Learning to use Mathematics vs Mastering Basics

Bukurie Gjoci

bgjoci@lagcc.cuny.edu

Department of Mathematics, Engineering & Computer Science

LaGuardia Community College of the City University of New York

31-10 Thomson Ave, Long Island City, NY 11101, USA

Abstract: *This paper bestows the literature support for the need of innovative curriculum and instruction approaches of College Mathematics courses offered to students in need of remediation. One such curriculum is presented here. Designed in agreement with AMATYC (2018)-IMPACT's vision, this innovative curriculum & instruction approach aims to create student-centered learning environments, engage students in written and oral communication and meaningful use of technology. It immerses students into subject's big picture, explaining its components implicitly and narrows the gap between school and out-of-school mathematics.*

This is Part 1 of the research project aiming to explore the use of top-down instructions when teaching mathematics thematically as a pedagogical way to introduce students to higher level mathematics regardless of their prior knowledge. The author's intention conducting this research is to substantiate mathematics teaching and learning pathways that do justice to the students and to mathematics as a subject.

Keywords: College Mathematics Teaching, Top-down Instructions, Teaching Mathematics Thematically, Learning to use Mathematics.

Mathematics is a way of thinking about problems and issues in the world.
Get the thinking right and *the skills come largely for free.*
(Devlin, 2010)

Many students entering higher education in general and especially junior colleges lack important prior knowledge and basic skills needed to enter the more advanced courses in their curriculum. Traditionally the remedial courses have been designed to prepare the students for higher level mathematics, but most of the students requiring remediation get caught in a cycle of not being able to complete the remedial sequence and eventually drop out of the college. The research suggests that, *for the most part, the traditional system of developmental education is not achieving its intended purpose: to improve outcomes for underprepared students* (CCRC, 2014), even though the amount of money spent per year in remediation coursework (Mathematics, English and Reading) is roughly $7 billion nationally (Scott-Clayton, Crosta, & Belfield (2012)).

The corequisite courses were lately introduced nationwide and across CUNY colleges as a reform to emend the remediation process. The aim of the corequisite courses is to enable underprepared students to be successful in their college-level mathematics courses by presenting developmental material relevant to the learning objectives of the credit-level course. The method of backward mapping is applied when identifying the pre-requisite knowledge required for the students to understand the college level content of the course and satisfy the set of learning outcomes. Usually the syllabus for the corequisite mathematics courses indicate the remedial portion, *the basic skills part*, to be taught at the beginning of the semester and then later the college credited curriculum continues. Research shows that corequisite remediation courses can provide a more effective alternative to traditional remediation and to quicker way to college level mathematics and graduation paths to underprepared students (Strother & Klipple, 2019).

While recognizing the improvement of the corequisite courses in developmental education we should point out that the hours per week assigned for these corequisite courses is basically the sum of the hours spend in the remedial course and the credited course combined. Also, the students go through *mastering the basic skills* which often results in a rote memorization of procedures, which all will be forgotten quickly, since they will never be used by the students once the mathematics requirements for graduation are satisfied. Why spend time *rediscovering the wheel*, when all algorithmic, computational, algebraic, geometric, logical and procedural skills that used to take ten years of effort to master are performed faster and in a more accurate way by the use of technology? The educational focus has to shift from procedural mastery to *understanding* (Devlin, 2010). This shift is a must especially for the college students needing remediation, it will do justice to them and to mathematics as a subject. The justice results by *Teaching Mathematics Thematically* using *Top-down Instructions*, which allows the introduction of high level mathematics to all students, regardless of their prior knowledge, through applications of mathematics around themes that interest the students. *Teaching Mathematics Thematically* emphasizes the use of mathematics applications around a central theme, narrowing the gap between *school* and *out-of-school* mathematics. *Top-down Instructions* focuses on providing students a large view of the subject, immersing them into the big picture of it. The explaining of the components that make up the subject will be *the implicit* part of teaching instructions. The use of technology will remove *the mastery of the computational skills* from being an entry barrier to the learning of mathematics as any other subject and will guide the students toward thinking mathematically. Devlin's (2010) statement bellow shows the benefit of teaching mathematics through relatable topics:

> "…when people of any age and any ability level are faced with mathematical challenges that arise naturally in a real-world context that has meaning for them, and where the outcome directly matters to them, they rapidly achieve a high level of competence. How

> high? Typically, 98 percent, that's how high. Also, those same people, when presented with the very same mathematical challenges in a traditional paper-and-pencil classroom fashion, perform at a lowly 37 percent level. … The evidence is clear. It's not that people cannot think mathematically. It's that they have enormous trouble doing it in a de-contextualized, abstract setting. Confusing mathematics with mastery of skills is the same as thinking architecture is about bricklaying, or confusing music with mastering the musical scale."

Teaching mathematical topics through applications that have meaning to the students personalizes the learning environments to the background characteristics of learners and their goals, increases their interest in the subject. Bernacki & Walkington, (2014, p.1) in their study of implementing a personalized intervention for Algebra I give insights of its impact, they state that "1) using a technology-based system for personalization that grounds algebra problems in students' out-of-school interests has the potential to elicit students' interest in the mathematics content to be learned, and 2) that personalization to well-developed individual interests can have a long-term effect on students' learning of algebraic concepts and their motivation to learn mathematics". During my first semester teaching remedial mathematics I experienced an Aha!Moment while trying to have students learn how to solve word problems. Introducing a new approach to Word Problem Solving, by asking my students "What if it was you?" (in the problem's situation) (Gjoci, 2015), I was applying the personalization of a mathematical situation, which eased the students into appreciation of the subject and its use and increased students' understanding.

While I strongly believe that every mathematical exercise performed properly increases intelligence, I also believe that trying to teach procedural skills that take ten years of effort to master to students in need of remediation (within one semester) when they enter college is a waste of time and resources. Handal & Bobis (2004) identified in their study some obstacles of implementation of a thematic approach to teaching mathematics as being instructional, curricular and organizational. While praising the humanistic goal of thematic instructions for its power of showing the students the usefulness of school mathematics, they stated in their results that teaching thematically requires complex and innovative pedagogical skills and a technology-based lesson setting. The college level setting minimizes the obstacles of implementing thematic instructions mentioned above, Handal & Bobis study explored mathematics teachers' believes about teaching and learning of mathematics thematically in secondary school. The curricular obstacle is basically inexistent since the college curriculum varies with respect to the major and designing major appropriate mathematical curriculums will help the students gain the useful mathematical knowledge that interests them. Providing professional development, networking possibilities and the sophisticated technology available fosters a pleasant environment for thematic instructions.

Innovative curriculum and instruction approach for college mathematics courses offered to students who need remediation will eliminate the entry barriers to the learning mathematics as any other subject. Mathematics courses should be designed according to AMATYC (2018) – IMPACT vision, aiming to: *deemphasize lecture, increase the use of active student-centered learning, make connections to other disciplines, and engage students in written and oral communication and the meaningful use of technology.* A combination of the following two teaching styles accommodates the implementation of such curriculum.

1. *Teaching Mathematics Thematically*, which emphasizes the use of mathematics applications around a central theme, narrowing the gap between *school* and *out-of-school* mathematics.
2. *Top-down Instructions* focuses on providing students a large view of the subject, immersing them into the big picture of it. The explaining of the components that make up the subject will be *the implicit* part of teaching instructions.

Blending the two teaching styles above will promote active learning. The research shows that the use of active learning engages students in more than just listening, the students are involved in higher-order thinking – analysis, synthesis and evaluation (Bloom's Taxonomy, Krathwohl, 2002). In the context of the college classroom, active learning can be defined as anything that involves students in doing things and thinking about the things they are doing, and they are engaged in activities like reading, discussing, and writing (Gjoci, 2018). The goal of the following curriculum is to display that the use of top-down instructions when teaching mathematics thematically is a pedagogical way to introduce students to higher level mathematics regardless of their prior knowledge. Designed in agreement with AMATYC(2018)-IMPACT's vision, this innovative curriculum & instruction approach aims to *create student-centered learning environments, engage students in written and oral communication and meaningful use of technology*. It immerses students into subject's big picture, explaining its components *implicitly* and narrows the gap between *school* and *out-of-school* mathematics. All of the topics in the following curriculum will be introduced through a real-life application problem. The learning process the students will experience is called the *situated learning*. While being guided to get the mathematical thinking right about the problem they are faced with, the need to piece together the mathematical skills required will be generated. The curriculum for a Calculus 1 class follows.

Calculus 1

1. Real Numbers – Introducing Set Theory
   1.1 Basics of Set Theory
   1.2 Important Sets of Numbers
      - The set of Natural Numbers, ℕ
      - The "whole numbers", 𝕎
      - The set of Integers, ℤ
      - The set of Rational Numbers, ℚ
      - The set of Irrational Numbers, 𝕀.
      - The set of real numbers, ℝ
   1.3 The Geometric description of the set ℝ of real numbers, the Real Number Line

While covering the above topics the following should be emphasized:

- The concept of infinity
- The concept of a well-ordered set
- The continuity of real numbers, the completeness property of real numbers.
- The introduction of the abstract representation of any number in the above sets by a variable x, which possesses the properties of the given set
- The abstract concept of a point without dimensions in the real number line
- The width-less dimension of a line

Many other advanced concepts may be introduced using the knowledge of real numbers: group theory, introductions of proofs etc.

2. Functions of Real Numbers
   2.1 Coordinate Geometry – The Cartesian Plane
   2.2 Definition of Function
   2.3 Properties of the most common (parent) functions
      - Linear Function
      - Absolute value Function
      - Quadratic Function
      - Cubic Function
      - Polynomial Functions
      - Rational Function
      - Radical Function
      - Exponential Function
      - Logarithmic Functions

- Absolute Value Function
- Greatest Integer Function

The use of a graphing device will be used heavily while studying the properties of the above functions accompanied by students' engagement of oral and written communication. The continuity property of the function will be emphasized and the change in the value of $f(x)$ as the value of x is changing will be explored.

3. Introduction to the Limit of a Function
   3.1 The Limit – Informal Definition
   3.2 Continuity – Revisited
   3.3 Finding the Limit of a function – Graphically and then algebraically.
      - At any value of x in the domain
      - As x approaches the point of discontinuity – Asymptotes Revisited
      - As x approaches infinity

The use of a graphing device will be used heavily while studying the properties of the above functions accompanied by students' engagement of oral and written communication.

4. The Derivative
   4.1 Average and Instantaneous Rate of Change
   4.2 Derivative – The definition of
   4.3 The meaning of derivative when it is positive, negative or equal to zero.
   4.4 Derivatives of the functions from Topic 2 above
   4.5 Rules of Differentiation

The use of a graphing device will be used heavily while studying the properties of the above functions accompanied by students' engagement of oral and written communication.

The forthcoming Part 2 of the research will describe the process and the results of teaching the above curriculum thematically using top-down instructions. The central theme(s) of the real-life applications used will be determined once the group of students volunteering to participate in this project is formed and their interests are known.

Author biography

Bukurie Gjoci is an assistant professor in the Department of Mathematics, Engineering & Computer Science at LaGuardia CC – City University of New York, USA. Her research in mathematics education includes standardized tests and knowledge measurement, assessment's use in teaching and mathematical curriculum design. She believes every mathematical exercise, when performed properly, increases intelligence. Gjoci holds a bachelor's degree from The University of Tirana, master's degree from Colorado State University, and PhD from Columbia University.

REFERENCES

AMATYC (2018). American Mathematical Association of Two-Year Colleges. *IMPACT: Improving Mathematical Prowess And College Teaching*. Memphis, TN: Author. https://cdn.ymaws.com/amatyc.site-ym.com/resource/resmgr/impact/impact2018-11-5.pdf

Bernacki, Matthew & Walkington, Candace (2014). The Impact of a Personalization Intervention for Mathematics on Learning and Non-Cognitive Factors. *Proceedings of the Workshops held at Educational Data Mining 2014, co-located with 7th International Conference on Educational Data Mining* (EDM 2014). *London, United Kingdom, July 4-7, 2014.* http://ceur-ws.org/Vol-1183/ncfpal_paper02.pdf

CCRC (2014). What do we know about the developmental outcomes? *Community College Research Center at Teachers College, Columbia University*. https://ccrc.tc.columbia.edu/media/k2/attachments/what-we-know-about-developmental-education-outcomes.pdf

Devlin, Keith (2010). In Math You Have to Remember, In Other Subjects You Can Think About It. *Devlin's Angle,* https://www.maa.org/external_archive/devlin/devlin_06_10.html

Gjoci, Bukurie (2018). Formative Assessment and Active Learning: Tools for Nurturing Higher

Level Thinking Skills. *Math*AMATYC *Educator*, February 2018, Vol. 9 Number 2 http://c.ymcdn.com/sites/www.amatyc.org/resource/resmgr/educator_feb_2018/Gjoci.pdf

Gjoci, Bukurie (2015). What if it were you? *Mathematics Teaching – Research Journal Online*, *Vol 8, N 1-2* (Fall and Winter 2015/16). https://commons.hostos.cuny.edu/mtrj/wp-content/uploads/sites/30/2018/12/v8n12-What-if-it-were-you.pdf

Handal, Boris & Bobis, Janette (2004). Teaching Mathematics Thematically: Teachers' Perspective. *Mathematics Education Research Journal, 2004, Vol. 16, No. 1, 3-18* https://files.eric.ed.gov/fulltext/EJ747861.pdf

Judith Scott-Clayton, Peter M. Crosta, and Clive R. Belfield (2012). Improving the Targeting of Treatment: Evidence from College Remediation. *NBER Working Paper No. 18457 JEL No. H75,I23,J24* https://www.nber.org/papers/w18457.pdf

Krathwohl, D. R. (2002). A revision of Bloom's taxonomy: An overview. *Theory into Practice 41*, 212–218. doi: 10.1207/s15430421tip4104_2

Strother, S., & Klipple, K. (2019). Corequisite remediation in math: A review of first year implementation and outcomes of Quantway and Statway. *San Francisco, CA: WestEd.* https://www.wested.org/wp-content/uploads/2019/10/Corequisite-Remediation-in-Mathematics.pdf