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# Solving Application Problems Using Mathematical Modelling Diagrams

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*This paper describes three simple practical problems solved by three university lecturers to demonstrate applications of mathematics or illustrate some issues in mathematics education. The problems were of different levels from primary to secondary school although the settings were at a university. All lecturers either explicitly or implicitly used the four step diagram for solving the problems: real problem => mathematical model => mathematical solution => real solution => real problem. While modelling the lecturers formulated inadequate mathematical models without the constraints of the variables involved. That led to contrasting the 'mathematical solution' and the 'real solution' which might have resulted in incorrect perception of the role of mathematics in real life among some students. The author suggests that the contrast could be avoided by using the three step diagram instead of the four step diagram for the process of mathematical modelling of such application problems: real problem => mathematical model => solution => real problem.*

There are many diagrams that illustrate the mathematical modelling process of solving real life problems. Some of the diagrams are very detailed and complicated. An example of such a diagram is represented in Figure 1:

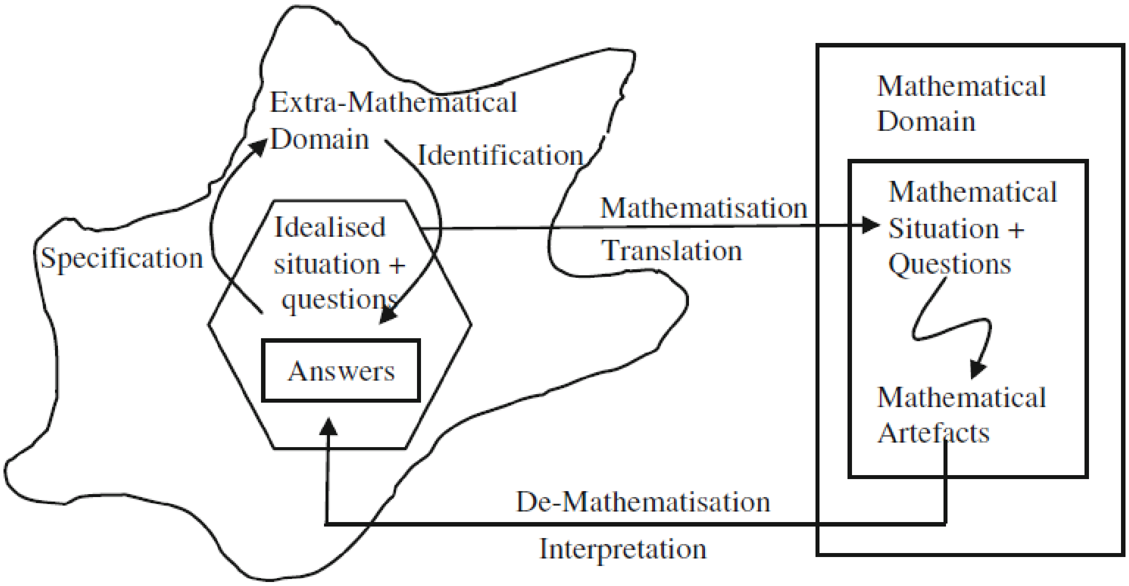


Figure 1. Mathematical modelling process (from Niss, 2010, p.44).

Many textbooks on calculus use much simpler versions like the one represented in Figure 2:

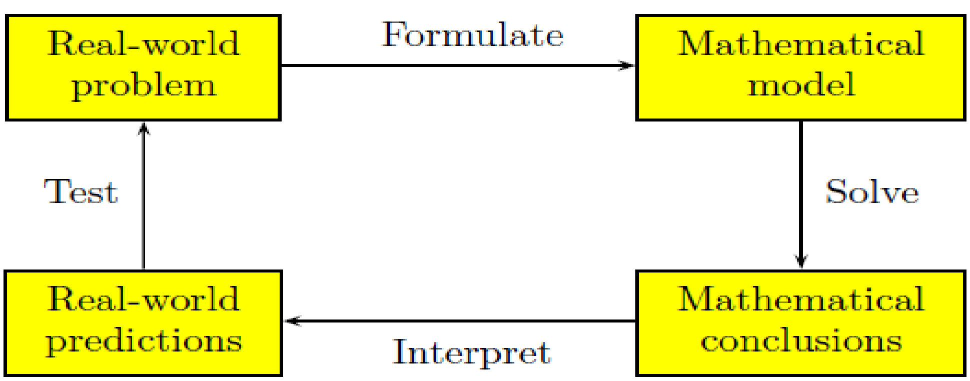


Figure 2. Mathematical modelling process (from Stewart, 2010, p.25).

Even a simpler version is very common in textbooks and in teaching practice:

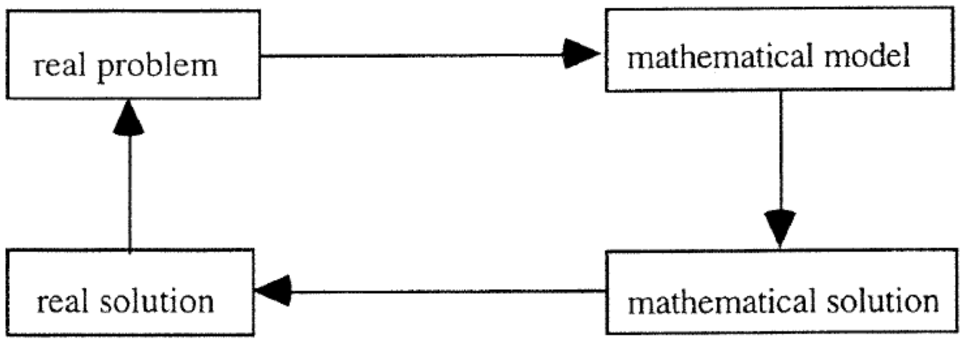


Figure 3. Four step mathematical modelling diagram.

The above diagrams differentiate the mathematical solution and real solution. Indeed, when we solve a real life problem we make assumptions, construct the mathematical model and solve it to obtain the solution of the mathematical model. After that we need to test that solution on the original real life problem and if it doesn't fit we need to adjust assumptions and/or refine the model and repeat the cycle again until we are satisfied with the outcome.

This paper is based on the author's observations of teaching mathematics and mathematics education courses while on sabbatical at a partner university. Within a month he came across three situations when the lecturers used the four step diagram represented in Figure 3 for solving simple application problems.

Problem 1. How many buses does it take to transport 145 people if each bus can take only 20 people?
The lecturer's commented that the 'mathematical solution' was 7.25 but it didn't suit us and the 'real solution' was 8.

Problem 2. There are three sorts of alloy bars. Their weights and contents of iron and copper are: A (10, 2, 1), B (15, 1, 3), C (15, 4, 3). How many of each of the alloy bars do we need to meet the following constraints: total weight 25; total amount or iron 9 and total amount of cooper 4.

After solving the system of the linear equations

$10x + 15y + 15z = 25$
$2x + y + 4z = 9$
$x + 3y + 3z = 4$

as a mathematical model and obtaining the answer (1, -1, 2) the lecturer commented that the ‘mathematical answer’ was not realistic and the ‘real solution’ of the problem did not exist.

Problem 3. A stone was thrown at an acute angle from the top of a 60 m high cliff. In 2 seconds the stone was in its highest point of 80 m. Assuming a quadratic relationship between the height and time, find the time when the stone lands on the ground.

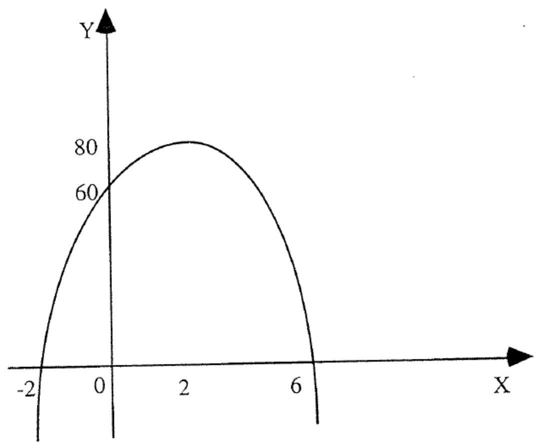


Figure 4. Sketch of the trajectory of the stone for problem 3.

Here the mathematical model was: find abscissas of the intersection points of the parabola with the *x*-axis. That led to two answers: -2 and 6. After the discussion, the ‘real solution’ (6) was chosen from the ‘mathematical solutions’ (-2 and 6).

It is clear that all three lecturers contrasted the ‘mathematical solution’ to the ‘real solution’ in the above problems. The range of the variables was taken into consideration only at the stage of adjusting the ‘mathematical solution’ to the ‘real solution’. These contrasts could be easily avoided if the constraints on variables were included into the mathematical models. For example, the mathematical models of the three problems might be:

Mathematical model of problem 1. Round up the number 145/20.
Mathematical model of problem 2. Find non-negative solutions of the system of the linear equations.
Mathematical model of problem 3. Find a positive *x*-intercept.

In such simple application problems there is no need to make assumptions, test and refine the models if they are constructed adequately – by taking into account the range of the variables involved. In most cases what is needed is just an interpretation of the solution by adding the units (kg, cm, etc.). In some cases care should be taken of reasonable accuracy of the solution (often students write the answer obtained from a calculator accurate to 5-7 decimal places).

For such simple application problems like problems 1-3 the following 3 step diagram might be an alternative:

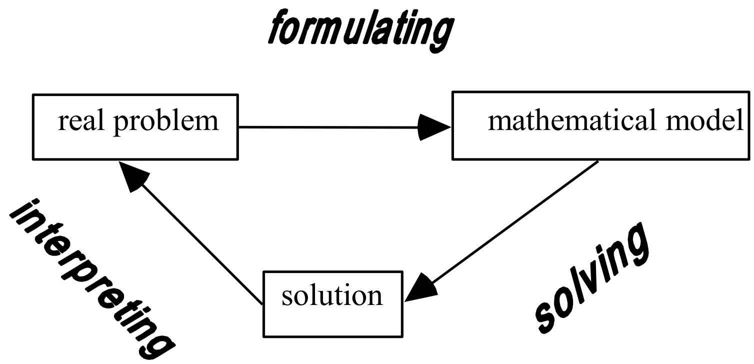


Figure 5. Three step mathematical modelling diagram.

The advantage of this diagram compared to the four step diagram is that it might prevent the psychological discomfort among some students caused by contrasting the ‘mathematical solution’ and the ‘real solution’. The matter of adequacy of a mathematical model is very important even in simple application problems especially for non-maths students. Otherwise we might face a situation that happened on a chemistry lesson when a student ‘proved’ that 1 plus 1 is not always 2. He mixed 1 litre of water with 1 litre of spirit and found that the total volume of liquid is not 2 litres! (It is a well-known fact that the mixture of water and spirit has more compact molecular structure).

References

Niss, M. (2010). Modeling a crucial aspect of students’ mathematical modeling. In R. Lesh, P. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modelling students’ mathematical competencies* (pp. 43–59). New York: Springer.

Stewart, J. (2006). *Calculus: Concepts & Contexts*. Thomson Brooks/Cole.