Mathematics Understanding of Elementary Pre-Service Teachers: The Analysis of their Procedural-Fluency, Conceptual-Understanding, and Problem-Solving Strategies

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Abstract: *Students in an elementary teacher preparation program at a Hispanic Serving Institution in deep South Texas were asked to solve non-routine, problem-solving activities. They were administered five tasks during one semester, as part of a mathematics methods course. Two experienced raters assessed the student's solutions to the non-routine problem-solving mathematical task using a mathematics understanding rubric that scores the Procedural Fluency (PF), Conceptual Understanding (CU), and Problem Solving/Strategic Competency (PS/SC). The research question was: What are the changes in procedural fluency, conceptual understanding, and problem solving-Strategic Competency in elementary preservice teachers after engaging in a series of non-routine problem-solving tasks? This is an ongoing research project, and preliminary results indicated that the teacher's candidates made improvements in each of the three measurements, demonstrating that they are able to successfully use procedures, and have adequate conceptual knowledge for problem solving.*

Keywords: Problem-Solving, Elementary, Preservice Teachers

Pre-service mathematics teachers'(PSMT) education programs are required to prepare candidate in both content and pedagogy (National Council of Teachers of Mathematics [NCTM], 2017). Mathematics education researchers have contended that it is very important for pre-service elementary teachers develop deep and connected understandings of mathematical ideas (Schoenfeld, 2007). One critical area is problem solving. Mathematics teachers are required to promote reasoning and problem solving with understanding among their students, while engaging them in productive discussions that elicit and enhance their learning acquisition (NCTM, 2014). Lam et al., (2013) suggested that problem solving should be infused in content courses. Thus, implying that problem solving is a critical ability to hone in mathematics. It is relevant that pre-service elementary teachers get involved in activities that foster their ability to engage in and teach problem solving (Olanoff, Kimani, & Masingila, 2009, p. 1299).

The purpose of this study was to examine pre-service elementary teachers' mathematical understandings when regularly provided opportunities to solve non-routine mathematics problems. To this end, we present preliminary results of an ongoing research project that looks to answer the following question: What are the changes in procedural fluency, conceptual understanding, and problem solving-Strategic Competency in elementary preservice teachers after engaging in a series of non-routine problem-solving tasks?

PERSPECTIVES

Typically, teachers have their students solve problems after introducing concepts and procedures that follow examples and prescribed algorithms that require memorization, rather than creativity and strategic competence to solve non-routine problems (National Research Council [NRC], 2000; 2001). The NCTM (2017) considers important that teacher preparation programs should provide opportunities to challenge their mathematical knowledge and ability through the use of high cognitive demand mathematical tasks, involving problem solving and reasoning, and where they are challenged to explore different strategies and solutions paths. It is necessary to engage pre-service elementary teachers in non-routine problem-solving mathematical activities, in which they have the opportunity to "understand [and reason] about problem solving processes" (Koray et al., 2008, p. 1). Hence, elementary teacher education programs should include opportunities to develop conceptual understanding through the use of non-routine problem-solving tasks, which would "significantly influence how and what [they] teach, and how and what their students learn" (Olanoff et al., 2009, p. 1299).

Many teachers, and in particular elementary teachers, have expressed their discomfort when it is necessary to implement problem solving activities with their students (Wilburne, 2006). However, to reduce the anxiety that it may produce, it is necessary that teachers have experiences with solving non-routine problems that help them build their confidence and ability. Mastering the art of problem solving requires extra time, which often is considered a barrier to its implementation, and a disposition to understand the potential of teaching mathematics through problem solving. Similarly, elementary teachers have shown to be uncomfortable teaching mathematics (Wilburne, 2006). This can be attributed to teachers' poor self-efficacy for teaching mathematics anxiety due to lack of knowledge or simply because of their negative attitudes toward mathematics (Bursal & Paznokas 2006).

Teacher-candidates need to experience and face the struggle of solving different types of problems, which develop, not only their mathematical concepts, but also their ability to address student solutions from different perspectives. Problem solving and reasoning were viewed as critical elements of mathematics teaching to the extent that Koellner, Jacobs & Borko, (2007) have incorporated a problem-solving approach into their design of a mathematics teacher professional development program, called the *Problem-Solving Cycle*. This involves teachers engaged in problem solving, video analysis of the implementation, and analyzing student work samples.

Historically, problem solving has been a part of the mathematical curriculum (Schoenfeld, 2011), and it becomes necessary to assess mathematics proficiency (Schoenfeld, 2007). Further, according to the NCTM (2012) problem solving skills are the main expectation of mathematics. Yet, teachers have difficulty implementing non-routine activities that are open-ended and require reasoning and problem-solving strategies. Phonapichat, Wongwanich, & Sujiva, (2014) argued that these may be due to the fact that teachers fail to connect real-life situations with the mathematical content, ask students to memorize algorithms and "keywords" to solve problems, do not deeply explain concepts behind textbooks problems, or they simple do not teach with understanding (p. 3171). All these affect students' knowledge acquisition and comprehension, which is later reflected in poor achievement in mathematics. Therefore, preparing teachers candidates in the mathematical content –in particular in non-routine problems– and pedagogy needed are essential to have a "positive effect on [their] students' learning" (Brabeck, et al, 2014, p. 5), as well as to increase their confident and self-efficacy.

METHODOLOGY

Setting

The study took place during the Spring semester of 2017, and an extension of the data collection is currently ongoing too. The participants were PSMT enrolled in a Hispanic Serving Institution in deep South Texas. The course in which the study took place had an enrollment of 28 PSMT, 100 percent were female. However, not all of them completed each of the tasks (See Table 3). This was due to students being absent when the task was administered. The tasks were administered at the start of class. They were allowed 15 minutes to individually solve the task. This was followed by sharing strategies and solutions with a partner. The instructor monitored the students as they were solving and discussing the tasks. The intent was to identify different solution strategies. Selected students were then asked to present their strategy to the whole group. During this time, connections were made by the instructor to the similarities and differences solution strategies. This allowed for all students to see different approaches that were mathematically efficient to less efficient solution methods.

Data Collection and Analysis

PSMT's solutions to the problem-solving tasks were analyzed during a fourteen-week spring semester of an elementary mathematics methods course. They were asked to individually solve each problem-solving task and then shared their solutions with others in small groups following this student work samples were selected for presentation to the class. The unit of analysis was the individual PSMT's responses to the mathematical tasks. Table 1 presents the name of each task, mathematical content addressed, and the date of implementation.

The solutions of the PSMT varied depending on the type of problem. They used different solution pathways in response to these non-routine problem-solving activities.

The Freckleham People Problem

> The people of Freckleham are interesting creatures. Every Frecklhamer is different from the other and has at least one freckle and one hair, but no more than three freckles and three hairs. Make a list of all the different Frecklehammers.
>
> Make a list of all the different Frecklehammers.
>
> The mayor of Freckleham decided to improve the manners of his townsfolk. He issued an order: When two Frecklehammers meet, the one with the most hairs or freckles will greet the other and say, “I have more ______than you have. A Frecklehammer might say, “I have more freckles than you have,” or a Frecklehammer might say, “I have more hairs than you have.” Or a Frecklehammer might not be able to say anything at all.
>
> At a town meeting of all the Frecklehammers, the greeting, “I have more ____than you have” was heard many times. How many times?

Figure 1 shows three different sample solutions for the FreckleHammer (Treffers & Vonk, 1987) task, in which they were asked to find the number of times a FreckleHammer said “I have more freckles or hair than you have.” PSMT one (Student 1) used a trial-and-error strategy, which resulted in a less clear and less efficient solution. PSMT two (Student 2) represented the data in a table, and later used an ordered pair to obtain a final solution. The PSMT three (Student 3) used a table and a tree diagram to represent the data.

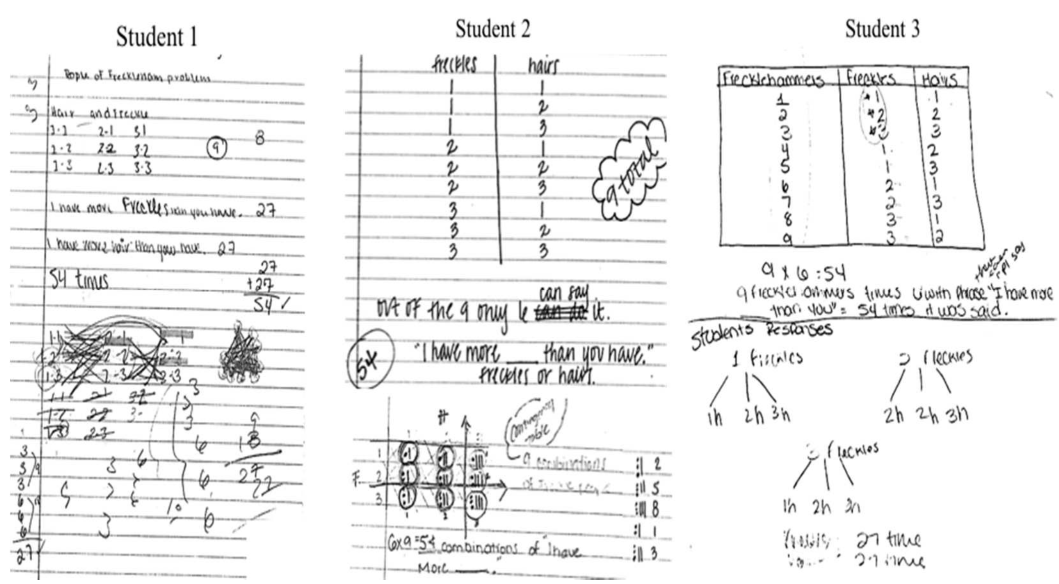

Figure 1. Example of student's solution for the FreckleHammer activity

Table 1
Task Names, Mathematical Content and Date of Administration

Task	Mathematical Content	Date Implemented
Freckle Hammer (Treffers & Vonk, 1987)	Logic	1/25/17
Rectangle Area*	Logic, area, perimeter	2/13/17
Vegetable Garden Fractions (Pelikan, DeJarnette, & Phelps, 2016, p. 332)	Fractions	2/20/17
Picking Pumpkins (www.mathwire.com)	Finding Patterns	3/22/17
Growing Caterpillar (Blanton, 2008)	Algebraic generalizations	4/12/17

* Retrieved from https://sites.google.com/a/arlington.k12.ma.us/ms-tomilson-750-math/

A rubric was designed that considers three mathematics proficiencies identified by The National Research Council [NRC] (2001): (a) Procedural Fluency (PF), (b) Conceptual Understanding (CU), and (c) Problem Solving-Strategic Competency (PS-SC). The reliability of the rubric was tested using Generalizability Theory (see Table 2). The G-coefficient for Conceptual Understanding was 0.86, Problem Solving was 0.88 and Procedural Understanding/Fluency was 0.92 (Telese, 1994). These coefficients indicated high reliability when rating each proficiency. The Mathematics Understandings Rubric (see table 3) was used to rate the solutions (Telese, 1994).

Table 2
Rubric's Generalizability Coefficients

Mathematical Proficiency	**Generalize Coefficient**
Conceptual Understanding	0.86

Procedural Fluency	0.92
Problem-solving/Strategy Competency	0.88

Table 3

Mathematics Understandings Rubric

Performance Level	Procedural Fluency	Conceptual Understanding	Problem Solving/Strategic Competency
0	No response	Lack of evidence to determine knowledge, or no attempt made	No response
1	Incorrect or very limited use of operations, more than one major error or omissions	Wide gaps in concept understanding, major errors made based on lack of conceptual knowledge	Unworkable approach, incorrect or no use of mathematical representations, poor use of estimation, evidence for lack of understanding
2	Some correct use of number operations but a major error or with several minor errors	Some evidence of conceptual understanding, but difficulty in using models, diagrams, and symbols for representing concepts or translating from one mode to another mode. Some evidence of the concept's properties	Appropriate approach, estimation used, implemented a strategy, possibly reasoned decision making, solution with observations
3	Appropriate use of number operations with possible slips or omissions, but without significant errors	Good evidence of conceptual knowledge. No major misconceptions; responses contain accurate use of models, diagrams, and symbols with evidence of translation from one mode to the other. Recognition of the meaning and interpretation of concepts. Some evidence of using concepts to verify or explain procedures	Workable approach, used estimation effectively, mathematical representation used appropriately, reasoned decision-making inferred, judge reasonableness of solution
4	Extended use of number operations	Clear understanding of concepts and associated procedures.	Efficient/sophisticated approach, estimation

without errors in calculations; appropriate use of models or representations	Effective use of models, diagrams, and symbols with broad translation from one mode to another. Recognition of the meaning and interpretation of concepts to explain or verify procedures or conclusions	used effectively, extensive use of mathematical representations, explicit reasoned decision-making Solutions with connections, synthesis or abstraction

RESULTS

The students became more confident in themselves than at the beginning of the semester. They hesitant at the beginning to engage with the task and often asked, “Where do I start?” The sharing of solutions assisted those who were less able to develop a strategy. The less creative students became more creative in their problem-solving approaches. Over the course of the semester, students engaged in problem solving; as a result, their confidence for providing similar opportunities to their future students improved, for example one student noted, “I learned about how a simple math problem can be solved in many different ways, and how we can help our students in the classroom when that happens." The student clearly indicates self-confidence in that when her future students are problem solving, she will know how to differentiate and compare strategies.

Inter-rater reliability was performed using percent agreement, where a difference of one was considered agreement. The raters conducted a calibration session prior to scoring. The percent agreement for each mathematics proficiency ratings had a low of 88 percent on the Vegetable Garden task’s conceptual understanding ratings to 100 percent on other tasks and proficiencies. Mean ratings were calculated for each of the three Mathematics Proficiencies from two raters. Table 4 presents the overall means and standard deviations for each task’s ratings for mathematics understandings.

Table 4

Tasks’ Means and Standard Deviations for Mathematics Understandings

Task Administered	Procedural Fluency	Conceptual Understanding	Problem Solving Strategic Comp.
Freckle Hammer	3.23	3.37	3.25
n = 26	(0.59)	(0.37)	(0.67)
Rectangle Area	3.37	2.94	2.94
n = 23	(0.53)	(0.73)	(0.79)
Vegetable Garden	3.5	3.72	3.52
n =26	(0.50)	(0.40)	(0.59)

Pumpkin Patch	3.21	3.02	2.78
n = 26	(0.45)	(0.48)	(0.51)
Growing Caterpillar	3.80	3.83	3.57
n = 15	(0.32)	(0.52)	(0.70)
Total	3.40	3.34	3.34
n = 116	(0.53)	(0.67)	(0.71)

Note: The number in parenthesis is the standard deviation.

When a MANOVA was conducted with the three dependent variables (PF, CU, and PS-CS) and the five tasks as the independent variables, the dependent variables were found to be highly correlated. A decision was made to conduct separate ANOVAs for each of the dependent variables because of the high correlation. The Levene's Test of Equality of Error Variances was found not to be statistically significant for Procedural Fluency and Problem Solving/Strategic Competency; hence, the Tukey Post Hoc test was conducted on the tasks. The Tamhane's Post Hoc test was used for Conceptual Understanding because equal error variance was not found. Statistically significant result was found for Procedural Fluency with $F(4, 111) = 4.80$, $p < 0.001$, $\eta^2_{partial} = 0.147$, for Conceptual Understanding with $F(4, 111) = 3.43$, $p < 0.001$, $\eta^2_{partial} = 0.262$, and for Problem Solving/Strategic Competency with $F(4, 111) = 6.56$, $p < 0.001$, $\eta^2_{partial} = 0.191$. The students' Procedural Fluency performance varied by task where they performed best on the 'Growing Caterpillar' task when compared to the 'Freckle Hammer' and 'Picking Pumpkins' tasks. Similar result were found for both Conceptual Understanding and Problem Solving/Strategic Competency ratings on the tasks. They performed best on the 'Growing Caterpillar' task when compared to the 'Rectangle' and 'Picking Pumpkins' tasks.

CONCLUSION

The PSMT displayed a variety of performance levels in relation to the task. The non-routine tasks were administered to provide practice at solving problems and to encourage modeling in order to demonstrate conceptual understanding. Also, meaningful mathematics discussions occurred to foster mathematics understanding. A limitation of the study was the small number of solutions to the 'Growing Caterpillar' task. The high performance may have been due to the smaller sample. Generally, the PSMTs scored higher on the 'Growing Caterpillar' where they had to express a pattern in algebraic terms, and the 'Vegetable Garden' task where they had to use knowledge of fractions. The PSMTs demonstrated a moderate level of Procedural Fluency. They could use appropriate number operations without significant errors, trending toward an extended use of number operations using appropriate models.

Regarding Conceptual Understanding, the 'Rectangle Area' task proved more challenging perhaps due the nature of the task being related to fractions, which had very little guidance embedded in the task to hint at a solution. They had to arrange areas and perimeters of rectangles when given particular conditions and may have held weak content knowledge associated with area and perimeter. Generally, PSMT' rating for Conceptual Understanding revealed that they held no major misconceptions, used accurate models, diagrams, and symbols while being able to move from one representation to another and tended to use concepts to explain or verify procedures. Taking all the tasks together, their Problem Solving/Strategic Competency indicated that they used workable approaches, used estimation effectively, mathematics representations were used appropriately and demonstrated reasoned decision making while judging the reasonableness of the solution. However, the overall mean for Problem Solving/Strategic Competency indicates that problem solving ability could be improved to the extent that they should be able to use more efficient or sophisticated approaches to solving problems. Although Conceptual Understanding was rated consistently, the error variance indicated that the preservice teachers had a wide range of conceptual understanding related to the tasks.

Consequently, the preservice elementary mathematics teachers demonstrated that they are able to successfully use procedures, and they have adequate conceptual knowledge for problem solving. Their problem-solving's capabilities need sharpening to reach the efficient and sophisticated approach to problem solving. This would come about through enhanced content knowledge. As Olanoff et al., (2009) noted, teaching mathematics through problem solving requires the teacher to have deep understanding of the mathematics and need to anticipate different approaches. Improvements in the way the problems are chosen and used are necessary too. The non-routine tasks should be selected with a common thread, whether content, or for encouraging the development of generalizations. Similar to the results of Olanoff et al's study, it appeared that the process supported the teacher candidates' problem-solving ability.

Finally, to specifically answer the research question, it has been shown that overall the PSMTs ability in all three dimensions improved throughout course of the semester. However, there was inconsistent performance on the tasks from the first implementation to the last task in all three dimensions. This means that on some tasks the PSMTs performed lower than the task previously administrated. The result could be due to the nature of the tasks; some tasks are less challenging than others; thus affecting performance. This study will continue to examine PSMTs performance in all the dimensions of the rubric, and their beliefs and perceptions toward problem solving.

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APPENDIX A: Problem Solving Task

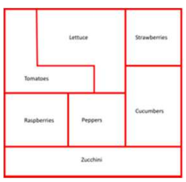
1. Freckle Hammer (Treffers & Vonk, 1987).

The people of Freckleham are interesting creatures. Every Frecklehammer is different from the other and has at least one freckle and one hair but no more than three freckles and three hairs. Make a list of all of the different Frecklehammers. The mayor of Freckleham decided to improve the manners of his townsfolk. He issued an order: When two Frecklehammers meet, the one with the most hairs or freckles will greet the other and say, "I have more __________ than you have." A Frecklehammer might say, "I have more freckles than you have," or a Frecklehammer might say, "I have more hairs than you have." Or a Frecklehammer might not be able to say anything at all. At a town meeting of all of the Frecklehammers, the greeting "I have more _________ than you have" was heard many times. How many times?

2. Rectangle Area.

Five rectangles are arranged from the least to the greatest area and named A, B, C, D, and E in order of increasing area. All dimensions are whole numbers, and no two rectangles have the same area. Determine the dimensions of all five rectangles using the following clues: The median area is 15 units^2. Rectangles B and D are squares. Rectangles C and D have the same perimeter. Rectangles A, B, and C have the same length. Rectangles D and E have the same length. Rectangles C and E have the same width.

3. Vegetable Garden Fractions (Pelikan, DeJarnette, & Phelps, 2016, p. 332).



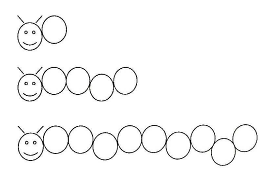
In the plot, what fraction of the garden is composed of Lettuce? What fraction of the garden do zucchini and cucumbers together use?

4. Picking Pumpkins (www.mathwire.com).

Allie was picking pumpkins for her school. She picked one pumpkin on the first day. She picked two pumpkins on each of the next two days. Allie picked three pumpkins for three days. Next she picked four pumpkins a day for four days and so on. If Allie continues this pumpkin picking schedule, on what day will she first pick 6 pumpkins? How many pumpkins will she have picked altogether for her school when she completes that first 6th day?

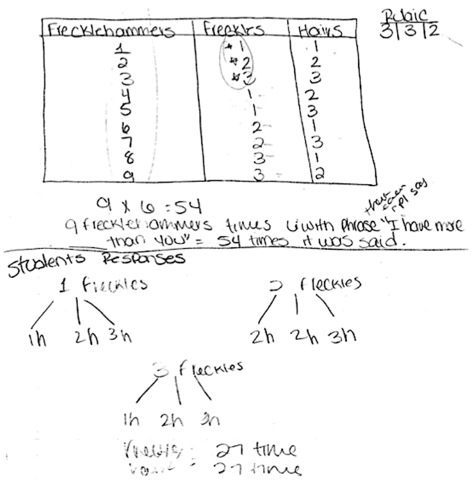
5. The Growing Caterpillar (Blanton, 2008).

A caterpillar grows according to the chart. If this continues, how long will the caterpillar be on Day 4? Day 5? Day 100? Day x? (Measure length by the number of circle body parts.)

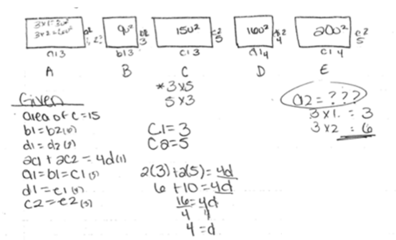


APPENDIX B: Student work sample

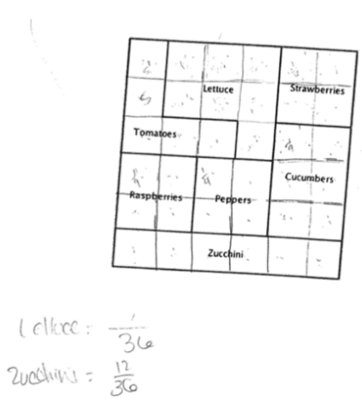
1. Freckle Hammer.



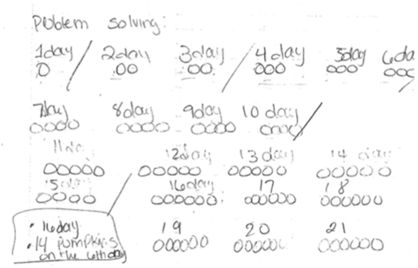
2. Rectangle Area.



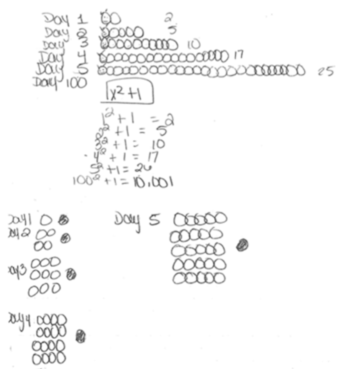
3. Vegetable Garden Fractions.



4. Picking Pumpkins



5. Growing Caterpillar



Book Review of "THE OUTER LIMITS OF REASON" by Noson Yanofsky

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Even if we do not admit it, or not aware of it, our daily existence is determined by the never-ending flows among conscious and subconscious, passive and active, yin and yang or what other names one may call them. In this constant flow of well-posed and ill-posed inquiries we often experience confusions, paradoxes, dilemmas and limitations, but do not pay attention to them for