Creating Meaning by Students within Class Discourse in Mathematics: Pre and Post the Transition to Online Learning due to Pandemic

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Abstract: This Teaching-Research article contains a Pre and Post observation of class discourse from two instructors applying constructivist principles in their classroom to support student insight and shared construction of knowledge before and after the Covid-19 pandemic necessitated on-line learning. One goal of this article is to present the difficulties with translating moments of shared construction of knowledge from in-class to on-line learning in an urban community college within the City University of New York (CUNY) system. The moments of student insight and learning within classroom discourse takes place within a theoretical framework that integrates constructivist’s theory and research on creativity. This paper proposes three criteria for analysis of student insights based upon the work of Arthur Koestler (1964) to understand moments of creative insight. These three criteria are then applied to teacher led discourse and student insight, responses, and reasoning at different levels of development. Thus, a second goal of this work is to bring creativity research closer to the everyday mathematics classroom through analysis of class discourse using these three criteria.

Keywords: Accommodation, Bisociation, Discovery of Hidden Analogy, Matrix, Piaget-Garcia Triad, Schemes, Shared Construction of Knowledge, Teaching Research Team

THEORETICAL FRAMEWORK

Creativity within Accommodation

This work is founded on the premise, noted by Prabhu (2016) that all students, even those struggling to accommodate simple mathematical structures must experience moments of insight to appreciate and learn mathematics as anything other than a collection of meaningless rules. In this view creativity takes place within a continuum of difficult to observe moments of recognition, recall, and internalization of new content to more dramatic ‘Aha Moments.’ These moments of insight, especially at initial levels of learning, are often subtle as student abstract meaning through interaction with the instructor, or follow peer-peer and teacher-peer discourse e.g. what Mason (1989) refers to as a “delicate shift of attention.” More dramatic ‘Aha Moments’ occur as one develops a domain of processes related to a situation and can search through an increasingly wider collection or toolbox of such processes. The discovery of a connections to previously unrelated process in a new situation is a foundational component in accommodation, understood as the modification of one’s schemes.
Accommodation as described may be viewed as taking place within a problem-solving environment, in which a person has a goal but the solution to that goal does not readily fit into any of his routine frames of reference (i.e., it cannot be readily assimilated). Note, the term goal is used broadly here and can perhaps better be interpreted as motive. Thus, the goal may include the motive to follow social discourse or math presentations (textbook, video etc.) or inquire into how new content relates to one’s existing knowledge structures.

Schemes

The constructivist term “scheme” or “action scheme” is understood as essentially two connections between three steps within a problem-solving environment, as set forth by Glasersfeld (1995). The first connection is between features of a new situation (step 1) and appropriate solution activity (step 2). This connection is often the result of a moment of insight--what Koestler refers to as the discovery of a hidden analogy. The second connection is between the activity and progress towards the goal (step 3).

Matrices and Hierarchies

The term matrix is used by Koestler (1964) to denote any frame of reference to interpret, and respond to a situation, if it follows a set of rules (the code). To analyze moments of insight within class discourse and whether such insight resulted in successful learning or accommodation, the terminology (“schemes”) of constructivists is integrated with the terminology (“matrices”) of Koestler (1964). We note, that the term matrix is used more broadly (outside mathematics and science) than the constructivist notion of an action scheme. Koestler (1964,pp.610-620) describes a hierarchy of matrices as the result of abstraction based upon two characteristics, first relevancy to any given situation (similar to an action schemes), and second, a conceptual network derived from the abstracted essence of the different situations it interacts with. ¹

Three Components of Moments of Insight

The three components of creativity leading to accommodation are as follows: first, the search process to resolve the goal in a new situation; second, a resulting connection (previously hidden) between the situation and a relevant scheme; and third, the novel process that results. The creation of a novel process or creative product is a defining characteristic of creativity research e.g. Leinkin & Pitta-Pantazi (2013). In this work, novel or original is taken as subjective (it is new for the learner), and a novel process is understood as a process born through reflection upon solution activity, e.g. process-object duality theories Dubinsky et. al. (1999). The notion of a connection between a new problem situation, and a previously overlooked scheme is central to Koestler’s notion of a hidden analogy and bisociation, and it is inherent in the previous definition of s’ scheme Glasersfeld (1995). The understanding that the search process within a new situation varies depending upon the level of development is a foundational characteristic of the Piaget-Garcia Triad (1983). We note that the degree of originality and depth of the resulting connection will also vary depending upon the level of development.

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Piaget-Garcia Triad Level 1

Students in level 1 view mathematical procedures as externally directed rules that must be memorized. Steffe (1991) present the first level as a transition from empirical reasoning (perceptual real-life objects) to internal reasoning (mental reference to perceptual objects required) and, finally, through interiorization (birth of abstract conception and process) to the ability to engage in activity independent of perceptual objects. In a classroom setting outside the use of ‘manipulatives’ to understand procedures, internalization often involves the solver’s creation of meaning not from manipulating perceptual objects, but from reproducing solution activity modelled by a mentor or peer. Norton and D’Ambrose (2008) suggest that social constructivist pedagogy based upon Vygosky’s notion of internalization involves reconstruction of modeled activity through conscious reflection and not simply imitation, yet they acknowledge that this explanation poses a dilemma for constructivist pedagogy, particularly with students at the first level of development. When does the teacher’s action promote consciousness as opposed to imitation? Vygotsky (1997) notes that “intelligent conscious imitation comes instantly in the form of insight, not requiring repetition.” (p.221).

Piaget and Garcia (1983) characterize the first level as one in which the search process is localized, that is on an existing scheme that is not sufficiently understood i.e., intuitive, semi-conscious or externally driven. Moments of insight are thus, often subtle, and undetected. In the first level, the connection and resulting activity are typically internal processes that becomes interior when one can perform them independently i.e., anticipate the results of an activity without the need to engage in such activity, Dubinsky et. al. (1999).

Piaget-Garcia Triad Level 2: Toolbox of Schemes

In the second level the search process extends beyond an existing scheme that one does not fully understand or exterior modeling (conscious imitation), and towards features in a new situations that require one to modify, adapt or generalize an emerging process. As the student transitions beyond the need to connect processes to a specific situation they simultaneously develop the ability to apply it to related situations, and thus develop a sense of ‘problem types.’ The collection of processes that apply to a situation develop into domain of processes for a problem type, what we have referred to as toolbox. The connections often integrate schemes appropriate for a given problem type or domain, including those learned in a previous math class. These connections may involve a synthesis of the reasoning of one process with another to form a new reasoning or code i.e. the bisociative synthesis of unrelated matrices Koestler (1964). At this level, the novel processes that result are typically new not only for the individual, but also for the other students in the class. The resulting novel connections are new not only to the individual but typically to the entire class, and to some extent the teacher as well.

As students transition between the second to third level, moments of insight are often distinct examples of ‘Aha Moments.’ Thus, they occur during the transition from incubation to illumination within the four stages of creativity developed by psychologist Graham Wallas (1926): preparation, incubation, illumination (insight), and verification.

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Piaget-Garcia Triad level 3 Connections that Transcend Domain

In the third level, the solver can increasingly transcend their toolbox of schemes, crossing domains when necessary. In this level, hierarchies of schemes (matrices) are being developed as connections are between problem types that cross domains even different fields. The novel processes that result are often new to the instructor as well as of interest to the mathematics community. Koestler (1964) provides copious examples of creative discoveries by research scientists and mathematicians. In mathematics education the third level is typically the domain of the study of ‘gifted students’ and it is at this level that the creative process is often characterized by notions such ‘flexibility’ and ‘fluency’ or ‘divergent thinking’ e.g. (Leikin & Pitta-Pantazi, 2013), (Singer et al.,2017).

The Role of the Teacher: Didactic Contract

Norton and D’Ambrosio (2008) characterize pedagogy, based upon Vygotsky’s notion of (social) internalization, as a cycle of “assistance” or guidance by the teacher modeling or explaining mathematical content and reasoning followed by “internalization” viewed as “…the internal reconstruction of an external process…” (Vygotsky, 1978, p.56). These authors suggest that pedagogy based upon Vygotsky’s work should present content within the upper reaches of a student’s ability, at which he or she can function only with assistance by a mentor—in other words, his or her zone of proximal development or ZPD.

The Didactic contract (Sarrazy and Novotná 2013). between the teacher and students is built upon a ‘less-is-more’ principle which underlies the tenets of constructivism and guided discovery methodology used within constructivist teaching experiments. This principle simply put states that, the less direct instruction, the more opportunity for the student to actively internalize content. As students begin to form interior constructions, instructional methods to assess whether the process have become interior include leaving time delays e.g. ‘the next-day-effect, introducing new problem features that require generalization, or coordination with related processes in the student’s toolbox. This approach begs the question of how to teach, assist struggling students, promote student insight, and share creation of knowledge in social discourse.

The Teaching Research Team

In this work, both the observer and instructors are members of a teaching research team at a community college in the CUNY system. The classroom instruction methodology of two instructors on the team were observed to analyze how they supported student insight, and shared construction of meaning and knowledge within social discourse both before (in-class) and after (on-line) instruction was necessitated by the Covid-19 pandemic.

Methodological and Theoretical Research Questions

1. What was the methodology of the instructors to motivate students and support the shared construction of knowledge in a flexible and creative learning environment, and how did the shift to online learning necessitated by the Covid-19 pandemic effect-limit this shared learning experience?
2. How useful are the three criteria developed within the Piaget-Garcia Triad in classifying and analyzing the continuum of student insight and responses from subtle shifts of attention to noticeable ‘Aha Moments’ within class discourse?

EMPIRICAL INVESTIGATIONS: TEACHING RESEARCH

In an earlier article Baker et. al. (2019) the teaching methodology of Professors Stachelek (College Algebra) and Wolf (Pre-Calculus) to promote student participation based upon their insights in the classroom learning process was reviewed.

Professor Stachelek, In-Class Lesson:

The first technique Stachelek employed was board work by volunteer students using their notes from the previous day. The students had to: name, sketch and find the domain/range of common ‘parent’ functions i.e. the linear function $f(x) = x$, the quadratic $f(x) = x^2$, the rational function $f(x) = \frac{1}{x}$ and the radical function. Scaffolding assistance included peer-assists from colleagues although these were limited to short directives e.g. do this, draw that… and hints from the instructor whose role The students readily and correctly completed the graph and found the domain-range of the linear and quadratic functions yet had much more difficulty with the rational and radical functions. The instructor seized on this opportunity to (re)introduce interval notation involving the concept of infinity.

This instructional methodology can be classified as level one (Piaget & Garcia Triad) because it was designed to give hands on practice of review of notes i.e. ‘conscious imitation’ and is a good exercise to promote internalization. In lower algebra courses these students clearly experienced many examples of linear and quadratic functions, and these are less cognitively demanding, especially the linear function. However, they struggled with the sketch of a radical, (it looks more like a sequence of line segments connecting points than a curve) finding the domain and range is especially challenging. Thus, the students consciously imitated the previous day work of instructor Stachelek from their notes, they began within their comfort zone (linear and quadratic) and moved to the upper level of their ZPD (radical and rational).

Inducing a Gestalt Experience

Then Professor Stachelek introduced a very creative methodology he titled “guess the parent function,” he asked the students for small reasonable domain values of $x$ and then gave them corresponding values of $y$ in an $x$-$y$ table format and asked them to guess the function. He presented two examples the first involved a linear function with an additive constant ($y = x + k$) and the students readily grasped that it was both linear and the constant. Then he introduced a quadratic function with an additive constant ($y = x^2 + k$) and the students struggled to understand it. This methodology was designed for the second level, as it required students to transition from an imitation of what was previously done (linear scheme) to a new less intuitive situation (quadratic scheme). The instructor scaffolded this challenge with clear and repeated instructions (e.g. “what parent function is this?”). The question-statement itself suggests to students it may not be the same as the previous linear scheme. However, for a while, it would appear most were trying
to interpret the new information using their linear schemes, finally in response to the instructors repeated questioning one student realized it was not linear as the previous one and responded that it was quadratic.

**Aha Moment**

Immediately after the first student suggested that it was a quadratic parent, a second student had an “Aha” moment, exclaiming “I got it” as she explained what the function was. It appeared that she had a Gestalt-like “Eureka” moment in which the correct solution came about only after she abandoned her linear scheme and the hidden “quadratic scheme” was presented by her peer. When Professor Stachelek asked the class to break into groups and present their results on the board, she was both a leader in her group and willingly assisted other groups when they had trouble presenting their results. Although she was clearly an advanced student, it was also clear that her realization resulted in positive affect and motivation to participate and assist other students. In summary, the technique of presenting a situation that readily assimilated into an existing scheme and immediately following this occurrence with a situation that superficially appears equivalent but requires a much less commonly used scheme was very effective in producing an “Aha Moment.”

According to Salamat et. al. (2018) “E-learning us utilizing electronic technologies to get access to educational curriculum” (p.2) These authors characterize e-learning as having potential to make learners independent and to bring about more equality in access to learning, it may motivate students and provide them with more flexibility and instructor access. However, Ariyanti (2020) reports statistical evidence that student performance on learning objectives decreased because of online learning during the pandemic. These results are not mutually exclusive e-learning may have much to offer, but not for unprepared instructors and students.

**Professor Stachelek, On-Line Lesson:**

The on-line lesson by Professor Stachelek was a creative format using the game show *Jeopardy* through “screen share”; the lesson objective was to review topics in elementary statistics for the final exam. The *Jeopardy* game board contained 4-5 columns-topics throughout the course and different levels of difficulty (i.e., rows-money). The topics included normal distribution, probability, linear regression, the binomial probability distribution, confidence intervals, and hypothesis testing. The game format was clearly exciting for the students and they enjoyed choosing topics at different ranges of money, yet they were often silent when presented with the *Jeopardy* question.

This review lecture would be classified as requiring second level development, in that the situations presented required the students to search for, recall, and connect schemes that had been learned weeks even months ago to appropriate solution activity. We review one example that highlighted the teacher-led discourse observed: the student chose a *Jeopardy* button under Probability, a table of data values with rows of student enrollment status (part-time PT or full-time FT), and the total. The columns displayed a tendency for smoking (none-NS, light-moderate LS or heavy HS) and the total.
The question was as follows: find the probability of any randomly selected student, either PT or HS.

1. A student replied that the answer is as follows: \((|PT| + |HS|) / |Total\ students|\).
2. At this point the instructor provides a scaffolding hint: “There is a little hitch here!”
3. The student replies, “We need to subtract 3 (the number of PT students who were also HS).
4. The instructor asks, “Why?”
5. The student replies, “Because it is in both” (3 is PT ∩ HS).
6. The instructor comments, “Nice job!” and reviews the formula for finding probability of PT or HS.

Although the online format seriously reduced the amount of peer-peer interaction (the audio did not readily allow for flexible group discourse), it did allow for limited back and forth dialogue. The focus of this on-line discourse was first the instructor-problem; then, when a student comments, the instructor stops as the audio switches to the student response.

**Level of Student Development**

The student had apparently internalized the conception that those elements counted twice as part of both a row and a column, and thus they need to be subtracted. However, in this review (after several weeks), this internal process was not adequately recalled. In this situation, one might say the internalization was in short-term, but not in long-term memory a classic manifestation of a level one participatory scheme (Tzur & Simon, 2004). Indeed, in the terminology of these authors one might say the student was in transition from a participatory to an anticipatory scheme as she required minimal assistance (i.e., the “OPPS-effect”).

**Professor Stachelek, Comparative Analysis:**

The on-line lecture was noticeably lacking in peer-peer interaction due to platform constraints and no moment of creativity was noticed within the in-class lecture. Thus, online learning makes the collective creativity referred to by Glăveanu (2011) more difficult. However, there was interaction between the instructor and peers that promoted a common type of accommodation in the math classroom, what might be called scheme reconstruction through s moment of recall. In this modification, an appropriate scheme is readily available, yet details have been forgotten. This accommodation as re-construction of forgotten details is frequently observed as students transition from “participatory” schemes that require assistance to more independent schemes (Tzur and Simon, 2004). The connection is between the situation and the newly recalled details of the existing scheme, and the novel process is a reconstruction of the forgotten original process.

**Professor Wolf: In-Class Lesson:**

The in-class lecture by Professor Wolf was more traditional, teacher led. It was, however, very motivational and student centered in that the instructor constantly paused, asking students for
input, and encouraging them to participate and seriously consider STEM careers. The lesson was designed to introduce Calculus students to antiderivatives using substitution as the reverse process of finding a derivative using the chain rule. The lesson began in a way uncharacteristic of constructivist pedagogy by initially presenting the antiderivative of the trig function \( \int \csc(x) \cot(x) \, dx \) and reviewing a formula sheet that had only the answer. The lesson plan then was to introduce students to the substitution technique (viewed as the reverse process to the chain rule derivative) and then to demonstrate how this substitution technique can be used to find the antiderivative of composite functions. The lesson concluded with finding the antiderivative of a trig function (equivalent to the one initially presented -using trig formulas) and finding the antiderivative using substitution.

**Building up Shared Knowledge: Instructor Flexibility**

Professor Wolf first reviewed finding a derivative that involved applying the chain rule: given \( y = \frac{2}{3} (1 + x^2)^{3/2} \), find \( y' \). After the students found \( y' = (1 + x^2)^{1/2} 2x \), the instructor asked them to find: \( \int 2x \sqrt{1 + x^2} \, dx \), the expectation being they would understand this procedure as the reverse process of the derivative they had just completed. Instead a student replied, “2x is the du!” Although this reasoning is not mathematically complete, Professor Wolf realized that the student had seen the substitution technique previously and paused to ask how she would proceed. The student explained how the substitution \( u=1+x^2 \) leads to \( du=2xdx \) and how this process can be used to find the antiderivative. Although this presentation was a teacher-led discourse, when it came to the new content in the lesson, Professor Wolf followed the lead of this student response. As the lesson continued with examples using substitution, it became evident that this instructional method resulted in the rest of the students readily grasping how to employ this technique. Note the search process for the peers involves internalizing the suggestion ‘\( u=1+x^2 \)’ and the result \( du=2xdx \) made by their peer and the connection involves relating this suggestion to resolve the situation. In this light student suggestions appears to assist with the process of internalization by peers i.e. conscious imitation of peer activity is a key learning process. In this manner, instructor flexibility promotes the shared learning experience although not necessarily student moments of insight.

**Professor Wolf, Online Learning:**

In this Pre-Calculus Course, the lesson plan was to introduce advanced topics (usually presented in Calculus) of using the limit to find the equation of the tangent line and finding the antiderivative by using the partial fraction decomposition techniques.

The lesson began with a review of a traditional Pre-Calculus topic, finding the difference quotient of a quadratic. The relationship between finding the limit of a difference quotient and the slope of the tangent line for the standard quadratic \( y = x^2 \) at the point (1,1) was discussed. Then, using straightforward algebraic techniques, the equation of the tangent line at this point was found. It can be difficult to determine the extent to which students comprehend the instructional method during an online observation as one cannot view their faces; on the basis of their comments, however, the students appeared to understand this presentation. The extent to which students followed became clear when the instructor explained the process of finding an antiderivative by
using partial fraction decomposition, a technique that involves an extensive amount of manipulation with signed numerical values. At some point in this manipulation, instructor Wolf, confused, asked, “Where did I mess up?” After a student pointed out the exact point in a long sequence of manipulations at which an incorrect rule of signed numbers had been used, the instructor responded, “You are a rock star! Did everyone get it?” and then continued.

Professor Wolf, Comparative Analysis:

In the first in-class lesson what came through was the spontaneous ability to pause the lesson trajectory and follow the lead of a student response. Although Professor Wolf valued and incorporated into the lesson student responses, this spontaneous incorporation of student responses was not evident in the online lesson. The online format was more tightly focused on the instructor presentation; the shortness of student comments and the brevity of these student responses made them more difficult to integrate into the lesson itself. The interchange between instructor and student or among students was much less flexible than in-person dialogue. However, the students (judged by their comments) readily followed the lesson plan, even making corrections at critical junctures, and the value that instructor Wolf gave to student comments translated into the on-line discourse.

INSTRUCTOR JOURNAL

The teaching research in this article focuses on student learning within class discourse; however, we believe that by sharing a summary of Professor Wolf’s journal entries from this period, the readers’ understanding of the transitional pandemic experience to on-line learning and how it affected the sense of a shared leaning environment will be enhanced.

The Transitional Teaching Experience:

When we heard that we might be sent home, I really did not believe that the transition was going to be a big deal or that it was going to be for long. I told my students that they should create a WhatsApp and in that way we could stay connected. I went home and did not take many of my personal belongings. However, it was obvious from the first week that we would not be able to go back to the campus, and subways and buses appeared to be dangerous.

I was already familiar with Blackboard, and all my textbooks were on my iPad in my i-Books. We were given a week to get our on-line act together. That week was fine for those who feel a natural connection to their students and for those who cared about their students and understood their needs. It was not long before we were up and running. The first experience teaching from home was a bust because Blackboard Collaborate (the College software platform) was constantly losing connection; furthermore, the students could not hear me, nor could I hear them. I instantly bought a subscription to Zoom, and we were up and running. The first thing I did was to reach out to all of my students, and those who didn’t respond I hunted down. Some of my students were essential workers, so they were called in to hospitals and essential businesses. Some of my students had Covid, or their family members did. In the midst of all the chaos, I managed to have an 85% retention rate.
My students and I were excited and scared. We faced many obstacles like no child-care and spotty internet or no internet at all. My research student got very sick, and we gave him advice and talked to him throughout his ordeal, and he came out unscathed. The best part for me was my first Zoom class in which one student was sitting in his kitchen eating, another was lounging on her bed, while others had several family members present, so their video was shut off. It was exciting to hear and to see them, with the knowledge that they were still learning and safe. I know that for many of my colleagues it was not the same experience. I know that our undocumented students and homeless students did not have an easy time. It has been strategically almost impossible for too many of our students to juggle housing, food insecurities, plus childcare, while trying to learn how to combine like terms or differentiate a function. For sure, as professors we gave many Incompletes.

We were so excited to reconnect and we succeeded; however, it was not easy to do so. I quickly decided I needed to assess their progress personally. Did they need to download an App to be able to send homework and tests? I bought many devices to try different ways of on-line communication; the first device was a x-pen notebook in which you write. One is supposed to be able to write on it and share, but the software in my older MacBook Air did not allow this process to work. Then I bought a giant whiteboard with a tripod, on which I would set my iPhone. It was great. Although I felt self-confident, the students got confused when I went back and forth between all of those devices; therefore, I kept going back to handwriting and sharing screens on my iPad. I would use Zoom from my IPad and had another IPad set up with the textbook. I eventually bought a OkioCam USB web camera which worked wonderfully with my lectures.

In the summer I continued this on-line experience as instructor of a college algebra course; however, I believe that the spring semester was more successful because the students had previously established a relationship with me before the on-line experience. Thus, during the summer, they were quiet and difficult reserved. It is difficult when you can’t look at your students and see whether they are lost; instead we have to assume that we are all on the same page.

SUMMARY COMPARATIVE DISCUSSION: PEDAGOIGY

We recognize that social interaction between peers is an essential component in developing a shared learning experience, and that instructor flexibility to promote and incorporate divergent student responses is critical for student creativity. Thus, we analyze instructor methodology to motivate students and support the shared construction of knowledge, and how the shift to online learning necessitated by the Covid-10 pandemic affected this experience

Professor Stachelek, Instructor and Student Creativity:

Professor Stachelek implemented creative instructional methods in both the Pre in-class and Post on-line lecture. During the in-class lesson however, there was more peer-peer interaction, and the instructor-peer interaction was followed more readily by the remaining students. Indeed, a clear “Aha Moment” as characterized by a clear search, a new (discovered) connection to an existing scheme, along with resulting novel process are all evident.

Professor Wolf: Instructor Flexibility:
The methodology of Professor Wolf in both the in-class and on-line lesson is highly motivational and her goal was to engage the students and get them to participate. The lecture method was traditional (i.e., based upon instructor-student dialogue). During the in-class lesson, when a new technique was introduced by a student, the instructor spontaneously changed the direction of the lesson trajectory to integrate the student response. This incorporation of student response motivated the class, promoting a creative learning environment. This flexibility was lacking in the on-line lesson due to the limitations of audio and lack of visual however, it was clear that students readily followed the new content of the on-line lesson and did participate with short comments when appropriate.

**CONCLUDING REMARKS**

**Research Question 1**

In this article, the first research question focuses on the quality of the instructor-student dialogue and the ways in which such dialogue supports construction of shared knowledge before and after the unexpected transition from in-class to on-line learning by the Covid-19 pandemic.

**Limitations of On-Line Dialogue**

The transition to on-line learning reduced peer-peer dialogue. Although students did follow the dialogue between their peers and the instructor, there was a degree of separation brought about by the on-line platform. This degree of separation was to some extent caused by the inability of students to see other students. In addition, although the on-line platform could handle an instructor-student dialogue, student comments were brief, and additional comments (peer-peer) outside a teacher-peer dialogue was not observed. Instead, they tended to result in distorted audio thus, all students were encouraged to mute themselves when not directly asking a question.

During the on-line lesson, the creativity of Professor Stachelek and the motivational style of Professor Wolf were both evident. However, the transition to on-line learning resulted in a more traditional instructor-student dialogue. This was due to the limitations of visual and audio. Although it was clear that students were learning from their mistakes and following the lesson, the audio and visual limitation restricted peer-peer interaction. This limited student creativity as well as instructor flexibility to incorporate student responses into the lesson.

The transition to online learning was made one day in March 2020 without warning, as the seriousness of the situation came upon NYC unexpectedly. Thus, the instructors had no preparation for this change to an online platform. Further study is required to determine if training and experience can overcome the audio and visual limitations to the collective learning experience especially that of students observing and learning from peer’s behavior. Perhaps technological improvements in group audio, or platform features such as “small break out group” may assist in this effort. On the other hand as noted in Professor Wolf’s journal entry it is equally possible that the sense of a shared learning experience will actually be more hindered in future classes that are completely on-line, as the students will never experience a social in class experience with their peers.
Research Question 2: Theoretical Considerations

The second research question focuses on how the three criteria of: search, connection and resulting novel process characterize the continuum of moments of student insight as they grow in understanding during class discourse.

During the in-class lesson of Professor Stachelek the ‘Aha Moment’ by a student demonstrated a clear (Gestalt) search process, as she abandoned her linear scheme in favor of a quadratic scheme, the resulting connection was to the new quadratic scheme. The novel process was her ability to translate raw data into her quadratic and later her radical schemes instead of only her linear scheme.

In the on-line lesson by Professor Stachelek the moment of insight was the result of a search to recall details of an existing scheme that had been forgotten, the connection was between the situation and these recalled details. The resulting activity was the result of reconstruction of the earlier forgotten activity. In the spectrum of accommodation this moment of insight would not be considered illumination of new content.

The in-class lesson of Wolf demonstrates students that appear to grasp a novel technique (substitution) suggested by a peer. However, as the search process is limited to interpreting what the peer or teacher presents, it is difficult to determine whether student’s conscious imitation was participatory or whether their connections would allow for independent activity, During the on-line experience the connections were those of a traditional lesson i.e., between the material presented by the instructor and individual students’ schemes.

In using these criteria to analyze moments of insight, clearly the dept of the connection and the resulting new process, must be considered, and more work is needed to trace out a continuum of moments of insight within class discourse. This continuum certainly includes the ‘Aha Moment’ observed in Professor Stachelek’s class where the connection was to a novel scheme, leading to novel solution activity, and during the on-line lesson where the connection was between forgotten material in which the process was novel only in that it had been recalled. In the in-class lesson of Professor Wolf the connection although not new for the student was novel for her peers, and it did result in new activity for these peer students.

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