Brief creative assignments in undergraduate mathematics courses: Calculus 3 and Linear Algebra

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Abstract: *This presentation contains brief creative assignments about mathematical convention and notation, real life examples, mathematical examples, mathematical terminology, and inquiries about mathematicians and their theorems. The theory of creativity that supports the concept of creative assignments in the classroom is supported by Graham Wallas (1926) description of stages of thought and by other well established psychological theories. Those assignments are serving as pilots for longer creative assignments and research projects aimed for improving students' satisfaction and performance in Calculus 3 and Linear Algebra. In addition, the creative assignments may improve students' graduation and retention of the underserved and underrepresented students in STEM. The classes where the assignments took place were taught at a large urban community college in small classrooms of ten to twenty-five students. The assessment is based on students' reactions in the classroom and on students' responses to a survey conducted at the end of the semester.*

Keywords: Calculus 3, Linear Algebra, creativity, creative assignments

Introduction

Creativity is probably the last concept that comes to mind when thinking about teaching or learning undergraduate courses such as Calculus 3 or Linear Algebra. This is a sad statement that undergraduate curriculum does not nurture knowledge by emphasizing creative assignments, but simply provides the material to be mastered. Unfortunately, without appropriate spirit of inquiry and without applicable context, the knowledge learned in classroom disappears within short time: often not building enough foundations even for handling comprehensive final exam questions and not building knowledge throughout multiple courses and semesters.

But it does not have to be that way. If instead of following the outline of the lecture rigorously, we introduce brief creative assignments in the classroom to bring educational enjoyment equally to the teacher and to the students. The goal of this enjoyment is to improve class atmosphere and student satisfaction but at the same time keep the quality of students' work, high standards of course outcomes, and integrity of students' knowledge. This article contains a few examples of such assignments to serve as inspiration for future course improvements.

Traditional Theory of Creativity

What is creativity? We are the most creative when we are children yet there is usually no scientific research involved. What is important is that being creative and playful is probably the most satisfying among all attitudes which somehow gets lost when we grow up.

The formal definition of creativity, or rather the lack of it, was presented in detail by Runco and Jaeger[1]. From the perspective of mathematical classroom, creativity happens when new things are created, or more precisely, creativity is a mental process that transforms old ideas, rules, patterns, relationships, etc., into relevant new ideas, forms, methods, interpretations, etc. For the purpose of this presentation, a brief creative assignment in is defined as a 15-minute question directed to students that reflects on the current class material and contains an element of students reaching towards subconscious and reflections on the outcomes.

The deep reasons for the need of creative assignments in the classroom was discussed by Savic[6]. Creative assignments may touch various aspects of the course. The main purpose of assignments presented here is to expose students to the joy of tapping into the subconscious while consciously following the lecture.

The benefits of such assignments include bringing joy, interest and students' involvement to the course. This helps observing students' attitudes towards creativity and unfettered thinking. At the same time, it provides assessment of actual connections among mathematical topics that students have built in their minds.

Stages of creativity

In his book "The Art of Thought" Graham Wallas[7] underlines the necessity of observing the thinking process while learning the actual subject (page 28):

> But behind the use of thinkers of rules and materials drawn from the sciences there has always been, since the dawn of civilization, an unformulated "mystery" of thought which has been "explained" by no science, and has been independently discovered, lost, and rediscovered, by successive creative thinkers. [Who] learnt from each other something which was neither logic nor accumulated knowledge.

This "mystery" from the quotation above describes connections between subconscious and conscious mental formations generated during the process of learning, and outside of it. Wallas indicates four stages of though: Preparation stage, Incubation stage, Illumination stage, Verification stage. The Preparation stage occurs when a student is learning about the problem. The Incubation stage happens when the subconscious mind is working on the inquiry. The most mysterious is the Illumination stage where the subconscious mind reveals its findings to the conscious mind and brings it to the attention of the student. The Verification stage occurs when the conscious mind analyzes the provided results with the conscious mind and verifies the results.

One observes that the process of problem-solving consists of multiple repetitions of those four stages. Indeed, anybody who ever solved a basic problem knows that the first solution provided by the subconscious mind may be incorrect or incomplete and the entire process becomes more of a cycle, where collecting information and incubating the answers, then illuminating them, and eventually revealing to the conscious mind, follow one another multiple times. An example of such a sequence encouraged by a mentor is, for example, revealed in "Minutes from a math meeting with an undergraduate student."[3] The focused state that involves those stages was already recognized and described as Creative Flow by Csikszentmihalyi.[1]

When the cycle does not work?

The theory brought by Wallas[7] and discussed later by multiple authors of psychology and philosophy, is not entirely aligned with the modern theories of the Inquiry Based Learning. But indeed, it simply assumes that the first stage "Preparation" is experienced effortlessly and naturally. However, without initial intentions for learning, or questions in mind, or desires for discoveries, this first stage would not take place. That is why the classrooms may be filled with students having no curiosity in their minds and who are not ready for the preparation stage. It is clear that without preparation stage the other stages will not take place.

If the cycle functions properly, it brings a sequence of satisfying events of inquiry, preparation, incubation, illumination, and verification followed by more inquiry, etc. However, if the cycle does not function, where shall we, as teachers and mentors, address the difficulty? Before responding to this question, one needs to observe that a deep inquiry is a subconscious stage, just like the stages of incubation and illumination. Thus, the process of creation is an internal discussion between conscious and subconscious states. When the cycle does not work, one would like to find methods of addressing the difficulty.

Wallas mentions mental tension between conscious and subconscious (page 34) and points out that this phenomenon was already discussed by H. Poincaré in *Science and Method*[4], who calls the force *'sensibilite'* translated as *'feeling.'* Is then this feeling that needs to be addressed in the classroom when students experience difficulties with solving problems or paying attention? Wallas, however, gives little hope for investigating methods of successful improvement of the process of creation in terms of doing more or working harder. He even indicates that being more conscientious of the process of thinking may disturb the thoughts and produce no results whatsoever.

Sample Assignments

This section contains sample assignments.

Justifying mathematical convention in Calculus 3 class

Often Calculus 3 students have difficulties understanding why the 3-dimensional space is represented with the yz-plane being drawn as the plane of the board or paper and the x-axis coming out of the board with the positive direction pointing towards the observer. To address this issue, I introduced a brief creative assignment, where each student made few sketches with suggestions how the three-dimensional space could be drawn. Then students presented their work on the board and plotted the point $(1,2,3)$. When the pictures were compared, it become evident that prior to understand each other's drawings at the first glance, we have to stay within one convention. At the same time students recognized positively and negatively oriented 3-dimensional spaces. Group work is very useful for this particular activity since students can look at each other's drawings to compare and verify whether two pictures represent exactly the same 3-dimensional space or its mirror image. The activity took about 15 minutes of class time. Figure 1 contains sample sketches.

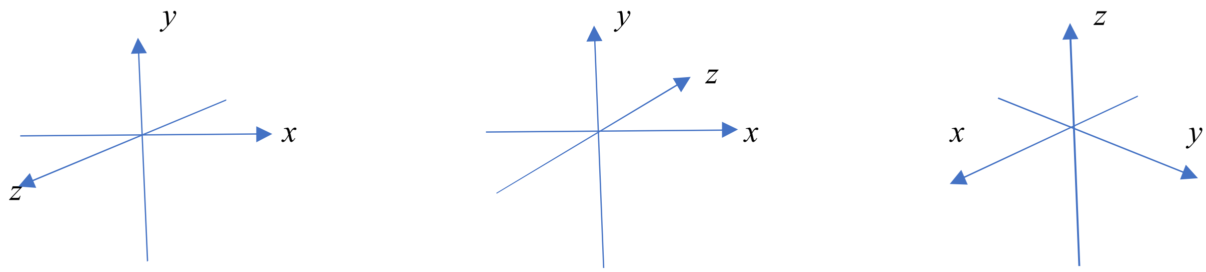

Figure 1. Sample students' work: positively and negatively oriented 3-dimensional spaces.

Justifying mathematical terminology in Linear Algebra

Usually terminology of math concepts reflects on their connections with life examples, name of a person honored or simply another idea. The general motivation for providing appropriate math terminology is to create a connection of the concept and vocabulary. But sometimes those ideas may be so particular and exquisite that modern education does not furnish students with suitable knowledge necessary to make this connection. In such a case, math terminology may become a true burden unless it is used as an excuse to expose students to new topics. The **echelon** form of a matrix is an example of perfectly placed similarity between the appearance of a reduced matrix and military formations known already in the Roman times. A brief creative assignment involves students searching on their cellphones for answers of few questions about the right echelon form, the left echelon form and their relationship to matrices. Students are asked to prepare a brief description what "echelon" means and should visualize their presentation with a picture or a photo as presented on Figure 2. The activity can be performed in class or assigned as homework.

Figure 2: Presentation of the echelon formation.[2]

Providing real life examples in Calculus 3 class

Frequently teachers of mathematics receive requests from students to clearly state the life applications of presented methods, concepts and theorems. Classroom methodology of responding to such an inquiry, involves asking students what examples they can provide based on their own experience. When introducing quadric surfaces and parametric curves, my students spend time in the classroom finding familiar shapes that are similar to those presented on the screen. Usually the examples are divided into those that appear in nature and those that are man-made. The most frequently analyzed curves are the helix and the spiral. When looking at them students recall car shocks, springs inside pens or side-spiral college notebooks. Often students while analyzing the physical models of the spirals claim that there is only one spiral, it just needs to "turned around." But after rotating the model several times, they eventually arrive to the illuminating conclusion that the two spirals are not identical. Figure 3 presents examples of different spirals.

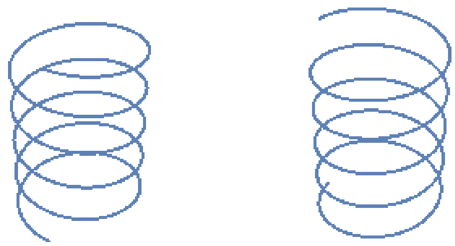

Figure 3: Samples of right and left sided spirals.

Among the surfaces the saddle is the most difficult to sketch but provides the most satisfying examples in a form of Pringles potato chips. When sketching a shape of a paraboloid $z = 8 - x^2 - y^2$ trimmed to the region $-1 \leq x \leq 1, 0 \leq y \leq 2$, a student realized that the graph reminds him of a segment of a parachute.

Similarly, when introducing triple integrals in cylindrical and spherical coordinates, we sketched a picture of the solid that lies outside of the cylinder $x^2 + y^2 = 1$ and inside the sphere $x^2 + y^2 + z^2 = 4$. Students were asked what real-life shapes come to their mind when looking at this solid. Students pointed out that some beads are shaped just like the solid. Figure 4 presents the surfaces and the solid considered in class.

Figure 4: Pringles surface, parachute-like surface, and bead-like solid.

Providing mathematical examples of typical regions in Calculus 3 class

In the past, before I made a commitment of introducing creative assignments in my classes, I prepared a neatly organized sequence of examples to introduce students to a variety of typical regions. Then I presented those examples in class and asked students to describe boundaries of the regions. When my perspective changed, the definition was followed by one example and then I asked students to produce their own examples. Students worked on the assignment for about 15 minutes and after receiving my feedback and encouragement, presented their work on the board. While observing the class I realized that some students immediately began producing examples but some had no trust in their mind and simply did not know what region to think of. I directed those students to the recent worksheet with an example of a domain of a function. As I observed, most students who began working promptly, started with a region in their mind and then tried to obtain functions that match the shape of their example. After reviewing lots of students' examples I realized that the variety, as presented in Figure 5 was just as good as the lecture prepared by me. At the end of the activity students sketched their regions on the board and provided their solutions. Then I analyzed all solutions and corrected the mistakes on the board. Students seemed to be more interested and involved in the topic than usual.

Figure 5: Sample typical regions provided by students: rhombus, trapezoid, semicircle, region between two circles.

Generating inquiries about mathematicians and their theorems mentioned in the textbook of Calculus 3 class: Fubini, Stokes, Green.

Doubting that a plain biography of a mathematician is placed properly in calculus class for engineers, I introduced creative assignments for students to become familiar with the historical background and personal life of few mathematicians mentioned in the textbooks. Sometimes the discovery when and where a particular mathematician lived brings additional questions. Students often realize that some theorems and observations are relatively young just like Fubini's Theorem from 1907, some other theorems are "slightly" older just like Green's Theorem from 1828, comparing to the concept of the tangent line already described by Euclid in 300 BC. The most surprising discussions are often created by the simplest and most innocent questions like "Where is Glasgow?" Since students represent highly heterogeneous group with various cultural and geographical experience, they often amaze each other with their questions and findings.

Observations and Challenges

While observing students during their creative work I realized that some of them did not get involved with the question and did experience the illumination stage. However, I would still consider the assignment successful from the perspective of those students' point of view since they were somehow exposed to the process when observing their peers going through all four stages of creativity. Moreover, the process was repeated multiple times in the classroom, and thus the observations were recurrent. I hope that during those brief assignments, the classroom offered safe and friendly environment for students to encourage and practice the habit of creativity while learning the material. Some students experienced illumination stage which was visible even from a distance in a form of a facial expression or even as a subtle jump of energy within the body.

As a teacher and a mentor of multiple research projects, I have been searching for methods of gently tapping into of students' subconscious minds to reveal the hidden potential of creativity. Personally, I believe that the habit of creativity can be trained just like other habits of mind. But the training requires suitable environment and appropriate circumstances to produce a genuinely creative illumination.

Assessment

Assessments for brief creative assignments are based on students' formal and informal responses. During the informal assessment proctored immediately after the brief creative assignment in Calculus 3 classes, I asked students whether they would prefer less or more of those assignments. Students agreed that they found this activity valuable and encouraging. During the formal assessment, I asked students from my Calculus 3 classes for feedback about all class modules: mini-lectures, worksheets, solving problems on the board, cardboard projects, etc. One of the questions asked for students' opinion about brief creative assignments. All students gave excellent marks to this assignment and did not suggest any improvements. However, since the sample group was very small and consisted of only five students, this assessment needs to be repeated for a larger class.

Conclusions

Students' involvement, their reactions to the brief creative assignments and the results of the survey indicate that this type of class activities are welcome in the classroom with joy. However, the most important benefit of the creative assignments comes from binding the conscious and subconscious mind by creating an everlasting conversation among their mental formations. Those conversations may seem insignificant to the entire course curriculum and marginal from the perspective of learning calculus but they are important or even crucial for the unity of knowledge.

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