student chose a *Jeopardy* button under Probability, a table of data values with rows of student enrollment status (part-time PT or full-time FT), and the total. The columns displayed a tendency for smoking (none-NS, light-moderate LS or heavy HS) and the total.
The question was as follows: find the probability of any randomly selected student, either PT or HS.

1. A student replied that the answer is as follows: \( (|PT| + |HS|) / |Total\ students| \).
2. At this point the instructor provides a scaffolding hint: “There is a little hitch here!”
3. The student replies, “We need to subtract 3 (the number of PT students who were also HS).
4. The instructor asks, “Why?”
5. The student replies, “Because it is in both” (3 is PT \( \cap \) HS).
6. The instructor comments, “Nice job!” and reviews the formula for finding probability of PT or HS.

Although the online format seriously reduced the amount of peer-peer interaction (the audio did not readily allow for flexible group discourse), it did allow for limited back and forth dialogue. The focus of this on-line discourse was first the instructor-problem; then, when a student comments, the instructor stops as the audio switches to the student response.

**Level of Student Development**

The student had apparently internalized the conception that those elements counted twice as part of both a row and a column, and thus they need to be subtracted. However, in this review (after several weeks), this internal process was not adequately recalled. In this situation, one might say the internalization was in short-term, but not in long-term memory a classic manifestation of a level one participatory scheme (Tzur & Simon, 2004). Indeed, in the terminology of these authors one might say the student was in transition from a participatory to an anticipatory scheme as she required minimal assistance (i.e., the “OPPS-effect”).

**Professor Stachelek, Comparative Analysis:**

The on-line lecture was noticeably lacking in peer-peer interaction due to platform constraints and no moment of creativity was noticed as occurred within the in-class lecture. Thus, online learning makes the collective creativity referred to by Glaveanu (2011) more difficult. However, there was interaction between the instructor and peers that promoted a common type of accommodation in the math classroom, what might be called scheme reconstruction through s moment of recall. In this modification, an appropriate scheme is readily available, yet details have been forgotten. This accommodation as re-construction of forgotten details is frequently observed as students transition from “participatory” schemes that require assistance to more independent schemes (Tzur and Simon, 2004). The connection is between the situation and the newly recalled details of the existing scheme, and the novel process is a reconstruction of the forgotten original process.

**Professor Wolf: In-Class Lesson:**

The in-class lecture by Professor Wolf was more traditional, teacher led. It was, however, very motivational and student centered in that the instructor constantly paused, asking students for...
input, and encouraging them to participate and seriously consider STEM careers. The lesson was
designed to introduce Calculus students to antiderivatives using substitution as the reverse process
of finding a derivative using the chain rule. The lesson began in a way uncharacteristic of
constructivist pedagogy by initially presenting the antiderivative of the trig function
\[ \int \csc(x) \cot(x) \, dx \]
and reviewing a formula sheet that had only the answer. The lesson plan then
was to introduce students to the substitution technique (viewed as the reverse process to the chain
rule derivative) and then to demonstrate how this substitution technique can be used to find the
antiderivative of composite functions. The lesson concluded with finding the antiderivative of a
trig function (equivalent to the one initially presented -using trig formulas) and finding the
antiderivative using substitution.

**Building up Shared Knowledge: Instructor Flexibility**

Professor Wolf first reviewed finding a derivative that involved applying the chain rule: given \( y = \frac{2}{3} (1 + x^2)^{3/2} \), find \( y' \). After the students found \( y' = (1 + x^2)^{1/2}(2x) \), the instructor asked them to find: \( \int 2x \sqrt{1 + x^2} \, dx \), the expectation being they would understand this procedure as the reverse
process of the derivative they had just completed. Instead a student replied, “2x is the du”!
Although this reasoning is not mathematically complete, Professor Wolf realized that the student
had seen the substitution technique previously and paused to ask her how she would proceed. The
student explained how the substitution \( u=1+x^2 \) leads to \( du = 2xdx \) and how this process can be used
to find the antiderivative. Although this presentation was a teacher-led discourse, when it came to
the new content in the lesson, Professor Wolf followed the lead of this student response. As the
lesson continued with examples using substitution, it became evident that this instructional method
resulted in the rest of the students readily grasping how to employ this technique. Note the search
process for the peers involves internalizing the suggestion “\( u=1+x^2 \)” and the result \( du = 2xdx \)” made
by their peer and the connection involves relating this suggestion to resolve the situation. In this
light student suggestions appears to assist with the process of internalization by peers i.e. conscious
imitation of peer activity is a key learning process. In this manner, instructor flexibility promotes
the shared learning experience although not necessarily student moments of insight.

**Professor Wolf, Online Learning:**

In this Pre-Calculus Course, the lesson plan was to introduce advanced topics (usually presented
in Calculus) of using the limit to find the equation of the tangent line and finding the antiderivative
by using the partial fraction decomposition techniques.

The lesson began with a review of a traditional Pre-Calculus topic, finding the difference quotient
of a quadratic. The relationship between finding the limit of a difference quotient and the slope of
the tangent line for the standard quadratic \( y = x^2 \) at the point \( (1,1) \) was discussed. Then, using
straightforward algebraic techniques, the equation of the tangent line at this point was found. It
can be difficult to determine the extent to which students comprehend the instructional method
during an online observation as one cannot view their faces; on the basis of their comments,
however, the students appeared to understand this presentation. The extent to which students
followed became clear when the instructor explained the process of finding an antiderivative by
using partial fraction decomposition, a technique that involves an extensive amount of manipulation with signed numerical values. At some point in this manipulation, instructor Wolf, confused, asked, “Where did I mess up?”  After a student pointed out the exact point in a long sequence of manipulations at which an incorrect rule of signed numbers had been used, the instructor responded, “You are a rock star! Did everyone get it?” and then continued.

Professor Wolf, Comparative Analysis:

In the first in-class lesson what came through was the spontaneous ability to pause the lesson trajectory and follow the lead of a student response. Although Professor Wolf valued and incorporated into the lesson student responses, this spontaneous incorporation of student responses was not evident in the online lesson. The online format was more tightly focused on the instructor presentation; the shortness of student comments and the brevity of these student responses made them more difficult to integrate into the lesson itself. The interchange between instructor and student or among students was much less flexible than in-person dialogue. However, the students (judged by their comments) readily followed the lesson plan, even making corrections at critical junctures, and the value that instructor Wolf gave to student comments translated into the on-line discourse.

INSTRUCTOR JOURNAL

The teaching research in this article focuses on student learning within class discourse; however, we believe that by sharing a summary of Professor Wolf’s journal entries from this period, the readers’ understanding of the transitional pandemic experience to on-line learning and how it affected the sense of a shared leaning environment will be enhanced.

The Transitional Teaching Experience:

When we heard that we might be sent home, I really did not believe that the transition was going to be a big deal or that it was going to be for long. I told my students that they should create a WhatsApp and in that way we could stay connected. I went home and did not take many of my personal belongings. However, it was obvious from the first week that we would not be able to go back to the campus, and subways and buses appeared to be dangerous.

I was already familiar with Blackboard, and all my textbooks were on my iPad in my i-Books. We were given a week to get our on-line act together. That week was fine for those who feel a natural connection to their students and for those who cared about their students and understood their needs. It was not long before we were up and running. The first experience teaching from home was a bust because Blackboard Collaborate (the College software platform) was constantly losing connection; furthermore, the students could not hear me, nor could I hear them. I instantly bought a subscription to Zoom, and we were up and running. The first thing I did was to reach out to all of my students, and those who didn’t respond I hunted down. Some of my students were essential workers, so they were called in to hospitals and essential businesses. Some of my students had Covid, or their family members did. In the midst of all the chaos, I managed to have an 85 % retention rate.
My students and I were excited and scared. We faced many obstacles like no child-care and spotty internet or no internet at all. My research student got very sick, and we gave him advice and talked to him throughout his ordeal, and he came out unscathed. The best part for me was my first Zoom class in which one student was sitting in his kitchen eating, another was lounging on her bed, while others had several family members present, so their video was shut off. It was exciting to hear and to see them, with the knowledge that they were still learning and safe. I know that for many of my colleagues it was not the same experience. I know that our undocumented students and homeless students did not have an easy time. It has been strategically almost impossible for too many of our students to juggle housing, food insecurities, plus childcare, while trying to learn how to combine like terms or differentiate a function. For sure, as professors we gave many Incompletes.

We were so excited to reconnect and we succeeded; however, it was not easy to do so. I quickly decided I needed to assess their progress personally. Did they need to download an App to be able to send homework and tests? I bought many devices to try different ways of on-line communication; the first device was a x-pen notebook in which you write. One is supposed to be able to write on it and share, but the software in my older MacBook Air did not allow this process to work. Then I bought a giant whiteboard with a tripod, on which I would set my iPhone. It was great. Although I felt self-confident, the students got confused when I went back and forth between all of those devices; therefore, I kept going back to handwriting and sharing screens on my IPad. I would use Zoom from my IPad and had another IPad set up with the textbook. I eventually bought a OkioCam USB web camera which worked wonderfully with my lectures.

In the summer I continued this on-line experience as instructor of a college algebra course; however, I believe that the spring semester was more successful because the students had previously established a relationship with me before the on-line experience. Thus, during the summer, they were quiet and difficult reserved. It is difficult when you can’t look at your students and see whether they are lost; instead we have to assume that we are all on the same page.

**SUMMARY COMPARATIVE DISCUSSION: PEDAGOIGY**

We recognize that social interaction between peers is an essential component in developing a shared learning experience, and that instructor flexibility to promote and incorporate divergent student responses is critical for student creativity. Thus, we analyze instructor methodology to motivate students and support the shared construction of knowledge, and how the shift to online learning necessitated by the Covid-10 pandemic affected this experience

**Professor Stachelek, Instructor and Student Creativity:**

Professor Stachelek implemented creative instructional methods in both the Pre in-class and Post on-line lecture. During the in-class lesson however, there was more peer-peer interaction, and the instructor-peer interaction was followed more readily by the remaining students. Indeed, a clear “Aha Moment” as characterized by a clear search, a new (discovered) connection to an existing scheme, along with resulting novel process are all evident.

**Professor Wolf: Instructor Flexibility:**
The methodology of Professor Wolf in both the in-class and on-line lesson is highly motivational and her goal was to engage the students and get them to participate. The lecture method was traditional (i.e., based upon instructor-student dialogue). During the in-class lesson, when a new technique was introduced by a student, the instructor spontaneously changed the direction of the lesson trajectory to integrate the student response. This incorporation of student response motivated the class, promoting a creative learning environment. This flexibility was lacking in the on-line lesson due to the limitations of audio and lack of visual however, it was clear that students readily followed the new content of the on-line lesson and did participate with short comments when appropriate.

CONCLUDING REMARKS

Research Question 1

In this article, the first research question focuses on the quality of the instructor-student dialogue and the ways in which such dialogue supports construction of shared knowledge before and after the expected transition from in-class to on-line learning by the Covid-19 pandemic.

Limitations of On-Line Dialogue

The transition to on-line learning reduced peer-peer dialogue. Although students did follow the dialogue between their peers and the instructor, there was a degree of separation brought about by the on-line platform. This degree of separation was to some extent caused by the inability of students to see other students. In addition, although the on-line platform could handle an instructor-student dialogue, student comments were brief, and additional comments (peer-peer) outside a teacher-peer dialogue was not observed. Instead, they tended to result in distorted audio thus, all students were encouraged to mute themselves when not directly asking a question.

During the on-line lesson, the creativity of Professor Stachelek and the motivational style of Professor Wolf were both evident. However, the transition to on-line learning resulted in a more traditional instructor-student dialogue. This was due to the limitations of visual and audio. Although it was clear that students were learning from their mistakes and following the lesson, the audio and visual limitation restricted peer-peer interaction. This limited student creativity as well as instructor flexibility to incorporate student responses into the lesson.

The transition to online learning was made one day in March 2020 without warning, as the seriousness of the situation came upon NYC unexpectedly. Thus, the instructors had no preparation for this change to an online platform. Further study is required to determine if training and experience can overcome the audio and visual limitations to the collective learning experience especially that of students observing and learning from peer’s behavior. Perhaps technological improvements in group audio, or platform features such as “small break out group” may assist in this effort. On the other hand as noted in Professor Wolf’s journal entry it is equally possible that the sense of a shared learning experience will actually be more hindered in future classes that are completely on-line, as the students will never experience a social in class experience with their peers.
Research Question 2: Theoretical Considerations

The second research question focuses on how the three criteria of: search, connection and resulting novel process characterize the continuum of moments of student insight as they grow in understanding during class discourse.

During the in-class lesson of Professor Stachelek the ‘Aha Moment’ by a student demonstrated a clear (Gestalt) search process, as she abandoned her linear scheme in favor of a quadratic scheme, the resulting connection was to the new quadratic scheme. The novel process was her ability to translate raw data into her quadratic and later her radical schemes instead of only her linear scheme.

In the on-line lesson by Professor Stachelek the moment of insight was the result of a search to recall details of an existing scheme that had been forgotten, the connection was between the situation and these recalled details. The resulting activity was the result of reconstruction of the earlier forgotten activity. In the spectrum of accommodation this moment of insight would not be considered illumination of new content.

The in-class lesson of Wolf demonstrates students that appear to grasp a novel technique (substitution) suggested by a peer. However, as the search process is limited to interpreting what the peer or teacher presents, it is difficult to determine whether student’s conscious imitation was participatory or whether their connections would allow for independent activity, During the on-line experience the connections were those of a traditional lesson i.e., between the material presented by the instructor and individual students’ schemes.

In using these criteria to analyze moments of insight, clearly the dept of the connection and the resulting new process, must be considered, and more work is needed to trace out a continuum of moments of insight within class discourse. This continuum certainly includes the ‘Aha Moment’ observed in Professor Stachelek’s class where the connection was to a novel scheme, leading to novel solution activity, and during the on-line lesson where the connection was between forgotten material in which the process was novel only in that it had been recalled. In the in-class lesson of Professor Wolf the connection although not new for the student was novel for her peers, and it did result in new activity for these peer students.

REFERENCES

Ariyanti (2020) The Effects of Online Mathematics Learning in the Covid-19 Pandemic Period: A Case study of Senior High School Students at Madiun City, Indonesia, MTRJ this volume


Covid-19 Pandemic and Its Impact on College Teaching: The Unexpected Benefits and their Consequences

Eric Fuchs, Doru Tsaganea
Metropolitan College of New York, USA
efuchs@mcny.edu, dtsaganea@mcny.edu

Abstract: The authors have been teaching mathematics and economics for more than 15 years at two nonresidential colleges in New York City: Metropolitan College of New York and Bronx Community College, which is part of the City University of New York (CUNY).

Until the first week of March 2020, the authors were teaching all their classes in-person. With the outbreak of Covid-19 epidemic, both colleges were immediately closed. The Spring 2020 semester was interrupted, and all professors were given less than 10 days to prepare their respective classes through distant education using the Zoom platform, or equivalent, for instruction.

While the transition was abrupt, the authors soon discovered that conducting some mathematics and business classes remotely yielded unexpected advantages to many students, professors and colleges alike. This article offers specific examples the authors used in teaching remotely on Zoom and discusses the improved pedagogy that can be applied only in distance learning.

With recruitment and retention of undergraduate students being a growing concern for U.S. colleges, economic survival will depend on an institution’s ability to adapt technology and flexible tuition structure in new post-pandemic future.

COVID19 EPIDEMICS: SOCIO-ECONOMIC AND EDUCATIONAL IMPACT

By the end of September 2020, more than 7,500,000 U.S. inhabitants were infected with the Covid-19 virus and about 210,000 of those infected died. The number of deaths is higher than the total number of deaths in World War I, Vietnam War and Korean War (134,512 deaths) and is equal to 2/3 of the number of deaths during World War II (291,557 deaths)¹. As a consequence, the number


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of infected people who have remained with chronic medical illnesses after recovery is probably comparable with the number of impaired veterans after World War II.

As a result of its extraordinary magnitude – comparable only with the magnitude of the Influenza (Spanish Flu) Pandemic of 1918 that caused an estimated number of 675,000 deaths in the United States\(^2\) – the Covid-19 pandemic has had and will have a very important economic, social and political impact. Revolutionary changes are taking place in the business models of some of the most powerful multinational corporations, in running political campaigns, and in all types of social activities.

Google’s employees will work from home at least until June 2021 (Fung, 2020), the Democratic and Republican national conventions during the U.S. presidential campaign were held online, and the leaders and parliament members of most countries are continuously wearing masks.

The retail and restaurant industries, the travel and leisure industry and the entertainment industry have been brutally affected. The number of scheduled passengers boarded by the global airline industry decreased from 4723 million before the pandemic to 2246 million by June 2020 (Mazareanu, 2020), and the volatility of the price of crude oil was extraordinary. It went from about 54 dollars on February 20 to 18 dollars on April 24 and to about 40 dollars currently.

These powerful and dramatic processes have been embodied in the temporary or final closure of tens of thousands of small and medium businesses. Consequently, the unemployment rate increased vertiginously from 4.4% in March to 14.7% in April, and although it has slowly decreased since May, it continues to be relatively high at about 7.9%.

In general terms the service sector of the economy has been the most vulnerable. It has been affected the most because it is more labor intensive and less capital intensive than the manufacturing sector. And this fact should be an alarm signal for higher education, because higher education is a service and is labor intensive. It is true that it is a service that uses highly educated and skilled labor, but this does not change its inclusion in the service sector or its characterization as labor intensive. A college or university is essentially defined by its professors and students, not by its buildings.

The demand for higher education is relatively elastic, and it depends as any demand of a service or commodity on quality, price and buyers’ income. Therefore, under the economic and social circumstances caused by the pandemic, the system of higher education is under high pressure.

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\(^2\) Data Source: CDC Centers for Disease Control and Prevention. *History of the 1918 Flu Pandemic.*  
Most current and potential students regard the strictly online, asynchronous, higher education (using only Moodle or a similar software) as being inferior to the traditional face-to-face, in classroom education. Subsequently, they are reluctant to pay the same tuition for a service they perceive as being relatively inferior, and expect tuition reduction in order to start or continue their higher education.

At the same time this expectative state is reinforced by the decrease in their personal income and/or in the income of their parents caused by the increase of unemployment. Finally, as a result of the conjuncture of these two factors – perception of cost quality as unfair and income reduction – naturally enrollment has the tendency to decrease.

In this context, adapting the models of higher education used by colleges and universities to the new economic and social conditions imposed by the pandemic and post-pandemic circumstances is more than a problem of efficiency improvement. It is a problem of survival for many colleges that requires the imagining of new models of higher education. It is especially serious for the small and medium colleges whose existence is strictly dependent on tuition.

TEACHING MATHEMATICS IN THE PRE-PANDEMIC WORLD

Before Covid-19 struck, the prevailing assumption was that in-person teaching is a must for good and effective mathematics pedagogy. Yet some colleges have been offering online teaching long before March 2020: correspondence classes have existed for more than a hundred years. With the advent of online technology, however, many colleges started offering online, asynchronous classes for some of their courses.

The financial benefits to a college are many: scheduling flexibility and lower overhead and maintenance costs are just two. With online teaching, colleges can consolidate several sessions, thereby reducing their reliance on contingent instructional staff.

In commuter colleges, some students have found that an online asynchronous class offers the advantage of flexible scheduling. A working student with children at home could, for example, wake up at 5 a.m. and study for a class before starting work. Other students might prefer to be at home early in the evening with their family instead of returning at 10 p.m. after attending a class that finished at 9 p.m. As great as that might sound in terms of schedule flexibility, reduced operational costs to the college and reduced transportation costs to the students, asynchronous online teaching is not the ideal way to teach college math classes.

In urban areas, many undergraduate students are unprepared for college work. Some lack basic study skills. They come to class expecting the professor to teach them topics that should have been mastered in their earlier education. For many urban students, these are the same basic mathematic
topics to which they were repeatedly exposed in high school or middle school but that they never understood sufficiently in order to do college work. Without a professor to adapt material to the students’ level of understanding, these students are totally lost.

It is not the purpose of this article to elaborate on the pedagogical tools available in an in-person class or to discuss the pedagogical methods the authors used in basic classes working with urban students. Over the years we have worked in mathematics with hundreds of students in both basic math classes and regular classes at the undergraduate, as well as graduate level. But invariably all our classes were in person, face-to-face.

We believed that urban students, as well as their professors, expected mathematics teaching to be in person, in a face-to-face class. From the students’ perspective it might have been acceptable for some of the classes to be conducted asynchronously, online, but mathematics? No way! The students were unaccustomed to working independently without a professor in the classroom. In addition, how can a student study with a couple of friends when everyone is locked in their respective apartments?

And how about using the library? For many students, the library was an oasis of calm that offered a wealth of resources, such as technology, books, proper lighting, and a quiet atmosphere away from home where students could focus on their studies. When the college closed, the library closed as well!

When the coronavirus struck with intensity in New York City, the old system of pedagogy was brought to its knees. With the college doors closed, with students and professors locked in their respective homes, we found ourselves in the middle of Spring 2020 semester with students and professors who could not go to college but expected to finish the semester and continue their academic studies, pandemic or no pandemic.

THE MOST CHALLENGING WEEK EVER

Starting on March 10, in one school after another in New York City students tested positive for the coronavirus. First the private schools closed, then the public schools. On March 16, 2020 the entire public school system in New York City, comprising more than 1.1 million students attending more than 1,700 schools, was shut down temporarily. What was supposed to be a temporary closing for the public schools in New York City ended up being a complete closing, until the start

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of the new school year in September. Following the school closings on March 10, remote or online instruction was to start on March 19.

Some college closings preceded the closing of the public schools by several days. New York University, Fordham University and Pace University announced the start of online remote teaching on March 1. SUNY and CUNY schools also cancelled all in-person classes on March 11. By March 12, both MCNY and Bronx Community College were closed, and the most challenging week of our pedagogical life began. We were given no more than 7 days to convert our pedagogy to remote, online.

During that week college professors started attending webinars and online training of how to use technology in teaching. Colleges offered training in Moodle or Blackboard Collaborate, to be used for asynchronous teaching, as well as training in Zoom or Blackboard Collaborate Ultra for synchronous teaching.

In addition to the ability to teach asynchronously, Moodle and Blackboard Collaborate provide the ability to post homework, upload lessons and other files, provide links to different websites, send and receive emails, keep attendance, etc. In the public school system Google Classroom was extensively used.

**TECHNOLOGY TO THE RESCUE: TEACHING MODALITIES**

Because institutions, articles and commentators have used different words and associated diverse concepts with online teaching, we have to briefly explain how we use these terms in this paper:

From a spatial perspective:

- Classes are considered *in-person* when students and professors are together in the same room.
- Instruction that does not take place in person is called *distant* or *remote*.
- When one part of the instruction is in person and the other part is done remotely, the class is called *hybrid*.

From a timing perspective:

- In-person classes are always *synchronous*.
- Distant/remote classes can be either synchronous (by using technology such as Zoom, Blackboard Collaborate Ultra, Google Classroom) or *asynchronous*, for example, when the class is taught with Moodle only.

Asynchronous teaching/learning is accomplished through the internet using Moodle, Blackboard Collaborate (not Ultra) software and many other platforms. Even traditional correspondence schools should be considered distance/remote, asynchronous teaching.
Courses like the ones provided by edX or Coursera, which are conducted through the internet, could be synchronous (even though they are distant/remote learning), asynchronous, or hybrid.

From the perspective of the use of technology, the terminology can get confusing, depending on the context. We are using the following definitions:

- **Hybrid** is as defined above, when instruction is partly in person and partly distant/remote.
- **Online**, or going online, usually means connecting to the internet. All classes conducted through Zoom, Moodle, Skype or Blackboard are online classes since internet access is required. It is preferable not to use that terminology, which can be confusing. In Zoom classes, for example, students must attend the class at a given time (**synchronous** teaching). This is in contrast to Moodle instruction, where students can log in whenever they want (**asynchronous** teaching). In both cases, these classes are **online**.
- **Hybrid** (capital -H) can be tricky. In many colleges, the term describes courses that are taught partially in-person and partially through distant learning.
- **Face-to-face** are classes offered in person on campus. Classes can also be face to face when offered through Zoom, Blackboard Collaborative Ultra, or Google Classroom as long as all students keep their computer, phone or camera on for the duration of the class.

Unlike Moodle, Zoom⁴ hadn’t been used in education for a very long time. Zoom was designed initially as a business teleconference tool, where a host could convene a meeting with up to 300 attendees. On their individual screen the participants have control of their microphone and camera (how a person is seen or heard by the others) as well as the way that participant sees the others (as a group or as an individual). In addition, participants can talk to each other privately or as a group using the chat mode.

The host controls who, and when, people can attend the meeting and controls the microphones and the camera of each person or the entire group. The host can connect to websites or spreadsheets, share the screen or allow participants to share theirs. The host can present slides or write on the whiteboard for everyone to see, record the entire session or parts of it, break the group into subgroups or even conduct polling among the participants. The host can even designate another participant to be the host.

Naturally, in the business world, Zoom was a godsend. The amazing thing, though, was how easy it was to adapt Zoom to education. The professor is the host; the participants are the students. The professor can share all teaching materials with the entire class or a subgroup. Instead of writing and then erasing the board to make room for new information, the professor writes or types on the

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⁴ Zoom is provided by Zoom Video Communications, Inc. For use of Zoom in education, the company’s link is [https://zoom.us/education](https://zoom.us/education)
Zoom whiteboard. But instead of having only one board, the professor has a multiple of them and can also easily return to a previous board when needed. With a bit of practice, most people can master Zoom in a few hours. Now you have the perfect replacement for in-person classes.

LIMITATIONS AND BENEFITS OF THE ONLINE TEACHING METHODS

After we have differentiated the meanings of terms, we have to describe the limitations and benefits of the methods of teaching online that have been employed.

The most usual and most frequently used method has consisted in using Moodle or similar software for two-way communication between professor and student. The advantage of this method over the old correspondence college was the speed provided by the internet. The disadvantage was the same as that of the correspondence college versus in-person college, the lack of interpersonal connection and socialization.

A second method of teaching has been the use of hybrid courses associating in-classroom instruction with online activities using Moodle or equivalent software and internet resources. For example, a three-credit course would be offered as a weekly two-credit in-person course and one credit online course, or as a course in which an in-person session will alternate with an online session. This method of teaching eliminates the deficiency of the previously presented one but cannot be used or can by only partially used during a pandemic.

The third method, which consists in using Zoom with Moodle or equivalent software, implies as we have specified before, two types of courses: asynchronous and synchronous. Its main advantage is that it allows face-to-face contact between professor and students and face-to-face student socialization without in-person contact. That is a huge advantage during a pandemic.

CLASSROOM TEACHING IN A VIRTUAL WORLD TECHNOLOGY

Starting with the third week of March, we taught a total of nine online courses. We taught seven additional online courses during the summer semester. Ten were graduate-level courses in business and education; the other six were undergraduate courses in mathematics. Approximately 250 students were enrolled in all these courses. There were some initial difficulties, such as a few students not having laptops or tablets, but these problems were quickly resolved. The colleges distributed laptops or tablets to all the students. The students also realized they could attend classes even when they were not at home simply by using their smart phone.

All classes were taught remotely, with both professors and students communicating online through Zoom platform. We found the Zoom platform to be the easiest and more adaptable to our needs.
A Zoom subscription allows up to 300 hundred people to communicate simultaneously. There is no time limit for each session. We were able to easily handle classes that last two hours or more.

For years we got used to the idea, we were convinced, that to teach mathematics to a classroom, you have to do it in person with a whiteboard and markers. Dispensing of the board when teaching mathematics was anathema to most mathematics professors. One of the authors was thinking of buying an easel and markers to direct the camera toward the easel. Maybe two cameras would be required, maybe two computers?

His initial thinking was to emulate the classroom in a virtual world. It was a truly quantum leap in our pedagogical thinking to abandon the physical whiteboard and teach the entire lesson from a laptop. We quickly entered a new era, into a revolutionary era of pedagogy, an era that we call the Zoom Revolution.

What we experienced was a true technological revolution in college education! The host, in our case the professor, controls who can attend the session, whether students can join the session before the class starts or whether they have to wait in a “waiting room.” The host decides whether to mute all or some students before class starts or at any time during the class. Attendees have control over their camera and their mute button.

Students can send messages to the professor, the entire class or each other. They can ask the professor to repeat an explanation or to slow it down. The host can present slides or write on the whiteboard for everyone to see, record the entire session or parts of it, break the group into subgroups or even conduct polling among the participants. Likewise, the host can even designate another participant to be the host.

In a similar vein, the professor can make another student a host, with all the privileges that position confers. The professor can share the computer screen with the entire class and allow the sharing of screens among individual students. The professor can also record the class session partially, or in its entirety, for distribution to the attendees or to students who missed the session. The recording file could be stored on the computer or in the cloud.

Zoom offers many additional features of the system such as ability to conduct polling or to separate the class into several groups. The professor, as well as the students, has access to a white board that can be shared with the rest of the class. The professor or a student can even type on the whiteboard, display an entire PowerPoint presentation and provide links to the internet.

With a bit of practice, most people can master Zoom in a few hours. Now you have the perfect replacement for in-person classes.
In this context, to get the most out of a Zoom session with our students, we recommend the following rules:

- Classes should be held at fixed times, just like in-person classes.
- Attendance should be mandatory for all classes and for the entire session.
- Students should have the microphone off unless they are invited to speak.
- Cameras should be on for the entire session.
- Students should use their complete roster name.
- Attendance should be taken 10 minutes after class starts and 10 minutes before class ends.
- A four- or five-minute break should follow any lecture longer that 25 minutes. Students complained that they lose concentration during long lectures/sessions.
- To increase effectiveness, each session should include interactivity, group work, guided discovery, and other group processes.
- All sessions should be recorded. This will help students who have poor internet access or students who miss a Zoom class.

By following these simple rules, a Zoom session is as or more effective than an in-person class.

**APPLYING TECHNOLOGY TO OTHER LEARNING ACTIVITIES**

Naturally, in addition to the scheduled class time, learning happens outside the classroom during the students’ spare time. In addition, in a college course there is a lot of communication that goes on outside the classroom between the professor and students (in both directions) and among the students themselves. This applies to homework and tests that have to be submitted on time, grading of assignments, discussion boards, group or individual e-mails.

We have used two platforms to achieve these objectives: in one college the Moodle platform, the Blackboard platform in the other college. With a little learning on behalf of the professor and the students, both platforms did the job very well.

By combining a synchronous technology platform (in our case Zoom) with an asynchronous technology (Blackboard or Moodle), we were able to teach in a virtual classroom and do all the pedagogical and administrative tasks. The students were able to complete the spring semester and to start and complete the summer semester as well.

As far as which technology is preferable to use, this article is not the place to recommend one technology over another. Suffice it to say that it was possible in a very short period of time to switch from in-person teaching to remote teaching and to complete two semesters of classes with a large number of students.
TEACHING MATHEMATICS AND BUSINESS IN A VIRTUAL WORLD

The specific courses that we taught (some of them more than once) since the pandemic started have been the following:

In MCNY School of Education:

- Teaching Mathematics in Grades 1-3 with technology. This is a master-level course.
- Teaching Mathematics in Grades 4-6 with technology. This is master-level course.

In MCNY School of Human Services:

- Basic Mathematics. This is an undergraduate, noncredit course.
- Mathematics for Finances. This is an undergraduate course.

In MCNY School for Business:

- Managerial Economics. This is a master-level course.
- Managerial Finance. This is a master-level course.
- Global Busines and International Practicum. This is a master-level course.

In Bronx Community College:

- Basic Mathematics (mathematics for grades 1-6). This is a noncredit course.
- Precalculus. This is an undergraduate course.

All the above classes were taught with Zoom for the virtual face-to-face sessions. All courses were taught on the same schedule as pre-pandemic courses used to be taught. We used both Moodle and Blackboard for the asynchronous tasks: posting assignments, students returning assignments, course announcements, discussion boards, posting reading materials and links to different websites.

Equipped with our students’ college email addresses, we were able to immediately invite all our students to the Zoom sessions, which we scheduled at exactly the same time as the regular classroom sessions. We used Zoom for the virtual face-to-face teaching, interaction with the students and students themselves during classes (visual, verbal, chat, and symbols) and for splitting the class into working groups.

Our students did not have to wait in a waiting room for us to open the sessions. Neither did they have to quit at the end of the class. That way Zoom bridged the geographical distance. Students who never met in person became friends and study buddies over the internet. We encouraged the students to get their own Zoom platform (free for each 40-minute session for a small group) and to use it outside the classroom time with their study groups.
The students even established times of the week at which the small group regularly met to study together. Students who before would have had at best a few minutes to talk to each other at 9 p.m. after an in-person class, were now spending time exchanging pleasantries and getting to know about each other’s family, children and even pets. Zoom became thus not only an instrument in teaching but also of socialization. From a mathematics or business perspective, the learning in small groups enhanced the efficiency of learning. We discovered that at times learning with a couple of students is much more efficient than listening to a professor’s lecture.

**SPECIFIC EXAMPLES ON TEACHING MATHEMATICS SYNCHRONOUSLY**

As mentioned already, all our classes were taught on Zoom. To write the equations or any notes instead of a physical whiteboard, the professor used the Zoom whiteboard or at times another electronic whiteboard.

Writing equations by hand on the whiteboard was accomplished through a Wacom tablet\(^5\) connected to the laptop through Bluetooth. The Wacom tablet costs less than $100, but requires some practice, similar to typing without seeing the keyboard while looking at the screen. In the endnotes we provide a link to a video demonstrating how to use the Wacom tablet for teaching mathematics online.

Other technological solutions exist such as using a touch-screen computer with a stylus. These computers come equipped with software allowing writing on the Zoom or other whiteboard. To display the graphs we used screen-sharing and we accessed a graphic calculator online. We used Desmos graphic calculator\(^6\), but other software programs could do the job as well.

For a professor to write on a physical board with colored chalk and colored markers, erasing the board the board each time the board fills up, it is a real discovery to use a virtual board containing literally an infinite number of pages. The professor can work on page number 5, for example, and return to page number 1 or 2 to answer a student’s question. Naturally, in the physical world the previous pages disappeared when the board was erased.

Since our classes are composed of students from different disciplines (business, education, human services, nursing, engineering), when working on the examples we discussed their applications in the different disciplines. For example:

In business:

\(^5\) The Wacom Tablet is manufactured by Wacom Co., Ltd. For how the tablet can be used in an interactive classroom, the link is [https://www.wacom.com/en-us/discover/educate/interactive-classroom](https://www.wacom.com/en-us/discover/educate/interactive-classroom)

\(^6\) Desmos Graphic Calculator is used in precalculus and calculus courses. The calculator is provided by Desmos, Inc. The link is [https://www.desmos.com/calculator](https://www.desmos.com/calculator)
o the intersection between supply and demand curves
o profit optimization under different constraints: the slope of the curves, the meaning of the asymptotes

In human services:
o number of cases of Coronavirus; maxima and minima; “bending the curve”
o future projections under different scenarios
o use of exponential functions to predict population growth

In engineering:
o combination of forces, each one represented by a different function
o composition of functions in microchip construction

The examples below show how the actual mathematics appeared on the Zoom screen of our students. The students used screen shots at different stages of solving a problem. By using Desmos graphing in conjunction with the whiteboard (on which we wrote or typed), we appealed to the students’ mathematical thinking. We did not provide them the solutions but helped the students to explore each step and to find the solutions themselves.

Example 1: Transformation of functions

We aimed for the students to understand the relationship between algebra and analytic geometry. We stressed that the same transformation concepts apply to any function, not necessarily polynomial, as will be seen in example #2 below.

We started by using the basic quadratic function, \( y_0 = x^2 \) (Figure 1). Students were asked how the function should be modified algebraically to obtain the following graph movements:

- \( y_1 \), by vertical translation, 3 units up (Figure 2A)
- \( y_2 \), by vertical translation, 2 units down (Figure 2B)
- \( y_3 \), by horizontal translation, 2 units to the left (Figure 2C)
- \( y_4 \), by horizontal translation, 3 units to the right (Figure 2D)
Figure 1: Graph of $y_0 = x^2$ in Desmos online graphing calculator.

Figure 2: Vertical and horizontal translations of the function $y_0 = x^2$. 

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Reflect the function $y_4$ from Figure 2D over the x-axis to obtain $y_5$ (Figure 3).

![Figure 3](image)

**Figure 3:** Reflection of $y_4 = (x - 3)^2$ over the x-axis.

Stretch the function $y_4$ from Figure 2D vertically by a factor of 2 (Figure 4A) and horizontally by a factor of 0.5 (Figure 4B). Find the zeros of each function.

![Figure 4](image)

**Figure 4:** Stretching of $y_4 = (x - 3)^2$ by a factor of 2 (Figure 4A) and 0.5 (Figure 4B).
Example 2: Transformation of functions – A non-polynomial function

Given the function $F_1 = \frac{1}{x^2}$ (Figure 5), move the graph first 2 units to the right (Figure 6A) and then 5 units up (Figure 6B).

![Figure 5: Graph of $F_1 = \frac{1}{x^2}$ in Desmos online graphing calculator.](image)

![Figure 6: Graph of $F_1 = \frac{1}{x^2}$ moved 2 units to the right (Figure 6A) and 5 units up (Figure 6B).](image)
Example 3: Rational functions

Given the rational function \( y = \frac{2x+1}{3(x-2)(x+1)} \), (Figure 7), examine the graph for the vertical and horizontal asymptotes.

- Find the vertical asymptotes by equating the denominator to zero.
- Why is the x-axis the horizontal asymptote?
- In a given interval, could a function have more than one horizontal asymptote?

Figure 7: Graph of \( y = \frac{2x+1}{3(x-2)(x+1)} \) in Desmos online graphing calculator with vertical asymptotes \( x = -2 \) and \( x = 2 \) indicated.

Example 4: Identification of a demand curve

This is an example of demand curve identification presented to students taking the Managerial Economics course to illustrate the theoretical concepts of demand function and demand curve.

The manager of a gas station decided to identify his customers' demand curve for gasoline in order to improve the profitability of his gas station. Subsequently, he collected data during a three-month interval of time and on this basis created the following table indicating the relationship between the price of a gallon of gasoline and the average number of gallons sold per day. Let’s determine
the demand curve using EXCEL and observe its shape depending on showing the price on the horizontal axis or of the vertical axis, considering that in economics the price is shown on the vertical axis although it is the independent variables.

Figure 8: Identification of a demand curve

TEACHING ONLINE VS. TEACHING IN PERSON- COMPARISON: COSTS AND BENEFITS

As we learned this summer, *synchronous* online instruction – regular Zoom classes in conjunction with an asynchronous platform (such as Moodle, Blackboard, or equivalent) – is as effective as, or better than, in-person instruction.
Within this framework our experience of seven months, from March to September 2020, of teaching exclusively online because of the pandemic have shown to us that the online teaching model has not only costs but also benefits.

The main costs have been as follows:

- impossibility to use library resources, computer laboratories and copy machines, the colleges being closed;
- limited contact between students and administrative services;
- very significant difficulties in advertising for recruitment caused by the impossibility to have face-to-face discussions between recruiters and potential students, and the drastic reduction of subway and bus ridership, making subway and bus advertising useless;
- the potential students’ reluctance to register to a college that is supposed to offer regular face-to-face courses but that offers in fact only online courses; and
- the steep intensification of the competition between private colleges used to provide traditional in classroom education and the exclusively online universities caused by the significant difference in tuition for providing the same type of service.

In parallel, the benefits of teaching exclusively online have been as follows:

- retention of those students who would have discontinued their studies for fear of coming to the college because of the pandemic;
- registration of new students who had agreed to take exclusively online classes but would have not agreed to take face-to-face courses during the pandemic;
- very significant time savings for students, faculty and administrative staff taking into consideration that the average time for two-way commutation in New York by using subway, bus or both is about one and a half hour (Kolomatsky, 2018);
- students’ results as reflected in grades as good than those obtained during face to face in classroom instruction;
- development of less formal, friendlier relations among students and faculty in the case of Zoom, the students having the feeling that they are invited in professor’s living room instead to be in the formal setting of a college classroom; and
- financial savings by the college due to reduction of some administrative staff and utilities.

In our opinion the synchronous courses using Zoom and Moodle or an equivalent software are the closest in efficiency to the in-person courses. They are undoubtedly superior to all other types of online courses, and although they do not offer the same level of socialization as in-person courses, they have some advantages in comparison with those courses that cannot be neglected.
Besides the obvious advantages of being offered during a pandemic without any risk of infection, the synchronous online classes allow considerable savings in commuting time in the case of the colleges and universities located in big cities and excellent higher education for students living at long distance or in remote areas. They can also attract a considerable number of foreign students who want to receive a very good higher education and are able to pay the tuition but cannot afford to come into this country because their financial means are limited.

Expressing our views on methods of teaching online and our appreciation for synchronous courses using Zoom and Moodle, we are also aware that the selection of one method or another should be dependent on courses’ specificity. In this way, it is probable that nearly all social sciences courses can be given synchronously, but the physics and chemistry courses requiring laboratory work as well as the medicine and polytechnical courses, should be offered as hybrid. The mathematics courses can be also offered as synchronous courses, but in a specific manner that is described below.

Taking into consideration these remarks, we recommend using Moodle (or Blackboard) for posting topics to be studied, readings and assignments, as well as for receiving completed assignment from students. But we stress that for students and professors, Zoom + Moodle is a greater teaching methodology considering:

- Savings in transportation time and costs.
- Better classroom attendance and a safer teaching environment during the pandemic.
- A more pleasant teaching atmosphere during the pandemic.
- Infinite white board space: no need to erase information and easy retrieval of information from previous boards.
- Ability of students to audit sessions in other courses.
- Opportunities for students to “meet” between classes for small-group study.
- Increased socializing between classes.

For the college, the many advantages of teaching through Zoom + Moodle include the following:

- Ability to consolidate classes and increase class size.
- Lower operational costs.
- More satisfied students, according to our discussions with many students and the fact that at MCNY most student most students registered for the fall semester.

**THE POST-PANDEMIC FUTURE**

The economic and social effects of Covid-19 pandemic are comparable to those of other major tragic events of American history. But as happened in the past, it is highly probable that this nation will be able to defeat this major crisis and to become more powerful in its aftermath.
After the Civil War the American system of social, political and moral values became unitary, and the economic growth of the republic was so high that in 1892 the U.S. economy became the largest in the world, this country’s GDP surpassing the GDP of the United Kingdom. (Haddow, 2008) In the aftermath of the Great Depression were established the bases of the welfare state and the creation of social security system. During and after World War II, the United States’ economic, military and political power grew rapidly, this country being recognized after 1950 not only as a great power, but as a super-power.

By the same token, during the tumultuous years of the Vietnam War, America was able to put humans on the Moon and 15 years after the humiliating end of that tragic war the communist system collapsed in Eastern and Central Europe. Two years later it also crashed in Soviet Union and the former communist superpower disintegrated. That brief period remained known in history as the “unipolar moment,” (Krauthammer, 1990) and since then the United States has been the sole super-power, its aggregate power – economic, military and political – being unmatched by the aggregate power of any other country.

Remembering these historical events and considering the American nation’s resilience and creativity, it is normal to expect that the changes generated by the Covid-19 pandemic will be remarkable. It is obvious that like the other pandemics, this one will also end – probably by the end of 2021 or during the spring of 2022. But the economic and social life after the pandemic will not be the same as before.

The driving force behind the changes – both economic and social – will be the modification in the structure of aggregate demand. Those corporations, institutions and private providers of goods and services who will respond to the new types of demand will survive and prosper. But the ones who will be waiting for the return of a completely unchanged pre-pandemic demand will decline, go bankrupt or perish.

CONSIDERATION FOR COLLEGES: TEACHING OTHER COURSES THAT REQUIRE MATHEMATICS

Our positive experience in teaching mathematics and business courses online suggest to us that other courses requiring mathematics can be successfully taught online. Among these are the courses on mathematical statistics, applied statistics and theoretical physics as well as mathematical modelling in various areas of scientific inquiry. These might be social sciences like macroeconomics, microeconomics, sociology, international security analysis or strategy, and physical sciences like meteorology or ecology.

Teaching advanced economics and international relations courses at master and doctoral levels requires good knowledge of differential and integral calculus, differential equations, dynamic
system theory, and mathematical methods of optimization. In parallel mathematical simulation and quantitative analysis require basic to medium knowledge of numerical calculus and computer programming.

But, unfortunately, in the contemporary United States, the mathematical education in high school and in many undergraduate programs is limited or very limited. (Klein, Rice & Levy, 2012) As a result, true advanced economics and international relations courses that necessarily require the use of advanced mathematics are not offered, or if they are given, they are “advanced” only with regard to their name but not to their content. They use numerical examples to prove particular, instead of general, properties and relations that would have been previously proved by using appropriate mathematical methods.

Under these circumstances, teaching online by using simultaneously Zoom and Moodle (or other similar software) could facilitate the offering and learning of courses requiring the knowledge of advanced mathematics for several reasons. Between the classes, students could meet and study in small groups using Zoom.

From the students’ perspective, taking courses online increases their familiarity with technology, and develops their propensity and ability to think in a rigorous logical manner. Consequently, they become interested in using computers for solving problems that they had not considered before as being resolvable by using quantitative analysis. In this way, understanding the utility of mathematical knowledge and reasoning, many students have the tendency to spend more time for increasing their proficiency in mathematics, and as a result they become able to take advanced courses requiring mathematics.

From the professors’ perspective, teaching online courses in economics and international relations requiring advanced mathematical knowledge might be also interesting and attractive for several reasons.

This method of teaching allows the split of a three-hour course in two equal sessions or in a lecture of two hours and a problem-solving seminar of one hour without any programming difficulties and addition of commuting time. Such a split is useful in courses requiring mathematics because the students have sufficient time between the two sessions to understand various mathematical subtleties and calculations techniques presented during the lecture and on this basis to work on applications during the seminar.

The method is also useful for computer simulation of domestic and global economic processes, strategic situations, economic or military crises, etc. Testing and analyzing various scenarios proposed by students usually required more time than that allocated to a usual in-person session. Therefore, doing the simulation at home, and combining a synchronous lecture with an
asynchronous seminar might be beneficial for both, professors and students. For professors, because they could give to their students all the knowledge that they want to convey to them, and for students because they could contact their professors easily.

Taking into consideration that there are often significant differences in students’ mathematical background, the professors can offer online tutorials or increase the duration or frequency of the online equivalent of office hours without too much additional effort.

**CONSIDERATION FOR COLLEGES: RECRUITMENT AND RETENTION**

Under these assumptions, institutions of higher education face and will face considerable challenges, the aggregate demand for higher education being essentially affected by three main factors. One is the change in students’ preferences determined by the experience accumulated in taking online courses using advanced online teaching methods. Another one is the development of psychological and social habits associated with working from home. And the third factor is the decline in income caused by the pandemic. The influences of these factors are conjugate, and they must be considered in analyzing recruitment, retention, tuition structure and advertising.

With regard to recruitment, the recruitment officers should take into consideration that during the last seven months most high school students were taking classes online and many employed people were working from home. Consequently, they became relatively well familiarized with learning and working online and are able to assess its advantages and costs.

Depending on the method of teaching online, they might assess their learning experience in itself by comparing it with in-person instruction. But regardless of how they evaluate their learning experience, they appreciate the time and cost savings caused by elimination of commutation to college, as well as the opportunity to supervise any children they may have.

The students also appreciate the ability to access the saved Zoom lectures at any time and as many times as they want, and the decrease of the formality of the professor-student relationship in the classroom. A Zoom lecture or a Zoom meeting attended by students and professors staying in their living rooms and being casually dressed is psychologically relaxing and allows in some cases a better concentration on the discussion topics than a traditional in-person session in a college classroom.

Concerning retention, colleges should consider that in nearly all cases their students had taken online classes for periods ranging from three months to six or seven months and have developed sufficiently clear opinions regarding the value of different types of online teaching and learning. As a result, they are comparing the financial and social costs of continuing at the same college with those implied by the transfer to another college. In this context, if they do not live in the city
where the college is located, the students consider not only the tuition but also room and board expenditures, and these expenditures are considerable in the big cities.

Subsequently the decision of continuing to study at the same college or to transfer to another one is based on a cost-benefit analysis, and the college administrators must do their best in order to understand students’ criteria of assessment and decision making. Proper understanding of these factors will help colleges maintain or increase retention level; neglecting them will result in lower retention.

CONSIDERATION FOR COLLEGES: FINANCIALS

Strongly associated with recruitment and retention is the tuition. The amount and structure are becoming today even more important problems for colleges and students than they had been before the pandemic. The main causes are the tough competition with the strictly online universities that offer lower tuition, and the steep decrease in income caused by pandemic. Under these circumstances, traditional colleges should increase programs flexibility and differentiate tuition depending on the programs’ characteristics.

For example, a college might offer in the same field of specialization the following types of programs, the tuition decreasing from top to bottom:

- all courses are given in person;
- some courses are offered in person and some courses online, using Zoom for lectures and meetings, and Moodle or equivalent software for posting syllabi, discussion topics, reading resources and assignments, as well as for receiving students’ completed assignments;
- all courses are given online using Zoom and Moodle or equivalent software, and complete access to all college physical facilities like libraries, reading rooms, computer rooms, laboratories, sporting halls and fields, etc.;
- all courses are provided online using Zoom and Moodle or equivalent software without access to college physical facilities.

In parallel both colleges and universities that have graduate programs besides the undergraduate ones might increase the attractiveness of their graduate programs by offering tuition reduction to their former undergraduate students interested in enrolling in graduate studies.

CONSIDERATION FOR COLLEGES: ADVERTISING

Finally, new advertising methods should be promoted, or the focus should be changed from one method to another. For example, observing that in New York City subway and bus ridership has decreased vertiginously during the pandemic, the colleges should concentrate on advertising online. At the same time more money should be spent for international advertising, taking into
consideration that many international students would like to have a degree from an American university and would be able to pay the tuition but would not be financially able to come and live in the United States.

Advertising flexibility and diversity should increase, and content should be enlarged to appeal to a wider audience. Besides the traditional objectives, advertising should also focus on the following ideas:

- the courses offered by using Zoom and Moodle or equivalent software are as good as the in-person courses;
- the students have the freedom to move from online programs to in-person programs and vice-versa without financial and administrative penalties;
- the Zoom meetings allow reasonable socialization; and
- the colleges take into consideration students’ financial interests, and subsequently promote fair tuition policies by charging different tuitions for in-person, online and hybrid courses.

CONCLUSIONS

These are a few suggestions that we would like to make on the basis of our experience of exclusively teaching online for seven months. We are aware that the colleagues from around the country have accumulated an interesting and challenging experience since the beginning of the pandemic. Consequently, we believe that the publication of their ideas and suggestions would be very valuable for the advancement of higher education, and we invite them to join the discussion.

References


Fung, B. (2020, July 27). Google will let employees work from home until at least next summer. CNN


Solving the Math Anxiety Problem Before It Starts

Patricia D. Stokes and Andrew Sanfratello

Barnard College, Columbia University and CUNY

“Can we do math instead of watching the movie?”

Kindergartener

Abstract: A student did ask that question. A whole class enthusiastically did math instead of watching the movie (Cardinale, 2019). Where? In a public school in Lodi, New Jersey, where kindergarteners learned how to think like mathematicians: in numbers, symbols, and patterns (DiLeo & Stokes, 2019). The learning was based on an early math intervention using an explicit base-10 count, a single manipulative, and deliberate practice (Stokes, 2014a, 2014b, 2016a), and its expansion from one school (the pilot) to five (the district). The intervention was designed using a problem-solving model of creativity/innovation (Stokes, 2006, 2016b). This paper reports on an equally successful expansion in first grade. It is the first to report success in two ways: acquiring the math and not acquiring the anxiety. Problems with, and implications for, early math curricula are discussed.

INTRODUCTION

Mathematical performance depends on working memory. Math anxiety – a negative emotional response associated with hyperactivity in the right amygdala (Young et al., 2012) – has been shown to impact mathematical performance by interfering with/compromising working memory (Ashcraft & Kirk, 2001; Beilock, 2008; Hembree, 1990; Vukovic et al., 2013) in tasks ranging from retrieval in elementary students (Ramirez et al., 2013) to computational span tasks in college students (Ashcraft & Krause, 2007). Given the strong relationship between a child’s working memory span and mathematical performance (Adams & Hitch, 1997), math anxiety is presumably a factor in low early math scores. Given the converse – that low early math scores are a factor in math anxiety (Meece, Wigfield, & Eccles, 1990) – Harari et al. (2013) examined several dimensions of math anxiety in first graders. Their results showed that one dimension (negative feelings) was related to lack of mastery in “foundational mathematical concepts” such as counting, while another (numerical confidence) was related to lack of mastery in computational skills, like addition.

Counting and computation are basic skills, which makes it puzzling that there are (to our knowledge) no studies relating math anxiety to what we perceive as its root cause: early math
curricula that do not foster mastery in numeric-symbolic patterns. What sorts of patterns? One example is the commutative property of addition: numbers can be added in any order to get the same answer. For example, \(4 + 2 = 2 + 4 = 6\). The commutative property of multiplication is similar: numbers can be multiplied in any order to get the same answer. For example, \(4 \times 2 = 2 \times 4 = 8\).

Mathematicians think in these kinds of patterns. The current intervention was designed to teach children how to think like mathematicians. Its name, *Only the NUMBERS Count*®, summarizes the strategy behind its success. The strategy is simple: *immersion in the strictly mathematical*, i.e., numbers, symbols, and patterns. Since the strategy was based, not on math education *per se*, but on expertise and problem solving, we begin with a brief discussion of expertise before describing the problem-solving model used to design the intervention.

**EXPERTISE**

Expertise is domain-specific. An expert is someone who has mastered the materials that define their area of expertise. Experts know more than novices, notice more than novices (Stokes & Gibbert, 2019), and construct more effective problem spaces than novices (Weisberg, 2006) because what they know is organized in ways that facilitate problem solving. That organization is based on patterns as well as on understanding the relationships that underlie those patterns.

**Expertise and Patterns.** Experts think and problem solve using large meaningful patterns in the *languages of their domains* (Chi, 2011; Chi, Glaser, & Farr, 1972). For diagnosticians, the patterns are represented by symptoms, behavioral and biological; for composers, by scales, pitches, rhythms, and sonorities; for chess masters, by legitimate board arrangements.\(^7\) For mathematicians, the patterns are represented by numbers and symbols.

These numeric-symbolic patterns are stored in long-term memory in what Chase and Ericsson (1982) called “retrieval structures.” The structures provide “slots” for rapidly storing relevant information in long-term memory. The structures are analogous to what are now called associative networks. The slots correspond to nodes connected to other nodes with related content. For example, a novice network for addition might contain the nodes and connections shown in Figure 1. The “flip” is what children (in our program) call the commutative property of addition, e.g., numbers can be added in any order. When they are more expert, the mathematically precise term would be added to their network.

---

\(^7\) A legitimate arrangement is a board position reached by correct moves of each chess piece (Chase & Simon, 1973).
Now, imagine the network expanding when our novice also learns that $2 + 3$ also equals $2 + 2 + 1$ which equals $4 + 1$ or $1 + 4$, all of which equal $5$. (Notice how, in this way, procedural knowledge generates conceptual knowledge: numbers are combinations of other numbers.) Imagine it expanding further when our novice learns that subtraction “un-does” addition. To “un-do” is analogous to the more mathematically precise process of finding the “inverse.” In each case, the result is taking the original output value and mapping the solution back to its original input value. An expert would have “inverse” (and its connections to addition/subtraction and multiplication/division) in their associative network.

Compared to our novice’s, an expert mathematician’s network would be extensively patterned and integrated, allowing the expert to readily access and retrieve those patterns\(^8\) (Ericsson & Kintsch, 1995), and to efficiently expand and elaborate them (Nokes, Schunn, & Chi, 2019). The more you know, the easier it is to know more.

**Expertise and Deliberate Practice.** In well-established domains, the patterns that constitute expert knowledge are acquired (in a semi-established order) in a process called deliberate practice (Ericsson, 2006). Deliberate practice is focused (on specific patterns), continuous (in developing those patterns) and variable (in elaborating them). Deliberate practice is how procedural knowledge is acquired and expanded.

---

\(^8\) The ability to overcome the capacity limits of short-term memory is called skilled memory or Long-Term Working Memory (Ericsson & Kitsch, 1995).
THE PROBLEM-SOLVING MODEL

The model is based on what Newell and Simon (1972) called a problem space. A problem space has three parts: an initial state, a goal state, and, between the two, a search space, in which paired constraints structure a solution path that changes the initial into the goal state (Reitman, 1965; Simon, 1973; Stokes, 2006). One of the paired constraints precludes something specific in the initial state; the other promotes a substitute. The process is called solution-by-substitution. The process is how the curriculum was created.

As shown in Table 1, the initial state was current early math curricula, characterized by elements in the preclude column of the search space. The goal state was a new curriculum with a very specific criterion, teaching children to think in numbers, symbols, and patterns. The new curriculum is characterized by the substitutions shown in the promote column. The promote column is the solution path. We consider each preclude-promote pairing in turn.

Table 1. Problem-space for Only the NUMBERS Count©

<table>
<thead>
<tr>
<th>Initial State: Current early math curricula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Search Space: Paired constraints</td>
</tr>
<tr>
<td><strong>Preclude</strong></td>
</tr>
<tr>
<td>Non-numeric</td>
</tr>
<tr>
<td>Current count</td>
</tr>
<tr>
<td>Multiple manipulatives</td>
</tr>
<tr>
<td>Split practice</td>
</tr>
<tr>
<td>Goal State: New curriculum</td>
</tr>
<tr>
<td>Criterion: Thinking in numbers, symbols, and patterns</td>
</tr>
</tbody>
</table>

**Primacy of Numbers, Symbols, and Patterns.** There are two reasons to preclude the non-numeric from early math education. First, we want children to begin thinking like mathematicians. Experts problem-solve in the language of their domain (Chi, 2011; Chi et al., 1986). For mathematicians, the language is numbers, symbols, and patterns, not words. Fluency in a language depends on time of introduction – early is important – and time spent practicing –
immersion is important (Johnson & Swain, 1997). Second, to solve a word problem, you need a mathematical model on which to map it. This is why we wait until later in the school year to teach children how to ‘translate’ word problems into math problems.

**Explicit Base-10 Count.** English number names were precluded because they obscure the base-10 pattern of the count. In their place, we substituted number names based on Asian language counts, which make the patterning explicit. Our English language version of the explicit base-10 Asian count is shown (in part) in Table 2. There are four things we want to emphasize. One, the first ten number names (1 to 10) combine in *iterative patterns* to form higher numbers. Two, this makes every number name *quantitatively concrete*. Three, ten appears in every number above ten up to 100 (e.g., 12 is ten-two, 22 is two-ten two). Four, ten is a unit. It is not ten ones, it is one ten.9

**Table 2. Explicit base-10 count in English.**

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tens</th>
<th>Twenties</th>
<th>Etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
<td>21</td>
<td>31</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>22</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>23</td>
<td>……..</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>24</td>
<td>44</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>25</td>
<td>45</td>
</tr>
<tr>
<td>6</td>
<td>16</td>
<td>26</td>
<td>……..</td>
</tr>
<tr>
<td>7</td>
<td>17</td>
<td>27</td>
<td>57</td>
</tr>
<tr>
<td>8</td>
<td>18</td>
<td>28</td>
<td>58</td>
</tr>
<tr>
<td>9</td>
<td>19</td>
<td>29</td>
<td>……..</td>
</tr>
</tbody>
</table>

**The Single Manipulative.** We precluded multiple manipulatives because they are distractions: the things counted (10 *straws*, 10 *paper clips*) are more salient than the commonality.

---

9 As a consequence of their count, Asian children think of numbers as combinations of 10s and 1s. This eliminates the place-value problem (Fuson, 1990).
of the count (10). Our substitute, called the Count-and-Combine Chart, remedies this problem by borrowing two things from the abacus: first, it makes the base-10 patterns visible, tangible, and concrete; second, it only represents numbers and patterns.

<table>
<thead>
<tr>
<th>1</th>
<th>=</th>
<th>One</th>
<th>=</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>=</td>
<td>Two</td>
<td>=</td>
<td></td>
<td>+</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>=</td>
<td>Three</td>
<td>=</td>
<td></td>
<td>+</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 2.** Three lines from the 1 to 10 Count-and-Combine Chart.

Figure 2 shows the first three lines of the 1 to 10 Count-and-Combine Chart, all parts of which were moveable. Children recited the rows this way: “Number 1 same as word one equals one block. Number 2 same as word two equals two blocks…” They arranged and re-arranged the blocks (laminated poster board) to make combinations, like the ones shown on lines 2 (1 + 1) and 3 (2 + 1). They also used bags of loose “blocks” to make addition combinations at their tables to supplement their work with the Count-and-Combine Chart. The word ‘combination’ was used to emphasize the fact that numbers are combinations of other numbers.

**Why is our count so complicated?**

After the first decade of numbers (from 1 through 9), it would make the most sense for the numbers 11 through 19 to linguistically mimic the numbers 1 through 9. This would make the obvious connection to emphasize our base-10 system. Unfortunately, due to the variances and evolution of language, the connection is not obvious at all.

The words eleven and twelve trace their etymologies back to the terms “one left” (over ten) and “two left” (over ten), respectively (and also reflect the remaining traces of a base-12 number system no longer utilized today). The words thirteen through nineteen are more or less bastardizations of three-ten, four-ten, five-ten, etc., which have evolved over generations. For the decades that follow there does exist a stronger connection to the first decade of numbers. Each two-digit number from 21 through 99 requires only the knowledge of the word for the tens digit (e.g., twenty, thirty, forty, etc.) followed by the single digit name word for the units digit. Each word for the tens digit needs to be committed to memory for the learner, which each bear some commonalities of their stem (e.g., two and twenty; three and thirty; four and forty; etc.), but once these are known, the pattern repeats itself each decade. Beyond two-digit numbers, the language gets more streamlined. We do not have a special word for, say, 600 or 6,000, the way we do for 60, merely the name for the single digit, followed by the place value.

---

10 Notice that the equals sign is alternatively called ‘same as’ and ‘equals.’ The first term defines the second.
This obfuscation is not unique to the English language either. Romance languages, American Sign Language, Farsi, and Hindi each behave similarly to English, in that for numbers below 20 there is complexity, while beyond the number 20, there is uniformity; though French has even more peculiarities in number names above 70. In German, the ambiguity lies between the numbers 21 and 99, where the numbers are read more or less from the units digit to the tens digit, so the number 54 is roughly four and fifty.

Fortunately, children learn language easily and, as shown in our classrooms, quickly become fluent in the explicit base-10 count.

Figure 3 shows the lines for 11 (ten-one), 12 (ten-two), and 13 (ten-three) in the Ten to Two-Ten (20) chart. Notice that ten is represented by a single block marked ’10.’ One combination appears on each line. For 13, the combination shown is ten-one plus two. As the school year progressed, children were also able to decompose the ten, for example, $6 + 4 + 3 = 13$. Once children mastered the combinations for 1 through 10, subtraction was taught\footnote{Children in two of the four expansion schools had already done subtraction (this way) in kindergarten.} conceptually (as un-doing addition) and procedurally (using green blocks for the minuend and red ones for the subtrahend). Children repeated the phrase “subtraction means taking away” as they simultaneously removed one red block and one green block until only green blocks were “left.” When solving problems with 10s and 1s, they learned to “take away” the 10 blocks first.

![Figure 3. Three lines from the Ten to Two-Ten Count-and-Combine Chart.](image)

**Deliberate Practice.** The precluded practice is aptly named. Split (intermittent) practice switches between kinds of problems (a little addition, a short introduction to subtraction, a little more addition…) and materials (those multiple manipulatives), making practice on any skill partial and interrupted. In contrast deliberate practice is highly focused on specific aspects of a skill to be continuously developed in highly variable ways (Ericsson, 2006). Decomposing (or to be more mathematically precise, partitioning) numbers provides a good example of deliberate practice.

As the numbers increase, so do the number of possible decompositions. The number 3 has four decompositions (3, 2 + 1, its flip $1 + 2$, and $1 + 1 + 1$). Students learn to associate $2 + 1$ and $1 + 2$ as “flips” of one another. We prefer this more colloquial term for the mathematical property of commutativity here.

The number 4 has eight decompositions ($4, 3 + 1$ and its flip, $1 + 3$, and also $2 + 2$, the latter of which can be further decomposed into $2 + 1 + 1$, or $1 + 1 + 2$, or $1 + 2 + 1$, all of which can be decomposed into $1 + 1 + 1 + 1$).

The number 5, has sixteen decompositions, and this, along with a more general formula, is seen below. Because the children learn all these decompositions (by physically constructing...
them\(^{12}\) early in the school year, and because each decomposition builds on previous decompositions of smaller numbers, none of the combinations in the previous paragraphs would be a problem – or a cause of anxiety.

Deliberate practice is how immersion happens.

---

**Decomposing = Partitioning**

The process of decomposing positive integers that students are practicing in this curriculum is known as “partitioning” in Number Theory. Partitioning has deep connections to a variety of areas of mathematics. In general, the number of partitions of a number, \(n\), is equal to \(2^{n-1}\). For example: The number \(n = 5\) has \(2^{5-1} = 2^4 = 16\) different partitions. These 16 partitions can be organized, like in the table below, by the amount of integers in the partition. Astute mathematicians will recognize the connections that exist between these partitions and the Binomial Theorem, and in fact, a bijection exists.

<table>
<thead>
<tr>
<th>Addends in the partition</th>
<th>Partition</th>
<th>Number of possible partitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>4 + 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 + 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 + 2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 + 3</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1 + 1 + 3</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>1 + 3 + 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 + 1 + 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 + 2 + 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 + 1 + 2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 + 2 + 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 + 2 + 1 + 1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>2 + 1 + 1 + 1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1 + 1 + 1 + 1 + 1</td>
<td>1</td>
</tr>
</tbody>
</table>

---

\(^{12}\) Learning combinations by making them is an example of Simon’s (1988) “learning by doing.”
THE PRESENT STUDY

We report on the expansion of Only the NUMBERS Count© to all first-grade classes in the Lodi, NJ school district. The first set of results covers classroom observations and teacher surveys. The second presents pre- and post-test results, along with examples and explanations, from two first grade classes in two different schools. The third presents first graders responses to our simple, direct math anxiety questions.

OBSERVATIONS AND TEACHER SURVEYS

Method

Participants. Participants in the expansion included four schools with 12 first grade classes, and 161 students. Three of these classes were designated special education: there were 10 students in these classes. First grade teachers in the fifth school, where the program was developed, had used it for several years. The fifth school had 4 first grade classes with 61 students. One class with 3 students was special education. The special education classes were not included in the observations. Only two special education teachers completed the teacher survey.

Procedure. There was a professional development day at the start of the school year. Teachers new to the program met with the PI and experienced teachers at one of the grammar schools. The new teachers were given how-to-work books with lesson plans, made their own materials (Count-and-Combine charts, poster-board ‘blocks’ for children to use) and learned how to use them. Instruction included how to talk about math using the explicit base-10 count.

School Visits. The number of visits was limited due to the number of schools, as well as days off and breaks (for both the district and the university). Visits did not begin until early October. By the end of the school year, all schools had been visited 4 times. Pre- and post-testing were done at the start and end of the school year by the PI and assistants. Teacher and student surveys were also done close to the end of the year. The teacher surveys were done online; the student surveys were conducted in the classrooms by the PI or the teachers.

Results

Observations. All classes began reviewing the 1-to-10 chart and making combinations with the blocks representing ones. Table 3 shows when more advanced tasks (using the 10-to-20 chart, breaking up/decomposing combinations, subtraction) were introduced in the four schools. Decomposing meant taking a combination that a child made (say, \(3 + 4 = 7\)) and breaking up either the 3 or the 4 to make the combination “longer” \((3 + 2 + 2 = 5)\). Children could then make a new “shorter” combination” by recombining the numbers \((5 + 2 = 7)\). When subtracting, children physically “took away” the same number of blocks from either side of the minus sign. Near the end of the school year, higher performing students worked on double-digit addition and subtraction, while lower performing students worked on single-digit problems.
Table 3. Task introduction by school.

<table>
<thead>
<tr>
<th>Task</th>
<th>January/February</th>
<th>February/March</th>
<th>April/May</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-to-20 chart</td>
<td>Roosevelt/Hilltop</td>
<td>Columbus</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Wilson</td>
<td></td>
</tr>
<tr>
<td>Decomposing combos</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Making “longer”</td>
<td>Roosevelt/Hilltop</td>
<td>Wilson</td>
<td></td>
</tr>
<tr>
<td>Making “shorter”</td>
<td>Roosevelt/Hilltop</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subtraction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With 10s and 1s</td>
<td>Roosevelt/Hilltop</td>
<td>Wilson</td>
<td>Columbus</td>
</tr>
</tbody>
</table>

The differences in the chart appear to be teacher specific. Roosevelt School had the advantage of having two teachers who had used Only the NUMBERS Count© in kindergarten and were familiar with the program. Teachers at Hilltop and Wilson enthusiastically embraced the program from the start. However, one teacher at Wilson only began substituting (and thus using the program) in the spring. One teacher at Columbus began quite slowly, the other didn’t begin until later in the school year.

At the end of the school year, we visited two first grade classes in Washington School where the program was piloted and, hence, well established. Both teachers said they had completed the first-grade curriculum and were preparing their classes for 2nd grade. One class had begun using the multi-operation chart developed for 2nd grade. The chart is designed to teach multiplication and division simultaneously.

Teacher Survey. All teachers in the non-special education math classes filled out the survey, which is shown in the Appendix. The most important results can be collapsed into two questions:

1. Do you like teaching Only the NUMBERS Count©?
a. 12 teachers answered yes; only 1 answered no. This means that 92% liked the program, only 8% didn’t. Liking suggests that teachers were not anxious about using the program.

2. What do you like about it?
   a. Most frequent answers were variations of:
      - The hands-on learning.
        - Students were excited/engaged.
        - “Doing things hands-on gives them great confidence.”
      - Students understood the meaning of what they were learning.
        - “They could visually see what they were solving.”
        - “They appear to think about their thinking.”
        - “They were thinking like mathematicians.”
        - “It helps them better visualize math.”
        - “Students were able to grasp the concepts it teaches.”
      - Students understood place-value.
   
   b. One teacher liked that “It is built upon. They had a foundation from last year.”

PRE- AND POST-TESTING THE CHILDREN

Since we only visited one school per week, there was insufficient time to pre- and post-test all children. Two classes in two different schools were selected randomly for this purpose.

Method

Participants. Twenty-seven children in two classes at two different schools in the district served as participants. Children were sorted into classes by ability. Both classes followed the New Jersey Math Standards. Both used material from Only the NUMBERS Count© for numbers and numeric relations, and materials from enVisionMATH for all other required topics. The time for math was equal in both groups. Descriptions of each group and its teacher follow.

Both teachers attended a preparatory workshop with the experimenter and teachers who had worked previously with the program. They made the materials needed (Count-and-Combine charts, etc.) at the workshop. To ensure fidelity of treatment, the PI and two to three assistants observed math lessons on a rotating basis. While the core elements of the program were pre-
planned, timing of the implementation depended on the teacher’s assessment of when their children were ready to move on to more advanced materials.

**Class 1.** At the beginning of the school year, there were 11 students (6 female, 5 male) in the class. Of these, 4 were Hispanic or Latino, 6 were White, and 1 was Black or African-American. One was classified as ESL; none as economically disadvantaged. Mean age at the beginning of the year was 78.8 months; range was 72 to 83 months. The teacher was experienced using *enVisionMATH*. She had 24 years total experience. In this school district, she taught fifth grade for 1 year, fourth for 11 years, kindergarten for one year, and first grade starting with this class.

**Class 2.** At the start of the school year, there were 16 students (11 female, 5 male) in the class. Of these, 10 were Hispanic or Latino, 5 were White, and 1 was Asian. Eight were classified as economically disadvantaged (eligible for reduced fee lunch); 1 as ESL. Mean age at the start of the year was 80.9 months; range was 72 to 95 months. The teacher also had experience using *enVisionMATH*. She had 12 years teaching experience. In this district, she taught second grade for 1 year, and first grade starting with this class.

**Procedure.** The study had three phases. Phase 1 included pre-testing to assess what students retained from kindergarten. Phase 2 included class visits. Phase 3 included post-testing to assess what had been learned. Testing was done by the primary experimenter and undergraduate research assistants.

**Phase 1. Assessing Prior Knowledge.** Pre-testing took place on October 12th and October 18th 2018. Table 2 presents items on the pre-test. The test was identical to that given at the end of kindergarten to two different classes that were also exposed to *Only the NUMBERS Count*© in kindergarten. This was used to see how much children retained over the summer.

**Phase 2. Observing the Classes.** Both were visited four times between October and May.

**Phase 3. Assessing New Knowledge.** Table 3 presents items on the post-test. Notice that there were more and more difficult items than on the pre-test.

**Table 4. Pre-test items.**

<table>
<thead>
<tr>
<th>Category</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting.</td>
<td>Children were asked to count as high as they could. Counting was coded as correct up to the first error (If a child counted 11, 12, 15, her score was 12, the highest correct number).</td>
</tr>
</tbody>
</table>
Number and symbol identification.

Children were asked to read aloud 10 written numbers (1, 2, 4, 5, 7, 8, 12, 15, 20, 32) and three symbols (plus, minus, equals) presented in problem format (e.g., $2 + 2 = 4$).

Correct responses for the + sign were: plus, and N more, add. Correct responses for the = sign were equals or same as. Correct responses for the – sign were: minus, less, and take away.

Place-value. Children were asked (a) to read aloud the written numbers 16, 25, 31, 56, 11; (b) tell the experimenter which of each pair was bigger and (c) explain their answers.

Addition. Children solved two single digit ($3 + 2$, $6 + 2$) and two double digit ($12 + 4$, $21 + 11$) problems. They were asked (a) to read the problem, (b) solve it, and (c) tell the experimenter how they did it.

Subtraction. Children solved two single digit ($5 - 3$, $7 - 5$), and three double digit ($10 - 6$, $10 - 10$, $22 - 12$) problems. They were asked (a) to read, (b) solve and then (c) explain their method to the experimenter.

Combinations. Children read two numbers (8, 12) aloud and were asked to make up addition problems (e.g., ___+___=8) and explain how they did each one.

---

Table 5. Post-Test Items

<table>
<thead>
<tr>
<th>Category</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting.</td>
<td></td>
</tr>
<tr>
<td>To 100</td>
<td>Identical to pre-test.</td>
</tr>
<tr>
<td>By tens</td>
<td>Children were asked to count by 10 to 100.</td>
</tr>
<tr>
<td></td>
<td>How many tens.</td>
</tr>
<tr>
<td></td>
<td>Children were asked how many tens there were in 30 and in 50.</td>
</tr>
<tr>
<td>Number and symbol identification.</td>
<td>Identical to pre-test.</td>
</tr>
<tr>
<td>Place-value.</td>
<td></td>
</tr>
</tbody>
</table>
Version 1  Identical to pre-test.

Version 2  Five different numbers (18, 27, 31, 58, 22) were used.

Children were asked (a) what is this number, (b) does it have any tens, (c) how many tens, (d) how many ones, and finally (e) which digit is bigger.

Addition.  Children solved three single digit (3 + 5, 6 + 6, 9 + 7) and three double digit (10 + 18, 21 +11, 17 + 35) problems. They were asked (a) to read the problem, (b) solve it, and (c) tell the experimenter how they did it.

Subtraction.  Children solved two problems with at least one single digit (5 - 3, 10 - 5, 13 - 6), and two double digit (20 - 20, 22 - 12) problems. They were asked (a) to read, (b) solve and then (c) explain their method to the experimenter.

Combinations.  Children read three numbers (8, 10, 25) aloud and were asked to make up addition problems, two with 2 addends and one with 3 addends, and to explain how they came up, with each one. The three-addend problem could be solved by decomposing/partitioning one of the addends in the previous problem

Word Problems.  Children read and figured out the answers to three addition problems and two two-step problems requiring (first) addition and (then) subtraction.

RESULTS

Given that the study involved a small number of students, we present descriptive statistics. We also present specific student solutions and explanations showing that their learning was primarily procedural.

How They Did:  Pre- and Post-Test Scores.

Table 4 shows pre- and post-test scores for the two classes. We look first at pre-test scores.

Pre-Test Scores.  Pre-test scores show that the children retained a great deal from kindergarten. The surprising exception was the relatively low place-value score in both classes. Relatively refers to post-testing at the end of kindergarten in two different classes. Means were 93.28 and 66.66. We attributed this decline to the time of testing. At the end of kindergarten, children would be reciting the 10 to 20 Count and Combine Chart (with 11 as ten-one) each day. At the start of first grade, they would not yet have reviewed this chart.
Table 6. Pre- and Post-Test Scores.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Class 1</th>
<th>Class 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre</td>
<td>Post</td>
</tr>
<tr>
<td>Count to 100 by 10s</td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td></td>
<td>85.63</td>
<td>32.20</td>
</tr>
<tr>
<td></td>
<td>100.00</td>
<td>.00</td>
</tr>
<tr>
<td>Tens in 30 &amp; 50</td>
<td>95.45</td>
<td>15.07</td>
</tr>
<tr>
<td></td>
<td>95.00</td>
<td>95.00</td>
</tr>
<tr>
<td>Number-Symbol</td>
<td>98.36</td>
<td>3.88</td>
</tr>
<tr>
<td>Place Value</td>
<td>49.09</td>
<td>45.04</td>
</tr>
<tr>
<td>Bigger</td>
<td>95.45</td>
<td>15.07</td>
</tr>
<tr>
<td>How many 10s</td>
<td>54.54</td>
<td>41.56</td>
</tr>
<tr>
<td>Addition</td>
<td>77.27</td>
<td>41.00</td>
</tr>
<tr>
<td>Double</td>
<td>54.09</td>
<td>30.65</td>
</tr>
<tr>
<td>Subtraction</td>
<td>90.90</td>
<td>20.22</td>
</tr>
<tr>
<td>Doubles</td>
<td>90.81</td>
<td>21.72</td>
</tr>
<tr>
<td>First</td>
<td>66.54</td>
<td>42.21</td>
</tr>
<tr>
<td>Second</td>
<td>100.00</td>
<td>.00</td>
</tr>
<tr>
<td>Triples</td>
<td>63.33</td>
<td>39.94</td>
</tr>
<tr>
<td>Combinations</td>
<td>69.36</td>
<td>27.86</td>
</tr>
<tr>
<td>Doubles</td>
<td>54.54</td>
<td>52.22</td>
</tr>
<tr>
<td>First</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triples</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Word Problems</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Addition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two step (add and subtract)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Post-test scores. We looked at each group alone to see where improvements occurred. We also compared performance between the two groups.

Class 1. This class was tested first. Scores increased in all categories except place value as tested using the “which digit is bigger” question. This could not be attributed to not being familiar with the explicit base-10 count. Since the children knew how many tens there were in 30 and 50, and had extensive practice using ten blocks and one blocks to add, subtract, and create combinations (all items on which they scored well), we needed to explain the anomaly.

To see if the problem was in the way the question was asked (which digit is bigger), we rephrased the question and re-tested this class at a later date. The re-phrasing, which reflected the class emphasis on breaking numbers into ten blocks and one blocks took this form:

What is this number?

Does it have any tens?

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How many tens?
How many ones?
Which is the ten?

With the re-phrasing, place value scores rose to 92.72%: 10 of the 11 children scored 100%, one scored 20%.

Class 2. The new place value questions were tested at the same time as the other items in this group. Three scores increased: counting to 100 correctly, making one addition combination, and place value with the re-phrased question. Scores decreased in double-digit addition, and both single- and double-digit subtraction

Between Classes. Class 1 did noticeably better on: number of 10s in 30 and 50, double-digit addition, single- and double-digit subtraction, two-step word problems (with addition and subtraction), combinations (doubles and triples), and place value (with the re-phrasing). We attribute this difference to teacher enthusiasm and utilization of the program. The teacher in Class 2 resorted and rarely used the program.13 Her class was eventually exposed to it by sitting in on math lessons in a class where the teacher was actively participating.

How They Did It: Explanations and Examples

Explanations. Since all testing was done one-on-one, we were able to ask each child how they solved each problem. Their explanations can be collapsed into two categories, shown by class, in Table 7. Counting took two forms: with the fingers or by drawing blocks. Notice that the class with the higher post-test scores (Class 1) had a higher percentage of students who said they “knew.”

<table>
<thead>
<tr>
<th></th>
<th>Class 1</th>
<th>Class 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>I knew</td>
<td>73%</td>
<td>33%</td>
</tr>
<tr>
<td>Counted</td>
<td>27%</td>
<td>66%</td>
</tr>
</tbody>
</table>

Examples. What they knew was both practiced and procedural. Practiced refers to retrieval, e.g., “knowing” and recalling the addition face that $9 + 7 = 16$. Procedural refers to knowing and performing the steps to find the sum. For some students, this meant mentally manipulating the ten and the one blocks, “knowing” to add or subtract the tens first, and then add or subtract the ones.14 For others, it made sense to them to make multiple combinations with the

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13 She was the sole teacher who (on the survey) wrote that she did not like teaching Only the NUMBERS Count©.
14 This procedure eliminates “carrying.” For example, with 23 (two-ten-three) and 19 (one-ten-nine), adding the tens yields three-ten, adding the ones gets get ten-two. Three-ten plus ten-two is four-ten-two (42).
blocks, “knowing” to use a “double” \((5 + 5 = 10)\) for a first combination, or to “flip” a first combination \((5 + 3 = 8)\) to make a second \((3 + 5 = 8)\).

Most impressive to us were the performances from students who were able to product a third combination. This ability to apply previous knowledge and to synthesize concepts indicated to us a deeper understanding of what it means to deconstruct numbers. Most students who showed this ability were in Class 1. One student decomposed one number in their second combination, turning \(2 + 8 = 10\) into \(2 + 2 + 6 = 10\). In the words of this student, “I broke apart the numbers and put in little numbers.” In another example, explaining how \(4 + 6 = 10\) became \(6 + 2 + 2 = 10\), the student told us “I knew that \(2 + 2\) equaled 4.”

The students who did well did so because they were fluent in numbers and symbols and patterns. They had learned to think like mathematicians.

**MATH ANXIETY: ASKING THE CHILDREN**

Math anxiety was not the original focus of this study. It was included at the end of the school year because virtually none was observed in our school visits. Rather, teachers (with the one exception) and children seemed to be engaged in, and enjoying, math. To quantify our observations, we reduced the MARS-E\(^{15}\) (Suinn et al., 1988) to a single question: how do the children feel about math? We report data collected from 11 classes in the four expansion schools, and from 2 classes in the pilot school.

**METHOD**

**Asking the Children.** Due to scheduling of visits near the end of the school year, we could only ask children in 8 classes to choose one of three cartoon faces (smiley, neutral, sad) to show how they felt about math. To cover all the classes, the teachers asked their classes (all, including the 8 surveyed by the experimenters) two questions: How many of you like math? How many of you don’t like math? Children who chose from the cartoon faces gave the same responses (smiley face = like, sad face = dislike) to their teachers’ queries, making their ratings reliable.

But what about math anxiety? Of the 200 first grade children surveyed, 171 liked math, only 29 didn’t. In percentages, this means that 86% **liked math**, only 14% **disliked it**.\(^{16}\) These results would, to researchers studying math anxiety (e.g., Ramirez et al, 2013), be very surprising.

We asked the children who choose from the cartoon faces why they picked the smiley face. Their reasons included:

---

\(^{15}\) Mathematics anxiety rating scale for elementary school students.

\(^{16}\) We were also able to ask three kindergarten classes (also using *Only the NUMBERS Count*) how they felt about math. Of 54 students, 46 liked math, only 8 didn’t. Put as percentages, 85% **liked math**, 15% **didn’t**, the results parallel those seen in 1st grade.
“The blocks make math easy.”

“It’s fun to learn.”

“I like plus-ing and minus-ing.”

“I like learning new things.”

The answer we liked best was “I like getting smarter.”

We also asked why the sad face was selected? One child simply said he didn’t like math facts. Another said the math was too easy. The others didn’t give any reasons.

**Asking the Teachers.** As already noted in the teacher survey section, 92% liked the program, only 8% didn’t. This suggests that the teachers were not anxious about implementing and using the program.

**DISCUSSION**

**Acquiring the Math.** Observations, tests, and children’s explanations show that the children did learn how to think in numbers, symbols, and patterns. This kind of thinking helped them meet all 1st grade standards for Operations and Algebraic Thinking (1.OA), and for Numbers and Operations in Base Ten (1.NBT). It also helped them learn things about numbers that they are not expected to learn (creating, decomposing, and recomposing combinations), or not expected to learn until 2nd grade – meeting all 2nd grade standards for Operations and Algebraic thinking (2.OA), and partially meeting those for Numbers and Operations in Base Ten (2.NBT.1 and 2.NBT.9)

**Not Acquiring the Anxiety.** Our survey asked if teachers liked teaching Only the NUMBERS Count©. One of the twelve teachers answered negatively. This meant that 92% liked the program, only 8% didn’t. One reason for the low math anxiety in the students may be low anxiety in their teachers.

With the children, we operationalized math anxiety as choosing the sad face when asked how a child felt about math, or when they told their teachers that they liked or didn’t like math. Only 14% of the children did not like math; 86% liked math. We underline this because it is markedly different from two recently reported studies with elementary students.

One study used an 8-item questionnaire and cartoon faces (calm, semi-nervous, nervous) to indicate (on a 16-point sliding scale) how anxious students felt about math. The mean math anxiety score was 8.07 (Ramirez et al, 2013). There were no 0s. All children experienced math anxiety. Another (Vukovic et al., 2013) did not report 2nd and 3rd grade scores on their math anxiety scale, but rather showed that those scores were negatively correlated to calculation and mathematical – but not to geometric – applications. The researchers then asked: why the difference? This is their answer. “Calculation skills and mathematical applications have in
common that they are both based in the symbolic number system… [suggesting] that math anxiety may specifically affect mathematical problems that involve understanding and manipulating numbers” (p. 8). Perhaps, then, the reason we report so little math anxiety is immersion in manipulating numbers. We elaborate on this idea in the next section.

REASONS FOR THE RESULTS

The goal of Only the NUMBERS Count© is to have children problem-solve like mathematicians, thinking not about numbers, but in numbers, symbols, patterns. To do this requires immersion, that is, constant practice in manipulating numbers in order to understand them. Understanding, taken here in its problem-solving sense, is primarily procedural (Zhu & Simon, 1987). It is also circular. Understanding numbers means knowing when and how to manipulate them. To understand how the understanding happened – and the anxiety didn’t – we look at the contributions of each of our core components.

The Explicit Base-10 Count. A basic problem with current curricula is not the often-blamed abstractness of math (Ashcraft & Krause, 2007), but the way the curricula obscure or ignore the concreteness and the connections. In contrast, using an explicit base-10 count teaches the children that numbers are specific real things that don’t change, which makes them more stable and more substantial than the things to which they are temporarily attached. In other words, nothing is more concrete than the count. As an unpublished poem put it:

You can count two snails or two pails,  
two trucks or two ducks
Two can be more than one \(1 + 1 = 2\)  
or less than three \(3 - 1 = 2\)  
and also one half of four \(4/2 = 2\)
But no matter what you do,

Two is always two
…and never more.

---

17 In this view, conceptual knowledge emerges from procedural knowledge. For example, practice making and re-making ‘combinations’ with the blocks taught children how to (procedural) do addition, and also that (conceptual) numbers are combinations of other numbers.

18 This definition of procedural knowledge is akin to the National Research Council’s (2001) version: “knowledge of procedures, knowledge of when and how to use them appropriately” (p. 12).

19 The poem goes through the count from 1 through 10. It was written by the first author.
Equally concrete are the count’s patterned iterations that facilitate mastery of place-value and double-digit calculation.

The Count-and-Combine Charts and the Blocks. The charts were designed – like the abacus – to make numeric-symbolic patterns primary as well as concrete. Notice in the poem, there are three different ways to get to 2. The different ways illustrate a foundational pattern in mathematics: numbers are combinations of other numbers. The blocks were designed to let the children concretely (visibly, tangibly) practice combining pairs of numbers like 3 + 4 and 5 + 2, both of which generate the number 7. They soon practiced a related pattern, which generates two more combinations for 7: 4 + 3 and 2 + 5. Mathematicians call this pattern (order doesn’t matter) the commutative property of addition. The children – thinking visually – call it the “flip.” Importantly, the 10-block – which represents 10 as a unit, rather than as a grouping of ten ones – externalized place-value.

Deliberate Practice. Deliberate practice is how immersion happens. It is continuous, focused, and variable. The focus is on base-10 patterns and relations. The variability is the result of sustained, incremental elaboration of those patterns. The children practiced the pattern of the base-10 count by reciting the charts. They elaborated on the pattern by creating, decomposing, and recomposing addition combinations using the 10s and 1s of the count. They practiced reversing the pattern, using subtraction to un-do addition.

This kind of practice exemplifies the idea of learning by doing (Papert, 1980) or, more specifically, learning by solving problems (Zhu & Simon, 1987). In this view, the problem solving process itself provided “a template” on which “knowledge of a correct solution provides information not only about the steps that have to be taken to follow the path but also about the cues present in the successive situations reached that indicate which next steps may be appropriate” (Anzai & Simon, 1979, p. 137).

Immersion in the strictly mathematical means continuous practice in manipulating numbers in order to understand them. Practice using the blocks to make combinations (addition) and to physically “take away” (subtraction) certainly contributed to the OA results; practice using the Count-and-Combine charts and the explicit base-10 count, to the NBT results. Deliberate practice made the (successively more elaborate) procedures procedural.

Expertise is procedural. Expertise solves the math anxiety problem before it starts. If you can do the math, there is no reason to be anxious about doing it.

CAVEATS AND CONCLUSION

Caveats. Unlike most math education research, we report on the expansion of a year-long program, not on the results of short-term, single, assessments of achievement and anxiety. The strength of those assessments lies in the rigor of their statistical analysis. The weakness lies in their paucity of practical application. Importantly, they do not address the effects of current
curricula on either the achievement or the anxiety they report. In contrast, the strength of our report lies precisely in application. We show that, in one school district, expansion of a quite different curricula can reduce anxiety by incrementing achievement.

There are two caveats. The first is that time constraints precluded pre- and post-testing of all classes. Since we visited each schools on a rotating basis, continuing professional development/demonstration/involvement in all the classrooms seemed more important to implementing the curriculum than testing all first graders.

The second caveat is that we could not compare classrooms with and without the new curriculum. It would have been unethical to leave any one school in the district without an intervention already proven (in the pilot school) to work so well.

However, an earlier study (Stokes, 2014b) can provide this comparison. First graders in the pilot group (using Only the NUMERS Count) outperformed the comparison group (using only enVisionMATH) on the (almost identical) post-test and on the district wide Renaissance STAR math test. On the latter, 71% of the pilot group scored above grade level. Only 30% of the comparison group did. That study did not include any math anxiety questions.

CONCLUSION: THE IMPORTANCE OF IMMERSION

Immersion – constant practice in manipulating numbers in order to understand them – was critical to the success of the current intervention. Success was measured in two ways: acquiring the math, not acquiring the anxiety. Immersed in the strictly mathematical, first graders in the four expansion schools – like those in the pilot (Stokes, 2014b) – had little difficulty with place-value, double-digit addition and subtraction, or composing, decomposing and recomposing addition combinations. Importantly, acquisition of the math did not include acquisition of anxiety. The results of both studies have three implications for early mathematical education.

- One, early immersion in the strictly mathematical can teach young students to think and problem solve like mathematicians, in numbers, symbols, and patterns.
- Two, early immersion can facilitate acquisition of procedural (how-to) knowledge and its product, conceptual (that/what) knowledge.
- Three, with early immersion, the math anxiety problem can be solved before it starts.20

20 We have one other suggestion, based on conversations with teachers. Teachers should have a say in selecting a math curriculum. They are the ones who have to implement it. Most teachers would not select the curricula they are currently teaching.
REFERENCES


APPENDIX

Questions for first grade teachers – June 2019

Name of teacher: __________________________
School: ________________________________

1. Do you enjoy teaching Only the NUMBERS Count?
2. If yes, what do you like about it? Please be specific.
3. What would you like changed or added to the program?
4. Do the children like leaning math with Only the NUMBERS Count?
5. If yes, what do they like about it? Again, please be specific.
6. Where are the children now with their math skills (e.g., single- or double-digit combinations, subtraction)?
7. Are you surprised at where they are (with learning math)?
8. Would you like Only the NUMBERS Count to be expanded as a complete math program? (e.g., with measurements, shapes, etc.)
9. What have you/have you told any other teachers about Only the NUMBERS Count?
10. Could you please ask your students the following questions, and indicate the numbers for each answer.
    a) How many of you like math? Number: ______________
    b) How many of you don’t like math? Number: ____________

ACKNOWLEDGEMENTS

Thanks to our Columbia University research assistants, Brian McIntyre and Emily Ringel, who did the observations and testing, to Lauren Launa, the curriculum coordinator who initiated the district-wide expansion, and especially to the principals, teachers, and children who made it both a success and a pleasure.
Effect of Using GeoGebra on Eight Grade Students’ Understanding in Learning Linear Equations

Dirgha Raj Joshi\textsuperscript{1,2}, Kailash Bahadur Singh\textsuperscript{2}

\textsuperscript{1}Mahendra Ratna Campus Tahachal, Tribhuvan University Nepal and Nepal Open University, \textsuperscript{2}Tribhuvan University Nepal

dirgha@nou.edu.np, bdrkailash@gmail.com

Abstract: GeoGebra is a dynamic geometric software developed for the support of mathematics teaching. The aim of this research was to study the effect of GeoGebra on mathematics achievement in relation to the linear equations and explore the perception of students towards the use of GeoGebra in teaching mathematics. Forty students were participating in this quasi-experimental study in which one group was assigned as experimental and another as control group. Data were collected by using self-constructed tools and analyzed by using percentage, mean, standard deviation and t-test as well as effect size was calculated. The result from the study revealed that the achievement of GeoGebra instructed students was significantly higher than the control group. Additionally, level of perceptions of experimental group students found to be high towards the use of GeoGebra. Hence teachers should use this software while they instruct graph of linear equations in their lessons and curriculum developers and textbook writers should incorporate such kind of activities into the curriculum and textbooks.

Keywords: GeoGebra, mathematics teaching, achievement, perception, effect, Nepal

INTRODUCTION

Mathematics is compulsory subject in school level in Nepal. Set theory, arithmetic, mensuration, algebra, geometry and statistics related content have been included under the curriculum of basic level (class 1-8). Linear equation is under algebraic portion. Many types of digital tools have developed for the support of mathematics teaching and learning, and GeoGebra is one of them. GeoGebra represents dynamic geometric software (Zengin et al., 2012) which is applicable and important for mathematics teaching. In this regard this experimental study focused on the use of GeoGebra in teaching linear equation at eight grades.

Mathematical digital tools are very convenient for effective, creative, collaborative and self-learning (Joshi, 2017). Digital resources encourage the learners towards multi-way of learning and supports to mathematical modeling, visualize mathematical shapes and figures. Concept of self-learning, technological based learning and artificial intelligence are modern pedagogical terms which are in practices in present context. GeoGebra is effective in teaching definite integrals (Tatar...
OBJECTIVES

The main concern of the study is the study of effect of GeoGebra on the mathematics achievement in relation to linear equation and to explore the perception of learners towards the effectiveness GeoGebra in mathematics learning.

METHODOLOGY

Materials and Methods

Quasi-experimental research design was used for the research. All the students of class eight (two groups) from Heartland Academy Kathmandu Nepal were assigned for the study. Both groups of the students were separated by school administration based on rule of the school containing 20 students in each group. Students were blind about the experiment. Additionally, the students of both groups were unaware about the GeoGebra software before the experiment. Experimental group and control group were declared by queen toss.

<table>
<thead>
<tr>
<th>Groups</th>
<th>Pre-test</th>
<th>Treatments</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>Q1</td>
<td>GeoGebra assisted teaching</td>
<td>Q2</td>
</tr>
<tr>
<td>Control</td>
<td>Q3</td>
<td>Traditional teaching</td>
<td>Q4</td>
</tr>
</tbody>
</table>

Table 1 Design of the study

Where, Q1=Pre-test given to experimental group, Q2=Post-test given to experimental group, Q3=Pre-test given to control group and Q4=Post-test given to control group

In the context of Nepal, application of GeoGebra have formally integrated in the curriculum of ICT in Mathematics Education in B. Ed and M. Ed. in Mathematics Education of Tribhuvan University Nepal. It has incorporated in short-term training programs of teacher training. There should be deep thinking about using GeoGebra application in linear equation teaching and how it effects in teaching linear equation? What is the perception of the students about the GeoGebra? This study will justify and find the answer of these inquiries.

In the study of Sariyasa, Tribhuvan University in Nepal, 2010; Zulnaidi et al., 2012; & Zengin, 2016), function (Zulnaidi & Zakaria, 2012), triangle (Dogan & Icel, 2011; Ozcakır et al., 2015), arithmetic (Kamariah et al., 2010), circle (Shadaan & Eu, 2013; Tay, 2018), trigonometric function (Ibrahim & Ilyas, 2016), derivative (Ocal, 2017), geometry (Jelatu, Sariyasa, & Made Ardana, 2018), fraction (Bulut et al., 2016), geometry (Kushwaha et al., 2014), trigonometry (Zengin et al., 2012), analytic geometry (Khalil et al., 2018), coordinate geometry (Saha et al., 2010), statistics (Emaikwu et al., 2015) and differential calculus (Diković, 2009). These studies show that GeoGebra is highly applicable and important tool for mathematics teaching yet, they have not addressed the issues of linear equations which is the focus of this current study.

Readers are free to copy, display, and distribute this article as long as: the work is attributed to the author(s), for non-commercial purposes only, and no alteration or transformation is made in the work. All other uses must be approved by the author(s) or MTRJ. MTRJ is published by the City University of New York. https://commons.hostos.cuny.edu/mtrj/
Study Setting and Sampling

Self-constructed tool "Achievement test' was implemented for pre-test and post-test. Where pre-test contain 10 and post-test contained 20 multiple choice items. Additionally, “Perception Scale” was implemented for the perception of students in experimental group. On the basis of national curriculum of mathematics, the unit was taught 10 days to both groups. On this regard ten episodes were developed for experimental group. Validity of tools were measured by content validity based on the suggestion of subject experts.

GeoGebra has several features like as animation, transformation of objects, calculation and graphical representation of 2D and 3D figures, algebraic, exponential, logarithmic and trigonometric functions. The problems related to equations solving, limits, derivative integration, probability and statistics can also be solved and visualized by this software. However, the main concern of this experiment was to teach linear equation at grade 8 hence the some examples related to concepts of linear equation used in experiment are given in Figure 1 to 4.
Definition of Variables

In this research, use of GeoGebra is independent variable and mathematics achievement and perception of students are considered as dependent variable.

Statistical Analysis

Two independent sample t-test was calculated to determine the significant different of mean score between two groups while percentage, mean and standard deviation were calculated under descriptive statistics. Additionally, effect size was calculated by using Cohen’s d formula for the accurate difference between groups.

RESULTS

Table 2 showed that experimental and control group have no difference in mean score of mathematics achievement. Which indicates that the groups were homogenous based on the average achievement score on pre-test.

<table>
<thead>
<tr>
<th>Groups</th>
<th>Sample</th>
<th>Mean</th>
<th>S.D.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>20</td>
<td>6.95</td>
<td>2.09</td>
<td>0.94</td>
</tr>
<tr>
<td>Control</td>
<td>20</td>
<td>7.0</td>
<td>1.95</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 Independent sample t-test result on pre-test between the groups

Table 3 showed that the significant difference was found on achievement scores between experimental and control group students in favor of experimental group and effect size (d=0.82) found to be high. Which indicated that the GeoGebra is best tool for teaching linear equation at school level.

<table>
<thead>
<tr>
<th>Groups</th>
<th>Sample</th>
<th>Mean</th>
<th>S.D.</th>
<th>p-value</th>
<th>Effect size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>20</td>
<td>16.60</td>
<td>2.26</td>
<td>0.02*</td>
<td>0.82</td>
</tr>
<tr>
<td>Control</td>
<td>20</td>
<td>13.80</td>
<td>4.30</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3 Independent sample t-test result on post-test between the groups

*p-value <0.05 (i.e. Significant)

Perception of Students Towards the Use of GeoGebra

After completing the experiment, students’ perception was measured by experimental grouped students. A tool having nine items were used to determine students’ perception towards the use of GeoGebra in linear equation and overall mathematics. All items were in the form of Likert scale as strongly disagree to strongly agree. The scoring technique is 1 for strongly disagree to 5 for strongly agree for positive items and reverse scoring in negative items.

Table 4 showed that the level of perception of students towards the use of GeoGebra in teaching linear equations found to be high. The result also indicates that the software is helpful to learn mathematical concepts, visualize mathematical content and make students more creative, enjoyable and confident. Additionally, it is essential and important for mathematics learning.
Table 4 Perception of students towards the use of GeoGebra in mathematics learning (n=20)  
(SA- Strongly Agree, A-Agree, N-neutral, D- Disagree, SD- Strongly Disagree and S. D. - Standard Deviation)

<table>
<thead>
<tr>
<th>Items</th>
<th>SA</th>
<th>A</th>
<th>N</th>
<th>D</th>
<th>SD</th>
<th>Mean</th>
<th>S.D.</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>I like GeoGebra while learning linear equation</td>
<td>45</td>
<td>50</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>4.4</td>
<td>0.59</td>
<td>High</td>
</tr>
<tr>
<td>GeoGebra helped me a lot to learn the mathematics concept</td>
<td>45</td>
<td>40</td>
<td>15</td>
<td>0</td>
<td>0</td>
<td>4.25</td>
<td>0.78</td>
<td>High</td>
</tr>
<tr>
<td>GeoGebra is essential and important for learning Mathematics</td>
<td>0</td>
<td>80</td>
<td>20</td>
<td>0</td>
<td>0</td>
<td>4.0</td>
<td>0.64</td>
<td>High</td>
</tr>
<tr>
<td>GeoGebra software helps easy to understand Mathematical problems</td>
<td>5</td>
<td>90</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>4.0</td>
<td>0.32</td>
<td>High</td>
</tr>
<tr>
<td>I felt confident to solve problems using GeoGebra</td>
<td>30</td>
<td>40</td>
<td>25</td>
<td>5</td>
<td>0</td>
<td>4.0</td>
<td>0.85</td>
<td>High</td>
</tr>
<tr>
<td>GeoGebra helps to make me more creative</td>
<td>25</td>
<td>60</td>
<td>15</td>
<td>0</td>
<td>0</td>
<td>4.1</td>
<td>0.64</td>
<td>High</td>
</tr>
<tr>
<td>I enjoyed learning mathematics much more using GeoGebra</td>
<td>20</td>
<td>75</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>4.05</td>
<td>0.82</td>
<td>High</td>
</tr>
<tr>
<td>GeoGebra helps to visualize mathematical content</td>
<td>10</td>
<td>75</td>
<td>15</td>
<td>0</td>
<td>0</td>
<td>4.0</td>
<td>0.45</td>
<td>High</td>
</tr>
</tbody>
</table>

DISCUSSION

The findings of this research indicate that GeoGebra is a very good tool for teaching linear equations. The finding indicated that the implication of GeoGebra in linear equation instruction has significant effect on students' achievement. Similar result were found by Tatar & Zengin (2016) in teaching definite integral among perspective secondary teachers in Turkey, Zulnaidai & Zakaria (2012) in teaching function among high school students in Indonesia, Ozcakır et al. (2015) in teaching triangle, Kamariah et al. (2010) in teaching arithmetic among secondary students of Malaysia, Shadaan & Eu (2013) in achievement nine grade students in teaching circle, Ibrahim & Ilyas (2016) in teaching trigonometric function at public high school in Istanbul, Arbain & Shukor (2015) in achievement on mathematics learning in Malaysia, Omer & Ozturk (2013) in in academic achievement, Bulut et al. (2016) in teaching fraction at primary level, Kushwaha et al. (2014) in teaching geometry to secondary level, Zengin et al. (2012) in achievement in teaching trigonometry, Khalil et al. (2018) in in teaching analytic geometry in class 11, Tay (2018) in teaching theorems of circle among senior high school students, Saha et al. (2010) in achievement on coordinate geometry teaching among low visual-spatial ability students, Emaikwu et al. (2015) in teaching statistics among secondary school students of Benue state. Additionally, the effect size found to be high in favor of experimental group (d=0.82). The finding of the study showed that
GeoGebra is highly applicable tool for teaching Linear Equation whereas literature verified that the tool is very good for teaching several dimensions of mathematics.

The participants highly agreed that the GeoGebra is a very good tool for learning mathematics, understand mathematical concepts, increase their confidence in solving problems, make them more creative, make learning more enjoyable and visualize mathematical content. Similar result were found by Shadaan & Eu (2013) and Arbain & Shukor (2015) in perception of students. Dikovic (2009), Ocal (2017) and Zulnaidi & Zamri (2017) showed that the tool is very good for developing conceptual understanding. Additionally, Lavicza & Prodromou (2017) and Wah, Kewangan, & Takaful (2015) showed that it is good resource for student attitude, attribute on attention, relevance and confidence. Hence the study verifies that GeoGebra is very good tool for students conceptual understanding and visualized the mathematical contents.

CONCLUSION

GeoGebra is a good tool for teaching linear equations and has a significant effect on students’ achievement in linear equations. GeoGebra instructed classroom students were agreed that the software is decent for learning mathematics, understand mathematical concepts, increase the confident, make them more creative, make learning more enjoyable and visualize mathematical content. The research was limited to small group of students of class eight so additional research is needed to different classes in similar content. The finding of the research suggested to all mathematics teachers that this software is highly beneficial and applicable to students’ achievement enhancement and conceptual understanding. Therefore, teachers have to use this software while they instruct the graph of linear equations in their lessons. In addition, it is recommended that the curriculum developers and textbook writers should incorporate such kind of activities which are based on dynamic mathematics software into the curriculum and textbooks for the graph of linear equations and other content.

REFERENCES


Effectiveness of Geometer's Sketchpad Learning in Two-Dimensional Shapes

Sugi Hartono
Mathematics Education Department, Universitas Negeri Surabaya, Indonesia
sugihartonounesa@gmail.com

Abstract: The present study was conducted to compare the effectiveness of Geometer’s Sketchpad (GSP) learning in two-dimensional shapes. This study was designed as a quasi-experiment which involved 60 students of class VII in SMP Negeri 1 Ngoro, Mojokerto, Indonesia. The sample in study was divided into two groups, mainly 30 students are the experiment class (GSP) and 30 students are the control class (conventional learning). There were three instruments used in this study namely, students’ responses, and pretest-posttests. The data were analyzed using analysis of covariance (ANCOVA). T-test showed that the effectiveness of student learning outcomes in GSP learning is higher than those in conventional learning. Based on the results of the student learning outcomes indicated that students in the experimental group outperformed those in the control group. In addition, a survey instrument was used to elicit students' perception on the use of GSP. Analysis of the questionnaire responses indicated a positive overall perception of using GSP in learning about two-dimensional shapes. Thus, it can be concluded that GSP learning was effective in two dimensional-shapes learning.

INTRODUCTION

One of the mathematics topics considered difficult by junior high school students is geometry (Battista, 1999). In the teaching and learning of geometry, it has been often realized that students still lack the cognitive and process abilities in the total understanding of two-dimensional shapes. It is important for students to be able to imagine, construct and understand construction of shapes in order to connect them with related facts.

In this context, Geometers’ Sketchpad (GSP) is one solution for understanding two-dimensional shapes (Dogan, 2010). GSP is a software that can be used in geometry to enhance teaching and learning (Kesan & Caliskan, 2013). Geometer’s Sketchpad (GSP) can be used by students and teachers as an instrument to help them in learning geometry.
According to Meng (2009), using GSP, the level of van Hiele's geometrical thinking of students about cubes can increase from level 0 to level 2. In line with that research, Idris (2009) also shows that learning using GSP can improve performance and van Hiele's level of geometrical thinking students in Malaysia.

The purpose of this research is to compare the effectiveness of Geometer’s Sketchpad (GSP) learning in two-dimensional shapes.

**REVIEW OF LITERATURE**

**Geometer’s Sketchpad (GSP)**

Teachers are expected to integrate Information and Communication Technologies (ICT) as a learning medium for all subjects (Muhammad & Powel, 2019; Kemendikbud, 2014). Therefore, teachers must master ICT to support learning in the classroom. One of the ICT-based learning used by teachers in mathematics learning such as Geometer's Sketchpad (GSP) (Berezny, 2015; Johar, 2015). GSP is software that can help students understand geometry starting from points, lines and angles to more difficult understandings such as arches, turns and transformations. Students can associate points and lines that are connected with angles through animation that are easier to understand. Geometer’s Sketchpad can also make learning more interesting and not boring, because this software can construct dynamic images so that they can be manipulated, analyzed, and processed into interesting learning (CITE).

In addition, the use of GSP can also help students think to solve problems, find ideas, and make the right decisions in learning geometry. In general, the use of GSP can be useful and realize healthy learning. Because students can see and imagine geometric shapes on the GSP. Kesan and kaliskan (2013) proposed some GSP characteristics, which are described below.

a. Accuracy in digitally painting and measuring
b. The process of visualization from the beginning with different dimensions of dimensions is easy to understand
c. Can be used to facilitate students conducting investigations, exploration and problem solving
d. Giving confidence and strong reasons for students in making conclusions can even provide motivation in doing proof
e. Has specific characteristics, animated images, trace images, and provides features to simulate various simulations
Steps to using GSP in two-dimensional shapes

To use GSP, the user must perform some basic steps.

1. Turn on the computer
2. Click the Start menu, if there is already a Sketchpad Icon, select All Programs, then select and click the mouse on Sketchpad like Figure 1

![Figure 1. Start to GSP program](image)

3. Select the Geometer 'Sketchpad, as shown in Figure 2

![Figure 2. Icon the Geometer’s Sketchpad](image)
4. Click on The Geometer’s Sketchpad, so that you get a view like Figure 3

![Figure 3](image1.png)

Figure 3. The view of the Geometer’s Sketchpad

5. Click any button of sketchpad field in left slide that the display of a worksheet in sketchpad, as in Figure 4.

![Figure 4](image2.png)

Figure 4. The worksheet of Geometer’s Sketchpad
Information:

- **Selection Tool**: Click, hold and drag to move the object.
- **Point Tool**: Click on the sketch pad to create points. To create a special point, click the command line and then drag the point.
- **Circle Tool**: Click, hold and drag on the sketch pad to create circles. Click the center of the circle to create a circle.
- **Line Segment Tool**: Click, hold and drag on the sketch pad to create line segments. Click the end points to create a line segment.
- **Text Tool**: Click, hold and drag on the sketch pad to create text boxes. Click the text box to edit the text.

On the left side of the sketch field there is a menu for creating images, namely the Toolbox menu, for example points, lines, circles, etc. The user just needs to click the desired mouse image then move the cursor to the sketch field, move the cursor while determining the desired image size.

6. If the image size has been determined. Click the icon print so that the image is printed.

7. To record work results, click the File menu, and then select and click the Save or Save as menu, such as Figure 5 and Figure 6, name the file to be saved, the file name will be given extension.gsp, for example the name of the file works 1. gsp

![Figure 5. Saving files on Geometer’s Sketchpad](image-url)
To exit the program, click the mouse on the file menu, as in Figure 2.5, then select and click the mouse on the Exit menu or press the Alt + F4 key.

**RESEARCH METHODOLOGY**

**Research Design**

This study is a quantitative study with quasi experimental design using one-group pretest-posttest design that conducted to compare the effectiveness of Geometer’s Sketchpad (GSP) learning in two-dimensional shapes. This research was conducted in class VII of SMP Negeri 1 Ngoro, Indonesia. This study was implemented for three months.

**Research Sample**

The participants of the study were two classes that were selected from cluster random sampling from nine classes in the same grade from a junior high school at Mojokerto city, Indonesia. The sample in this study comprised two groups, 30 students are in the experimental group (GSP) and 30 students in the control group (conventional learning). All students are in grade VII and aged between 12 - 13 years.

**Data analysis**

Achievement test scores were analyzed using inferential statistics. Specifically, the t-test was executed using the Statistical Package for Social Sciences Version 22.0 (SPSS 22.0) software. The
t-test was used to test for statistical significance difference between the control and experimental groups at the beginning of the study and at the end. Descriptive statistics were used to analyze the data from the survey questionnaire.

RESULTS

Effectiveness of using GSP on students’ understanding of two-dimensional shapes

To determine whether any significant differences existed between the pre-test mean score of both the control and experimental groups, an independent sample t-test was done.

<table>
<thead>
<tr>
<th>Group</th>
<th>Post test</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>S.D.</td>
<td>t-value</td>
<td>Sig (2 tailed)</td>
</tr>
<tr>
<td>Experimental</td>
<td>6.17</td>
<td>2.16</td>
<td>-1.265</td>
<td>.188</td>
</tr>
<tr>
<td>(n = 30)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control</td>
<td>6.94</td>
<td>2.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(n = 30)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

t-value significant at p < .05

Table 1. Results of the independent t-test on the pre-test for both groups

Table 1 shows that the control group obtained a mean score of 6.94 while the experimental group obtained a mean score of 6.17. The mean score difference between the groups was 0.81 with a t-value of -1.265. Nonetheless, the p-value was 0.188 (p > .05) indicating that the difference in the mean score of the two groups was not significant. This result illustrated that both the students in the control and experimental group were similar in abilities before the treatment was administered.

<table>
<thead>
<tr>
<th>Group</th>
<th>Post test</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>S.D.</td>
<td>t-value</td>
<td>Sig (2 tailed)</td>
</tr>
<tr>
<td>Experimental</td>
<td>15.17</td>
<td>3.21</td>
<td>3.278</td>
<td>.000</td>
</tr>
<tr>
<td>(n = 30)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control</td>
<td>13.94</td>
<td>4.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(n = 30)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

t-value significant at p < .05

Table 2. Results of the independent t-test on the post-test of both groups

This table shows that the control group obtained a mean score of 13.94 while the experimental group obtained a mean score of 15.17. The mean score difference between the groups was 5.12 with a t-value of 3.278. Furthermore, the p-value was low (p < .05) indicating that the difference in the mean score of the two groups was significant. Thus, the students in the experimental group performed better using GSP than the control group using the conventional learning method. The students in the experimental group performed better in the post test compared to the control group.
Table 3. Results of the paired sample t-test

Based on the above table, the result of the one sample t-test shows t-test of control group is 15.931, while t observed of experimental group is 34.655. If the result of the calculation is compared with t table (2.042) then t test count is greater than price t table. Because t observed > t table then there are differences in student learning outcomes control group and experimental group in learning two-dimensional shapes.

For the other results showed that the questionnaire of students’ response completed by 30 students after following GSP learning on statistical materials obtained as follows:

<table>
<thead>
<tr>
<th>No</th>
<th>Responded aspect</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Agree</td>
</tr>
<tr>
<td>1</td>
<td>I was excited about using Geometers’ sketchpad (GSP) software</td>
<td>98</td>
</tr>
<tr>
<td>2</td>
<td>I was very engaged in the learning process</td>
<td>95</td>
</tr>
<tr>
<td>3</td>
<td>I was able to visualize and answer the questions after each activity</td>
<td>96</td>
</tr>
<tr>
<td>4</td>
<td>I enjoyed learning mathematics much more using GSP</td>
<td>90</td>
</tr>
<tr>
<td>5</td>
<td>I learnt a lot using GSP</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>TOTAL</td>
<td>91.8</td>
</tr>
</tbody>
</table>

Table 4. The results of response students toward GSP design

Based on the criteria of students' responses, it can be concluded that the overall percentage of students' responses to learning tools amounted to 91.8 % which means that students' responses are positive to follow GSP lessons in two-dimensional shapes materials.
DISCUSSION

Based on the results that technology is a great motivational tool as students’ understanding of geometric improved when GSP was used to enhance the students’ learning process. This was especially beneficial for the lower ability students. Technology acted as a scaffold which enabled learners to reach their zone of proximal development (Vygotsky, 1978). The improved cognitive process is supported by Dogan’s (2010) study where he observed that computer-based activities encouraged higher order thinking skills, and had a positive effect in motivating students toward learning.

Besides that, the student's response in this study is positive such 91.8% such that can improve students' mathematical understanding. Then, this results also shows that difference of learning result of control and experimental group given can be shown with average value is 5.12. The result indicated that students in the experimental group outperformed those in the control group. For test result observed pre-test and post-test value show bigger than t table, that is t count pretest = 15.931, t count posttest = 34.655, and t table = 2.042. Thus it can be concluded that there are differences in student learning outcomes between control group and experimental group after learning GSP, so that GSP can give effect to students’ mathematical understanding so that student learning outcomes increase compared to control group.

CONCLUSIONS

Based on the above results, we can conclude that the student's response in this study is positive so that in using of GSP in mathematics learning make to improving students' mathematical understanding. Besides that, this results also show that difference of learning result between control and experimental group which students in the experimental group outperformed those in the control group.

References


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This notion of a hierarchy of matrices is similar to Vygotsky’s (1997,p.199) notion of a hierarchical structure of concepts, which also contains two defining characteristics “coordinates” one is the degree of abstraction of the concepts and the second is the collection of situations in which it is relevant. The notion of a systems or hierarchies of matrices or concepts based upon relevancy and abstraction bridges the gap between “action schemes” related to a specific situation, and the notion of a collections or toolbox of schemes relevant in any given domain.