

Al-Khwarizmi (Algorithm) and the Development of Algebra

David M. Nabirahni, Brian R. Evans, Ashley Persaud

dnabirahni@pace.edu, bevans@pace.edu, ap13061p@pace.edu

Pace University

Abstract*: The purpose of this article is to provide some background on the life and contributions of Muhammad ibn Musa al-Khwarizmi to the development of algebra, in particular. The authors would like to share this information with algebra teachers at the high school and college level because mathematics history has the potential to engage students and provide a "human face" to the subject. Consequently, this may lead to higher achievement for students. This article contributes to mathematics history and algebra.*

INTROCUCTION

Muhammad ibn Musa al-Khwarizmi *aka* Algorithm was an intellectual from the 8th through the 9th centuries who contributed significantly to the development of algebra (Evans, 2014). Al-Khwarizmi was born in 783 CE in Khwarizm in then Persia of Southwest Asia, which is presently the city of Khiva in Uzbekistan (Aksoy, 2016; Stewart, 2017). Al-Khwarizimi was highly influential in the development of algebra throughout Southwest and Central Asia, North Africa, and Europe, which thus subsequently influenced the development of algebra and medieval and modern mathematics throughout the world. It would serve any high school or college algebra instructor very well to be able to impart some of this historical knowledge to students in the classroom. The authors are interested in sharing background on al-Khwarizmi with other mathematics teachers since it is believed this will make mathematics more interesting and engaging for students, which may lead to higher level of achievement for students along with an increased appreciation for the contributions of Persian mathematics.

HISTORICAL BACKGROUND

As the Islamic Caliphates of Baghdad had only invaded the Khwarizm for mass conversion to Islam just a few years before his birth, and as in part evidenced by al-Khwarizmi's father's name Musa in then a predominantly Zoroastrian Persian world, some speculated him to be of Mizrahi Persian Jewish heritage. Moreover, although the newly established Islamic Omayyad followed by Abbasid caliphate dynasties had overshadowed the Persian and Egyptian Empires, and the Iberian peninsulas in Europe, and thus as a result Baghdad had become the center, and the Arabic language had become the *lingua franca* as English is today; nonetheless, the use of the Islamic world is a misnomer. It is true the Persians had by and large become Muslim but had and still continue to retain their historical multiethnic Persian/Iranian identity. Al-Khwarizmi's early studies took place in various prominent madrasas called Khiva, Kiat, and Gurgench (Aksoy, 2016). Al-Khwarizmi was able to study texts in multiple languages including Persian, Arabic, Syrian, and Sanskrit (Aksoy, 2016). The topics al-Khwarizmi studied in depth and later contributed toward their advancement were geometry, astronomy, and geography among others (Aksoy, 2016). Eventually, he became widely known as a top scholar of the region of Khorazm (Aksoy, 2016).

Al-Khwarizmi lived during an era called the Golden Age of Islam, between the 8th and 13th centuries (Evans, 2014; Faruqi, 2015). Contributing to the Persian Golden Age, al-Khwarizmi led a very pious and intellectual religious life (Faruqi, 2015). By 830 CE, he began studying under the reign of Caliph al-Ma'mun, who ruled from 813 to 833 CE (Ramsden, 2017). Al-Khwarizmi's studies took place in the *Bayt al-Hikma*, or the House of Wisdom, in Baghdad (Stewart, 2017). The House of Wisdom, founded by al-Ma'mun, contained a research library and observatory where many scholars studied (Aksoy, 2016). Al-Ma'mun envisioned the House of Wisdom as a place where all Greek texts could be translated into Arabic and or Persian (Aksoy, 2016). Baghdad became a central location for science and trade, which attracted many scholars mostly from Persia and as far as China and India (Stewart, 2017).

Al-Khwarizmi worked with many prolific mathematicians and scientists in the region, such as the Banu Musa brothers, also known as Sons of Moses, and Al-Farghani (Aksoy, 2016; Ramsden, 2017). He worked with others to calculate the circumference of the Earth and determined that the Earth was spherical. This conclusion was soon accepted by the scientists of this time (Aksoy, 2016).

Around 825 CE, al-Khwarizmi wrote the manuscript, *al-Jam' wa al-Tafriq bi Hisab al-Hind*, translated as *The Book of Addition and Subtraction According to the Hindu Calculation* (Evans, 2014; Gillispie, Holmes, Koertge, & Gale, 2008; Stewart, 2017). The text was al-Khwarizmi's most influential work and remained so for the several hundreds of years that followed (Stewart, 2017). During the medieval European era, the text was translated into Latin and shared widely because it showed Europeans a novel method to carry arithmetic operations. The original version of this text has been lost, but the Latin version, *Algorithmi de Numero Indorum*, still exists (Gutstein & Peterson, 2013). In this work, al-Khwarizmi introduced the decimal system that was created by Hindu mathematicians in the 6th century and added zero to the system to complete it (Aksoy, 2016; Baharuddin & Wan Abdullah, 2014). The concept of zero as both a number and placeholder should not be taken lightly. The concept of zero may be one of the most important ideas in mathematics as related to numeral systems. Evidence can be found for the development of zero both in Indian and Mayan mathematics with the Indian concept influencing the work of al-Khwarizmi and other scholars of the time.

Al-Khwarizmi explained how to add, subtract, multiply, and divide using this numeral system (Gillispie et al., 2008). Al-Khwarizmi provided solutions as sequenced steps, thereby introducing the concept of the algorithm and thus leading to the creation of the word algorithm the precursor for today's computing (Aksoy, 2016). The word "Algorithmi" in the Latin title of the work became known as "Algorismi" and the mathematical methods using the numeral system as described in the work became known as "algorisms" (Stewart, 2017). Among the Europeans, the phrase "dixit Algorismi," or "thus spoke al-Khwarizmi" became an arguing point in mathematical disagreements to send the message that any words written by al-Khwarizmi are final, true, and must be followed, not argued against (Stewart, 2017).

Around 830 CE, al-Khwarizmi wrote another significant mathematical work in an effort to give the Persian hemisphere and Muslims mathematical aids to solve issues of inheritance, partition, lawsuits, legacies, and trade (Stewart, 2017). His work, *al-Kitab al-mukhtasar fi hasab al-jabr wa-l-muqabala*, translated as *The Compendious Book on Calculation by Completion and Balancing,* is considered to be the first book on algebra (Rashed, 2015; Stewart, 2017). At this time, algebra was solely the operation of restoring an amount that was subtracted when solving for an unknown (Hannah, 2015). Al-Khwarizmi began the book by writing about six algebraic equations of the first and second degrees in which linear and quadratic equations can be reduced in order to find a solution (Gillispie et al., 2008). The solutions for reducing any problem to these equations involve the operations of balancing and completion (Gillispie et al., 2008). A geometric model for a quadratic case is that x^2 can be thought of as a square with side of x units. More complex quadratic equations can also be thought about from a geometric perspective by added additional rectangles such as 2x with length of 2 units and width of x units to model $x^2 + 2x$.

The word "al-muqabala" in the title of the book referred to the operation of balancing. The word algebra comes from "al-Jabr" (literally means to *enforce*) in the title of the book, which al-Khwarizmi used to describe the operation of completion (Aksoy, 2016; Faruqi, 2015). In the second part of the book, al-Khwarizmi writes about mensuration. He provides steps for solving the area of plane figures such as the circle and solving the volume of solids such as the truncated pyramid (Gillispie et al., 2008). The third part of the book is the longest and consists of solved problems regarding legacies. The solutions involve arithmetic and simple linear equations. However, knowledge on Islamic juris prudence on inheritance laws is needed in order to understand the problems (Gillispie et al., 2008).

In conjunction with the algebraic solutions of the equations in al-Khwarizmi's book, there are geometric proofs (Aksoy, 2016). Al-Khwarizmi was the first scholar to provide geometric proofs to quadratic equations (Aksoy, 2016). Al-Khwarizmi's original ideas in this text, such as those pertaining to geometry, are believed to be inspired by Euclid's *Elements* (Ramsden, 2017). In all of his mathematical texts, al-Khwarizmi did not use the standard mathematical symbols we use today (Stewart, 2017). Expressions and solutions were described verbally in sentences without symbols. Al-Khwarizmi used the word "unit" to describe a number, "x" or to describe a root, and the word square "Morabba" to describe x^2 (Aksoy, 2016). For example, the equation $x^2 + x = 12$ would be expressed as "square plus root equals twelve units" (Stewart, 2017, p. 31). In his equations, al-Khwarizmi only used whole numbers, and the solutions only included positive numbers (Aksoy, 2016).

Not only is al-Khwarizmi recognized for his seminal contribution to mathematics, but that he is also regarded for his contribution to the field of astronomy. Al-Khwarizmi was one of the first scholars to draw the world map and create an astronomical table (Baharuddin & Wan Abdullah, 2014; Faruqi, 2015). His schematic tabulation was used to find the positions of stars and planets using calculations (Faruqi, 2015). Al-Khwarizmi's most famous astronomical work is *Zij al-Sindhind*, translated as *Astronomical Table of the Sindhind* (Gillispie et al., 2008; Stewart, 2017). The table was written around 820 CE and was in part based on the Indian methods of studying astronomy (Stewart, 2017; Yazdi, 2011). The text includes astronomical tables for determining the magnitudes of solar eclipses and finding solar and lunar velocities and apparent diameters (Yazdi, 2011).

Since al-Khwarizmi was interested in astronomy, he wrote a treatise on the Jewish calendar titled *Risala fi istikhraj ta'rikh al-yahud*, translated as *Extraction of the Jewish Era* (Gillispie et al., 2008; Stewart, 2017). In this text, al-Khwarizmi explains features of the Jewish calendar such as the 19-year Metonic cycle, the process of determining which day Tishri should fall on, and the steps for using the calendar to figure out the mean longitude of the sun (Gillispie et al., 2008). The text also includes a calculation of the time period between the Jewish era and the Seleucid era (Gillispie et al., 2008). It took three more centuries when another Persian mathematician-philosopher-poet Omar Khayyam corrected the solar calendar called *Jalali* down to 365 days and six hours (Akrami, 2017). Al-Khwarizmi also wrote two texts on the astrolabe, *Kitab 'amal al-asturlab*, translated as *Book on the Construction of the Astrolabe*, and *Kitab 'amal bi'l-asturlab*, translated as *Book on the Operation of the Astrolabe* (Gillispie et al., 2008).

Al-Khwarizmi's geographical text, *Kitab surat al-ardz*, translated as *Book of the Velocity of the Earth*, was written in 833 CE (Stewart, 2017). In this text, al-Khwarizmi provided latitudes and longitudes for 2,402 places and divided the places into six sections which are seas, mountains, islands, rivers, cities, and regions. In each section, all of the places are arranged within certain climata (Gillispie et al., 2008; Ramsden, 2017). The climata are seven sections of the world divided by longitude as seen in ancient Greek works (Gillispie et al., 2008). Al-Khwarizmi discerned information from Ptolemy's geographical work, but al-Khwarizmi's geography on Southwest Asia was more detailed and more accurate than Ptolemy's work (Gillispie et al., 2008; Ramsden, 2017). Ptolemy's length of the Mediterranean Sea was too long, and al-Khwarizmi corrected it (Stewart, 2017). Ptolemy surrounded the Atlantic and Indian oceans by land, portraying them as seas. Al-Khwarizmi in contrast, did not border the oceans by land (Stewart, 2017).

Al-Khwarizmi wrote another compendium that did not survive called *Kitab al-ta'rikh*, translated as the *Chronicle*; it was a record of events that took place in Southwest Asia in accordance to astrology (Gillispie et al., 2008). It has been noted that al-Khwarizmi used the astrological methods of this text to figure out the hour in which the prophet Mohammed was born according to the astrological events of his life (Gillispie et al., 2008). Another text that did not survive is *Kitab al-rukhnama*, translated as *On the Sundial* (Gillispie et al., 2008). Only the title of this text is known, but the subject of the title appears to match al-Khwarizmi's interests (Gillispie et al., 2008).

CONCLUSION

Al-Khwarizmi lived until 850 CE, but his work lived on much longer (Evans, 2014; Ramsden, 2017). Other Persian philosophers who followed al-Khwarizmi and his work were: Farabi Omar Khayyam, Nasir-E-Din Tusi, Avicenna, Razes, Averroes, Biruni, and al-Kindi, to name a few. There are few historical figures who carry as much influence on the development of mathematics, and nearly none on the development of algebra, as al-Khwarizmi had achieved.

The first two authors are faculty in the chemistry and mathematics departments at the college level, respectively, and both integrate the life of al-Khwarizmi into their own instruction. Additionally, the second author teaches the history of mathematics and spends time connecting the development of algebra to al-Khwarizmi. It is recommended that high school and college instructors of algebra incorporate some of the historical knowledge within instruction in order to motivate students, to inspire them to feel its relevance, and to place a human face on the subject; otherwise, many young students do not directly correlate the relevance of mathematics to their own lives.

REFERENCES

Akrami, M., (2017). *The development of Iranian calendar: Historical and astronomical foundations*. Retrieved from https://arxiv.org/ftp/arxiv/papers/1111/1111.4926.pdf

Aksoy, A. G. (2016). Al-Khwarizmı and the Hermeneutic Circle: Reflections on a Trip to Samarkand, *Journal of Humanistic Mathematics*, *6*(2), 114-127. Retrieved from http://web.a.ebscohost.com.rlib.pace.edu/ehost/pdfviewer/pdfviewer?vid=1&sid=9cea40b1-b1fc-4de1-ac6e-cb7eef92df42%40sessionmgr4009

Baharuddin, M., & Wan Abdullah, W. (2014). The Book al-Jabr wa al-Muqābalah: A Research on Its Content, Writing Methodology and Elementary Algebra by al-Khwārizmī. *AFKAR: Journal Of 'Aqidah & Islamic Thought, 15*(1), 135-162. Retrieved from https://ejournal.um.edu.my/index.php/afkar/article/view/5743

Evans, B. R. (2014). *The development of mathematics throughout the centuries: A brief history of mathematics in a cultural context.* Hoboken, NJ: Wiley.

Faruqi, Y. (2015). Role of Muslim intellectuals in the development of scientific thought. *Jurnal Ilmiah Peuradeun, 3*(3), 451-466. Retrieved from http://journal.scadindependent.org/index.php/jipeuradeun/article/view/80

Gillispie, C., Holmes, F. L., Koertge, N., & Gale, T. (2008). *Complete Dictionary of Scientific Biography.* Detroit, MI: Charles Scribner's Sons. Retrieved from http://go.galegroup.com.rlib.pace.edu/ps/retrieve.do?tabID=T003&resultListType=RESULT_LIST&searchResultsType=SingleTab&searchType=BasicSearchForm&currentPosition=1&docId=GALE%7CCX2830902300&docType=Biography&sort=RELEVANCE&contentSegment=&prodId=GVRL&contentSet=GALE%7CCX2830902300&searchId=R2&userGroupName=nysl_me_pace&inPS=true

Gutstein, E., & Peterson, B. (2013). *Rethinking mathematics.* Milwaukee, WI: Rethinking Schools: Retrieved from https://eclass.edc.uoc.gr/modules/document/file.php/PTDE110/MATH%20AND%20SOCIAL%20JUSTICE.pdf#page=34

Hannah, J. (2015, December). Book review [Review of the book *Taming the unknown: A history of algebra from antiquity to the early twentieth century,* by V. J. Katz & K. Hunger Parshall (2014)]. Retrieved from http://www.ircps.org/sites/ircps.org/files/aestimatio/12/2015-03_Hannah.pdf

Ramsden, J. (2017). Snapshots from the history of mathematics: Al-Khwarizmi. *Mathematics in School.* Retrieved from http://web.a.ebscohost.com.rlib.pace.edu/ehost/pdfviewer/pdfviewer?vid=0&sid=52ae26c6-7cd7-43d2-bc1f-aac1dd498d0a%40sessionmgr4008

Rashed, R. (2015). *Classical mathematics from al-Khwarizmi to Descartes.* New York, NY: Routledge. Retrieved from https://books.google.com/books?hl=en&lr=&id=qfRTBAAAQBAJ&oi=fnd&pg=PP1&dq=Al-Khwarizmi&ots=KW3LkdZZvn&sig=OJHN9ridOTr40fodtwdMfSvObtQ#v=onepage&q=Al-Khwarizmi&f=false

Stewart, I. (2017). *Significant figures: the lives and work of great mathematicians.* New York, NY: Basic Books.

Yazdi, H-R. G. (2011). Al-Khwārizmī and annular solar eclipse. *Archive for History of Exact Sciences., 65*(5), 499-517. Retrieved from http://web.a.ebscohost.com.rlib.pace.edu/ehost/pdfviewer/pdfviewer?vid=0&sid=dd573e51-ab80-418e-9a88-477d6584fd05%40sessionmgr4007