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Using Common Sense in a Mathematical Modelling Task

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Abstract

The paper compares responses of students (novices) and lecturers (experts) to questions regarding differences in predictions from 3 different mathematical models of a real-life problem. The problem was based on the data of the spread of SARS (Severe Acute Respiratory Syndrome) in Hong Kong in 2003. The models were based on the same data but they gave very different predictions of the spread of the disease. Although the majority of the students used common sense compared to the lecturers who used their knowledge and experience in explaining the differences, the proportions of correct answers were not far apart. It might suggest that the use of common sense in modelling real-life problems can be a good starting point in dealing with some modelling issues.

Introduction

We believe that even simple mathematical modelling activities can be beneficial for students. We agree with Kadjievich who pointed out that "although through solving such … [simple modelling] … tasks students will not realize the examined nature of modelling, it is certain that mathematical knowledge will become alive for them and that they will begin to perceive mathematics as a human enterprise, which improves our lives" (Kadijevich, 1999).

In many cases a major purpose for mathematical modelling of a phenomenon is to make predictions. Taking into account uncertainty, variety of possible models and a number of assumptions in each model the task of prediction cannot have the "correct" answer. This fact alone can confuse many students. This paper investigates students' opinions regarding differences in predictions from 3 different models based on the same real data. The task given to the students might look very simple. They neither needed to build a model nor to solve the given models. All they needed to do was to read the given real life problem, look at the predictions from 3 different models and give their reasons for the differences in the predictions. We tested one of the modelling competences described by Kaiser in (2007): "Relating back to the real situation and interpreting the solution in a real-world context". We also gave the same task to university lecturers who teach mathematics or mathematical modelling courses. Our idea was to compare the responses of the students and lecturers. The main research question was to investigate possible patterns within each group and also similarities and differences between the two groups when they do the same modelling task. In particular, to which extent the two groups use their intuition, common sense and past experience explaining the differences in predictions from 3 familiar models.

The theoretical framework of this study was based on the works of Haines and Crouch (2001, 2004). A measure of attainment for stages of modelling has been developed in (Haines & Crouch, 2001) The authors expanded their study in (Crouch & Haines, 2004) where they compared undergraduates (novices) and engineering research students (experts). They suggested a three level classification of the developmental processes which the learner passes in moving from novice behaviour to that of an expert. One of the conclusions of that research was that "students are weak in linking mathematical world and the real world, thus supporting a view that students need much stronger experiences in building real world mathematical world connections" (Crouch & Haines, 2004). This was consistent with the findings from the study by Klymchuk & Zverkova (2001) on possible practical, not cognitive reasons for students' difficulties linking mathematics and real world. Referring to that study Crouch and Haines wrote: "…students across nine countries all tended to feel that they found moving from the real world to the mathematical world difficult because they lacked such practice in application tasks" (Crouch and Haines, 2004).

The Study

Three easy models of the real epidemic of SARS in Hong Kong in 2003 - linear, exponential and logistic - were offered as a student project in calculus in (Hughes-Hallett, et al., 2005). Although the models were based on the same data, they gave very different predictions of the spread of the disease. We asked two groups of people, students and lecturers, to explain the differences in predictions from the three models in an unfamiliar (for students) context. The students' group consisted of first-year undergraduate students majoring in engineering from a German university and second and third-year students majoring in applied mathematics from a New Zealand university. Ninety questionnaires were distributed and 48 responses were received so the response rate was 53%. It was a self-selected sample. We systematized and grouped students' answers into different categories according to the nature of their responses. We used either the key words or exact quotes to name the categories. Some students gave multiple responses to some of the questions and some students did not answer all the questions. The lecturers' group consisted of university lecturers from different countries who teach mathematics or mathematical modelling courses. Some of them were involved in research on teaching mathematical modelling and applications. Some of the lecturers were from the same universities as the students participated in the study. Thirty eight questionnaires were distributed and 23 responses were received so the response rate was 63%. It was a self-selected sample. We systematized and categorized the lecturers' answers in the same way as the students' answers.

The questionnaire given to the participants of the study is below.

The Questionnaire

Please read the case below and answer the questions. You don't need to solve anything.

In 2003 a highly infectious disease SARS spread rapidly around the world. Predicting the course of the disease – how many people would be infected, how long it would last – was important to officials trying to minimise the impact of the disease. A number of mathematical models of the spread of SARS were developed to make the predictions. Below are three simple models of the spread of SARS in Hong Kong. We measure time *t*, in days since March 17, the date the World Health Organization (WHO) started to publish daily SARS reports. Let $P(t)$ be the total number of cases reported in Hong Kong by day *t*. On March 17, Hong Kong reported 95 cases. We compare predictions for June 12, the last day a new case reported in Hong Kong (87 days since March 17). The constants in the differential equations were determined using WHO data from 17 to 31 March (15 days).

A Linear Model $\frac{dP}{dt} = 30.2$, $P(0) = 95$. The prediction for June 12 was 2722 cases.

An Exponential Model $\frac{dP}{dt} = 0.12P$, $P(0) = 95$. The prediction for June 12 was 3,249,000 cases.

A Logistic Model $\frac{dP}{dt} = P(0.19 - 0.0002P)$, $P(0) = 95$. The prediction for June 12 was 950 cases.

The actual number of cases on June 12 was 1755.

Please answer the following questions:

1. What were possible reasons for the differences in the predictions from the three models above?
2. On what were your reasons from question 1 based (e.g. your experience in modelling, common sense, etc.)?
3. What could make the predictions more accurate?

Students' Responses

The students' categorized responses are presented below.

1. What were possible reasons for the differences in the predictions from the three models above?

Different models (16), lack of biological factors (10), different ideas of the speed of spread (8), isolation of infected people (8), population density (6), different assumptions of cases per day, report of cases is not correct (3), different infection rates (3), counter actions, for example pharmaceuticals, different side conditions (1), different assumptions for each model (1), probability of onset (1), people developed immunity (1), the predictions are theories, which are different from the reality (1), not enough data (1).

2. On what were your reasons from question 1 based (e.g. your experience in modelling, common sense, etc.)?

Common sense (19), mathematical knowledge and experience in modelling (7), both modelling experience and common sense (3), the given information (1), idea of spread of disease (1), I have never seen such problems in mathematical context before, so I don't know exactly, how to solve it (1), reality, never a constant number of persons will be sick (1), my knowledge about curves of elementary functions (1).

3. What could make the predictions more accurate?

Use experiences from studies of other epidemics, in other regions (14), use more data (7), more knowledge of the virus (3), look for preventive steps, compulsory registration (2), improve data collection (1), average value of cases from 7 days (1), a constant showing the rate of infections (1), side effects like number of travellers to and from Hong Kong (1), information of medical doctors or scientists for the course of disease (1), a study of people behaviour and their health state (1), more facts (1), evaluation of the models (1), the logistic model looks more realistic and it could be improved by using more variables (1), set up a limit of resources (1), adjust the models results to the reality all the time (1), compare the first 2-3 days to find the initial condition (1).

Lecturers' Responses

The lecturers' categorized responses are presented below.

1. What were possible reasons for the differences in the predictions from the three models above?

The models (19), different ideas of the spread of the disease, certain factors were not considered (2), the models were developed for other epidemics, SARS does not fit (1), the assumptions are not the same in all three models (1), did not consider the spread style of the disease (1), infinite number of predictions exist (1).

2. On what were your reasons from question 1 based (e.g. your experience in modelling, common sense, etc.)?

Experience in modelling (13), common sense (5), both modelling experience and common sense (3).

3. What could make the predictions more accurate?

More data (6), a better model (3), better parameters estimation (3), knowledge about infection mechanism and other factors e.g. travelling routes, social patterns (2), more accurate analysis of influencing factors (2), a deeper understanding of how infectious disease spread (1), the parameters in all the models must be the same (1), distribute the observing time in intervals and use different models in different intervals (1), use learning methods (1).

Analysis of the Responses

After consultations with professional mathematicians specialising in epidemic modelling we estimated percentages of appropriate answers to questions 1 and 3 in both groups. The results are presented in the table below. 'CS' means 'common sense' and 'Exp' means 'experience'.

		Question 1	Question 2				Question 3	Question 4	
	N	Appropriate	CS	Exp	Both	Other	Appropriate	Yes	No
Students	48	73%	56%	20%	9%	15%	74%	9%	81%
Lecturers	23	92%	24%	62%	14%	0%	90%	64%	36%

Table 1. Summary of the findings from the questionnaire.

The majority of the students had no or very little experience in mathematical modelling. The closest activity to real mathematical modelling for them was solving application problems. To our surprise the students did well in both modelling questions 1 and 3. They were not much behind the lecturers giving 73% appropriate reasons for the differences in the predictions from the models versus 92% given by the lecturers. They were not much behind the lecturers giving 74% appropriate ways to improve the accuracy of the predictions in the models versus 90% given by the experts. This is consistent with the findings by Haines and Crouch (2001, 2004) where the authors found that sometimes novices exhibited aspects of expert behaviour although they were not consistent in doing so. In particular, in their study on self-assessment and tutor assessment they found that students were almost as good as tutors in assessing group (project) presentations on modelling and so they could recognize modelling behaviour in others. It is the consistency demonstrating expert behaviour that perhaps puts the lecturers ahead.

In question 2 the reverse polarity on the answers by the students and the lecturers was anticipated: the students relied more on common sense (56%) rather than on experience (20%) compared to the lecturers (24% on common sense and 62% on experience). Apart from lack of modelling experience by the students one of possible reasons for that reverse polarity might be elements of the lecturers' behaviour where they were reluctant to put their responses down to common sense, preferring to classify it as experience. After all they have invested a great deal of time in mathematics/modelling.

Based on the participants' comments in the questionnaire and follow-up interviews with some of them we attempted a comparison of the processes used by the students and the lecturers in terms of links between the mathematical world and the real world in a similar way it was done in (Crouch & Haines, 2004). We took the first "level a) where there was clear evidence that the participants took into account the relationship between the mathematical world and the real world" (Crouch & Haines, 2004). The students referred explicitly to that relationship in 65% of cases (though not always in a correct way) whereas the lecturers in 20% of cases. The lecturers tended to concentrate more on the mathematical aspects of the models probably implicitly assuming that relationship. One of the possible reasons might be that the lecturers used their experience in modelling and knowledge in mathematics much more than their common sense whereas the students relied more on their common sense and life experiences lacking the experience in mathematical modelling.

Conclusions

This study indicates that in spite of lack of experience in real mathematical modelling, students can effectively use their common sense and general knowledge of mathematics to evaluate some modelling issues dealing with prediction. The responses at a more general level indicated that both students and lecturers would have preferred to include more parameters in the model to make the modelling more realistic and intuitive, i.e., to have a theoretical basis for the modelling that included hypothetical rates of spread, infection mechanisms, etc.

We are very aware of the limitations of the study. It was intended as a pilot study to check our assumptions and share the findings with the mathematics education community. Future work should explore students' and lecturers' (or novices and experts according to Haines and Crouch, 2004) responses to more sophisticated mathematical models that allow for the adjustment of parameters to optimize the output from the model.

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References

Crouch, R. & Haines, C. (2004). Mathematical modelling: transitions between the real world and the mathematical world. *International Journal on Mathematics Education in Science and Technology,* 35 (2), 197-206.

Haines, C. & Crouch, R. (2001). Recognizing constructs within mathematical modelling. *Teaching Mathematics and its Applications*, 20 (3), 129-138.

Hughes-Hallett, Gleason, McCallum, et al. (2005). *Calculus: Single and Multivariable*. 4th edition, Wiley.

Kadijevich, D. (1999). What may be neglected by an application-centred approach to mathematics education? *Nordisk Matematikkdidatikk*, 1, 29-39.

Kaiser, G. (2007). Modelling and modelling competences in school. In Haines C., Galbraith P., Blum W. and Khan S. (Eds) *Mathematical Modelling (ICTMA 12): Education, Engineering and Economics*, Chichester: Horwood Publishing, UK, pp. 110 - 119.

Klymchuk, S., Narayanan, A., Gruenwald, N., Sauerbier, G., & Zverkova, T. (2011). Modelling of infectious disease with biomathematics: Implications for teaching and research. In G. Kaiser, W. Blum, R. B. Ferri, & G. Stillman (Eds.), *Trends in Teaching and Learning of Mathematical Modelling* (Vol. 1, pp. 489-498). Springer Verlag.

Klymchuk, S., Zverkova T. (2001). Role of mathematical modelling and applications in university service courses: An across countries study. In Matos J.F., Blum W., Houston S.K. and Carreiara S.P. (Eds) *Modelling and Mathematics Education: ICTMA-9: - Applications in Science and Technology*, Chichester: Horwood Publishing, UK, pp. 227-235.