

EDITORIAL

The philosophy of mathematics education can be traced back to the work of Plato. In his *Republic* Plato considered deeply the role and purpose of mathematics in teaching and learning. His enquiries were founded on ethics, for questions of meaning and purpose within a social context inevitably bring in the Good. At the same time, he was interested in the epistemology and ontology of mathematics and its relations with the Truth and Beauty. Overall, Plato displayed great interest in the subject of mathematics throughout his philosophical work, and he is an inspirational godfather and patron saint of the philosophy of mathematics education.

Current mathematics education research is mostly concerned with two questions, one epistemological and the second methodological. The epistemological questions asks what is mathematical truth and how do we justify and explain it, and above all, how to we come to know it? The methodological questions concern how we can best and most effectively teach and facilitate the learning of mathematics. Research in the philosophy of mathematics education also addresses epistemological questions of mathematics and its teaching and learning, but it does so more explicitly, more theoretically. In addition, it considers the ontological, aesthetic and ethical issues of mathematics with respect to education and society.

The philosophy of mathematics education is an interdisciplinary area of research that incorporates many questions.

- What are the goals and purposes of mathematics education?
- What can we learn from deep analyses of the methods and means of teaching and learning mathematics, as well as from studying the underlying theories and philosophies?
- What new insights are revealed by the application of deep theoretical approaches including Phenomenology, Hermeneutics, Complexity, Embodiment and Critical Theory within research in the philosophy of mathematics education?
- What are the relationships between and, the mutual influences on each other, of the philosophy of mathematics and mathematics education?
- How do the personal philosophies of mathematics and mathematics education of learners, teachers, teacher educators and researchers impact on practice?
- How are the different actors of interest including students, teachers, researchers, theorists, philosophers and mathematicians linked together professionally within the fields of mathematics education research and practice?
- How do mathematics and the philosophy of mathematics impact on the nature, structure and content of mathematics for teaching?

- What do deep analyses of mathematics itself tell us about its structures, processes and fundamental concepts and about their relationships with its teaching and learning?

This special issue offers a range of partial answers to these question, and suggested pathways to further extend research in the areas. It represents work in progress from worldwide scholars including both northern and southern hemispheres, and from the east and the west. Every continent barring Australia is represented, thus providing a truly global representation of work in progress.

This volume is divided into four parts, a tetrahedron whose faces represent different directions of inquiry in the philosophy of mathematics education.

First, there are papers about the philosophical foundations of mathematics education research including foundational issues of research and methodologies and paradigms, meaning and the semiotics of mathematics and mathematics and the special mathematical issues of rigor and axiomatics.

Second, there are the philosophical dimensions of teaching, learning and teacher education including: teachers' epistemologies and the use of incompleteness theorems in teacher education, critical examinations of constructivist theory, mediation in Vygotskian theory, the use of French Didactique and French notions of knowing in China, as well the crowning purposes of learning mathematics.

Third, there are future oriented visions of how the theory and practice of teaching and learning mathematics might be developed. This includes questions such as how can we theorize sustainable development in mathematics education? How can we incorporate philosophy of mathematics education and theories of imagination for normal students? How can or should we introduce the theory of infinity and George Spencer-Brown's laws of form in the teaching and learning of mathematics?

Fourth, philosophy and the humanities are home to many grand and widely used theories beyond our own multidisciplinary field. How can we bring them into mathematics education to extend its philosophical foundations and the underlying research paradigms? In what ways can we utilize Phenomenology, Hermeneutics, Complexity Theory, Embodiment, Critical Theory and Critical Pedagogy in our research and teaching practices? The papers in this section offer tentative accounts of the achievements so far, in pursuing these directions of research.



All these papers were due to be presented in Shanghai July 2020 at the *International Congress of Mathematical Education*. This, of course, needed to be postponed because of the Covid-19 pandemic. Instead, the papers have been rewritten, extended and improved for publication here, without waiting until after the conference. So, something positive has come out of this crisis. We present to you, the philosophy of mathematics education 2020, the current state of the art, for your edification and enjoyment!

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