The "act of creation" of Koestler & theories of learning in math education research.

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Abstract: *The presentation explores the relationships between Koestler's theory of the Act of Creation and learning theories used in mathematics education, in the context of the call by Prabhu & Czarnocha (2014) To provide a suitable background Koestler's understanding of creativity as a transformation from habit to originality is integrated with Vygotsky's understanding of concept development within the common learning environment of problem solving. The presentation provides an instructional example of bridging Koestler's insistence on the individual, "un-tutored" nature of bisociation with Vygotsky's concept of ZPD within properly structured social learning environment. In this instructional sequence Koestler's bisociation is seen as a type of reflective abstraction that provides a mechanism of concept development to transition the solver through the Piaget & Garcia Triad (1989).*

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Bisociation as the definition of creativity in mathematics education

Prabhu & Czarnocha (2014) call for the adoption of Koestler bisociation as the definition of creativity in mathematics education in response to the conviction of the leaders of the field that there is no single, authoritative perspective or definition of creativity (Mann, 2005; Sriraman, 2005; Leikin, 2009, Kattou et al, 2011). The theory of the Act of Creation (1964) by Arthur Koestler, defines "bisociation" as the "creative leap [of insight], which connects the previously unconnected frames of reference and makes us experience reality at several planes at once" (p.45) an Aha moment.

In following words Koestler clarifies the experience [of] reality at several planes at once: "The pattern… is, the perceiving into situation or Idea, L, in two self-consistent but habitually incompatible frames of reference, M1 and M2. The event L, in which the two intersect, is made to vibrate simultaneously on two different wavelengths, as it were. While this unusual situation lasts, L is not merely linked to one associative context, but bisociated with two" (p.35).

Prabhu and Czarnocha (2014) state two reasons for their call:

1. Democratization of the research into mathematical creativity, which, in their opinion, is presently biased towards giftedness.

2. Separation of the definition of creativity from the concept of fluency since recent reports indicate that emphasis on fluency might negatively impact creativity itself, resulting in the lowering of originality as reported by Leikin, (2009).

An important component of Koestler theory, which makes it particularly useful for democratization in learning mathematics by all students is the relationship between creativity, originality and habit, in often quoted words of Koestler, "*Creativity is the defeat of habit by originality*." Stated in more positive and precise terms, bisociation can help transform habits that hamper learning of mathematics into original creative insights of our students (Prabhu & Czarnocha, 2014). It's helpful to know that our colleagues in Computer creativity branch of informatics have identified already three types of bisociation: bisociation by a common concept, bisociation within a concept map, and bisociation by the sub-structural similarity. Thus, the groundwork for a teaching-research investigation of bisociativity in our classrooms has been prepared (Berthold, 2012).

To ease the process of adoption of bisociativity into the mathematics education discourse we present here elements of a very rich interaction between Koestler's theory and different theories of learning utilized in mathematics education: Approaches of schema development based upon Piaget's work, and Vygotsky theory of concept development. This short expose will provide a unifying impetus for these diverse theories of learning based upon the relationship of bisociation with the theme of consciousness during reflection and abstraction within problem solving.

Koestler believed that creativity, specifically bisociation took place within a problem solving environment as a result of intense reflection and conscious deliberation. Vygotksy would affirm that concept development takes within a problem solving environment as a result of "reflective consciousness." Educators who follow the work of Piaget understand concept development and schema formation as the result of "reflective abstraction" within a problem solving environment.

Koestler and Piaget

Bisociation is the process, which can take place in the context of the construct called dialectical Triad of Piaget and Garcia formulated in the profound but little known book, Psychogenesis and the History of Science (Piaget & Garcia, 1989). This Triad of stages for concept formation was constructed on the basis of the comparative analysis of the development of physical and mathematical ideas in history of science on one hand, and the psychogenetic development of these concepts in a child, on the other. It is defined as the process of concept development through intra-operational, inter-operational and trans-operational stages.

> "Intra-operational stages are characterized by intra-operational relations, which manifest themselves in forms that can be isolated"
>
> inter-operational stage is "characterized by correspondences and transformations among the forms that can be isolated at previous levels…"
>
> "The trans-operational stages are characterized by the evolution of structures whose internal relationships correspond to inter - operational transformations." (pp.173-184)

Despite its somewhat opaque language, the described process is not very complicated and its components can be traced out in the simple Paradigmatic Example below. The development of a concept, according to Piaget & Garcia (1989) starts with its isolated manifestations of the intra-stage. The comparison of isolated cases, search for similarities and differences based on the quality of discernment leads to the formulation of relationships between them. Formulations of relationships carries the inquirer into the inter – stage. Bisociations can be observed (1) during the formulation of these relationships and (2) during the transition from inter- to trans- stage as the moment of understanding joining all relationships developed in the previous stage into one structure. This leads us to conjecture that bisociation is a type of an instantaneous reflective abstraction – a mechanism of thinking formulated by Piaget involving the coordination of concepts, processes and entire structures to synthesize and develop new matrices of thought or schema. (Dubinsky,1991)

Paradigmatic Example

Consider the square root domain question in the classroom of a teacher researcher, demonstrating the interaction between student and instructor, in which the latter is able to get the student engaged in the thinking process and hence to facilitate student creativity. The domain of the function $\sqrt{X+3}$ is at the centre of the dialog.

Note that it is the spontaneous responses of the student from which the teacher-researcher creates/determines the next set of questions, thus balancing two frames of reference, his/her own mathematical knowledge and the direction taken by the student. Similarly the student has her own train of thought and prompted by the teacher-researcher's questions, she must now balance two frames of reference to determine her next response.

Note that Koestler would define a matrix as, "any pattern of behaviour governed by a code of fixed rules," (p.38) and the first line 1 the limitations of the students' internal matrix or problem representation is demonstrated. The teacher adjusting to the students' limited matrix provides two examples (line 6, 8) that provide a "perturbation" or catalyst for cognitive conflict and change, "...perturbation is one of the conditions that set the stage for cognitive change." (Von Glasersfeld, p.127).

In lines 6-9 the student reflects upon the results of the solution activity, through comparison of the results (records) they abstract a pattern, "the learners' mental comparisons of the records allow for recognition of patterns" (Simon et al, 2004). Thus, in this example the synthesis of the student's matrix for substitution and evaluation of algebraic expressions with their limited matrix of what constitutes an appropriate domain for radical functions (bisociation) resulted in the cognitive growth demonstrated in line 10.

In line 11,12 the perturbation brought about by the teacher's question lead to the student entering into the second stage of the Piaget & Garcia Triad as they understood that the matrix or domain concept modifications for radical functions learned previously extended to this example as well. They were then able to reflect upon this pattern and abstract a general structural relationship in line 14 characteristic of the third stage of the Triad.

The problem starts with the function *f(X)* = $\sqrt{X+3}$

0 The teacher asked the students during the review: "Can all real values of be used for the domain of the function $\sqrt{X+3}$?"

1 Student: "No, negative X's cannot be used." (The student habitually confuses the general rule which states that for the function $\sqrt{X}$ only positive-valued can be used as the domain of definition, with the particular application of this rule to $\sqrt{X+3}$.)

2 Teacher: "How about $X=-5$?"

3 Student: "No good."

4 Teacher: "How about $X=-4$?"

5 Student: "No good either."

6 Teacher: "How about $X=-3$?"

7 Student, after a minute of thought: "It works here."

8 Teacher: "How about $X=-2$?"

9 Student: "It works here too."

A moment later

10 Student adds: "Those X's which are smaller than -3 can't be used here." (***Elimination of the habit through original creative generalization.***)

11 Teacher: "How about *g(X)* = $\sqrt{X-1}$?"

12 Student, after a minute of thought: "Smaller than 1 can't be used."

13 Teacher: "In that case, how about h(x) = $\sqrt{X-a}$?"

14 Student: "Smaller than "a" can't be used." (***Second creative generalization***)

The creativity of the teacher manifests itself in the scaffolding which led the student to the cognitive conflict between the two frames of reference. In the first case, the data driven results obtained through the matrix-process of substitution was synthesized with their limited matrix of the possible domain of a radical function, this bisociation and the resulting abstraction lead to a more complete understanding of the possible domain for specific functions. This represents a transition for the first to second stage of the Piaget and Garcia Triad. A continuation of this questioning process leads to further creative moments of understanding, in which the student was able to synthesize their understanding of the domain for two separate special cases of radical functions, this bisociation and the resulting abstraction into the structural understanding (line 14), suggests the student had crossed the ZPD from the second to third stage of the Triad.

We propose the method of scaffolding presented above as the teaching-research inspired guided discovery method of creating a bridge between Koestler's insistence on the "un-tutored' nature of bisociation with Vygotsky emphasis on the socially structured nature of learning environment.

Concept Formation within Problem Solving: Vygotsky & Koestler

Vygotsky, like Koestler treats learning, in particular concept development within the framework of problem solving, "concept formation…is an aim directed process…for the process to begin, a problem must arise that cannot be solved otherwise than through the formation of a new concept (Vygotsky, 1997, p.100). The problem solving environment that leads to concept formation is described by Vygotsky using the work of the Gestalt psychologist Max Wertheimer. In this framework concept formation results from productive thought which, "is based upon insight i.e. instant transformation of the field of thought. The problem X that is a subject of our thought must be transformed from the structure A within which it has been first apprehended to the entirely different structure B, one must transcend the given structural bonds and this […] requires shifting to a plane of greater generality, to a concept subsuming and governing both A and B" (Vygotsky,1997, p.205).

The distinction that Koestler, and Vygotksy make between routine and insight problems solving is the degree of consciousness or awareness required. For Vygotsky and followers of Piaget cognitive change involves reflection and the abstraction of patterns for actions and their effects. Thus, cognitive change (accommodation) requires first cognitive conflict as an individual applies their existing matrices to a new problem situation, i.e. it is triggered whenever "an individual try to understand, organize or make sense of a new situation" (Steele & Johanning, 2004) and second, the situation requires synthesis of a new concept with this matrix. As this synthesis requires judgments of comparison and differentiation on how the new concept relates to this matrix, this process breaks the routine automatic thinking of proceduralization and can lead to generalization of the matrix and concept formation. “But in original discoveries, no single pre-fabricated matrix is adequate to bridge the gap. There may be some similarities with past situations, but these may be more misleading than helpful […] here the only salvation lies in hitting upon an auxiliary matrix in a previously unrelated field one may call this […] reasoning from a parallel case.” (Koestler, p.201)

Vygotsky would allow for concept formation or cognitive change with the help of instructor’s (or a peer group’s) facilitation of understanding while Koestler would make a subjective distinction between such tutored instruction and untutored learning. Koestler would allow the synthesis of concepts related to previously unrelated matrix-structures to be considered as ‘creative’ only when this synthesis occurred organically, without structural guidance i.e. untutored learning. “Familiar situations can be dealt with by habitual methods; they can be recognized at a glance as analogous in some essential aspect to past experiences which provides a ready-made rule to apply to apply to it.” In contrast, problems that are unfamiliar to the solver provide the opportunity creativity

> “the more new features as task contains the more difficult it will be to find the relevant analogy, and thereby the appropriate code to apply to it…one of the basic mechanisms of the Eureka process is the discovery of a hidden analogy; but hiddennes is again a matter of degrees. How hidden is a hidden analogy” (Koestler, p. 653).

An example that involves a synthesis of matrices in elementary algebra is the use of fractional exponents to represent roots. In this situation, the matrix or structure associated with fraction notation is used in an entirely new situation that has little to do with the concepts of part-whole, ratio, quotient or even the fraction as numerical value on the number line (measure) concepts that typically underlie fraction notation. Thus, both Koestler and Vygotsky would view a tutored social situation presenting the topic of fractional exponents for the first time as leading to concept formation however, Koestler as a constructivist would insist that originality and creativity would only come when the situation was untutored i.e. learning based upon student discovery.

In contrast to Koestler, Vygotsky has a strong focus on the role of education in concept-schema development, “school instruction induces the generalizing kind of perception and thus plays a decisive role in making the child conscious of his own mental processes. Scientific concepts, with their hierarchical system of interrelationships, seem to be the medium within which awareness and mastery first develop” (Vygotsky, p.171). This consciousness as noted by Vygotsky arises with adolescence, during the middle school years of education when students are required to and many struggles to learn fractions, proportional and algebraic reasoning.

The role of education in Vygotsky’s framework is to present problems on the upper structural level of the individual’s ZPD and then provide them with guidance in reaching that goal. To the constructivist like Koestler who believes that learning only has meaning when the individual reinvents or relives the process of discovery, Vygotsky would counter that guided learning even when students follow instruction without grasping the essential processes is valuable within the individual's ZPD. "To imitate, it is necessary to possess the means of stepping from something one knows to something new."

Our Teaching Research approach to the Koestler/Vygotsky contradiction is to find a compromise between the two positions. As the previous section Paradigmatic Example suggests, the Teaching Research methodology incorporates successful facilitation of bisociation into student-teacher interaction during which student's ZPD of the concept is traversed from the initial to the final stages of the Piaget & Garcia Triad. Thus, it is possible to facilitate student creativity and grasping conceptual connections in the context of guided discovery method within student ZPD.

In this situation, the synthesis of student's matrices seen as their internal representation of the domain of a radical function with their reflection and abstraction required to resolve the cognitive conflict brought about by the teacher's data driven examples.

The reflection required to accommodate this new information for the specific radical function analysed and the resulting conceptual awareness of the effect the numerical values under the radical had on the domain of the function were then transferred to another similar function and their understanding of these situations was synthesized to form an abstract new structural understanding in the manner Vygotsky describes concept development.

Conclusion

The aim of this report has been to show the mutual interactions between the theory of Act of Creation and learning theories in mathematics education of Piaget, and Vygotsky, in explaining concept and schema development within a problem solving environment. The integration of their work focuses on bisociation as a mechanism of conscious reflection to promote concept development in Vygotsky's theory and as a type of reflection and abstraction i.e. the mechanism of growth through the Piaget & Garcia Triad for schema development. As such this article provides impetus for further research to integrate creativity into mathematics education and comparative analysis of the mechanisms of learning based upon reflection and abstraction with bisociation.

One of the central issues to be resolved in the process of integrating Koestler and Vygotsky approaches is in the interaction of social environment of instruction with bisociation discussed above. We have provided an example a teaching-research method for such scaffolding the teacher-student interaction, which leaves the student with sufficient amount of intellectual freedom to facilitate a creative leap of insight, the bisociation. This example suggests a more general question: how to construct such instructional dialogues, which provides for an optimal amount of student intellectual freedom to promote the occurrence of this creative leap of insight?

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