MATHEMATICS TEACHING-RESEARCH JOURNAL ONLINE
VOL 7, N 3
Spring 2015

CREATIVITY AND BISOCIATION

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Abstract. *Creativity research has its roots in reflections upon eminent mathematics creating original and useful mathematical products. Its extension into the mathematics classroom leaves open many definitions and understandings of its value and role in education of gifted students. The extension of creativity to ordinary students who do not see themselves as gifted or the demoncratization of creativity is explored through the lens of Koeslter's theory of bisociation and the relationship between bisociation and reflective abstraction the mechanism of learning proposed by Piaget is put forth.*

Key words: bisociation, reflective abstraction, incubation, illumination

HISTORICAL DEVELOPMENT OF CREATIVE RESEARCH

The historical development of creativity is typically traced to the original and creative work of eminent mathematicians. Inherent in this view is measurability through the end result or the product definition (Liljedahl,2011, 2013) in which the community of mathematicians is to verify the truth of as well as to judge the usefulness and originality of the idea presented. The extension of creativity to the mathematics classroom is often traced to the work of Jaques Hadamard who synthesized the stages the Gestalt psychology Wallas to explain the setting in which the "Aha" moment occurs through a process-stage that begins with incubation in which the solver tries diligently but unsuccessfully to obtain the goal to the instantaneous insight of the illumination stage. Hadamard it would appear believed that the distinction between the creative genius and a creative individual learning mathematics that is known to the community but not him or herself is only a matter of degree. Yet research on incubation and the transition to illumination is scarce in mathematics education and the mechanisms poorly understood (Sriramen et al, 2011) Perhaps one reason is the view that concepts of incubation and illumination are considered, " …arachaic Gestalt constructs…" (Juter & Sriramen, 2011) which as Piaget (1977) and Koestler (1964) would note has fallen from fashion due in part to its overly strict focus on not only an instantaneous but also complete insight into problems and structures that in cognitive development takes places in repeated flashes of insight or eureka moments that build on one another and occur as a result of continued effort by the individual learner. Another related reason is that the affective component of creativity is too difficult a concept for educational researchers and psychologists to measure objectively, indeed the intuitive nature of the creative experience is often tied to a self transcendent force in which inspires a spontaneous “Aha” or Eureka moment s simultaneous with a complete solution to the problem. One can only imagine the comments a pedagogy based upon complete solutions mystically popping into students head would receive.

Transition from genius to gifted students

The transition from the so-called genius understanding of creativity, to a more inclusive definition and understanding that allows for research into creativity in the mathematics classroom especially in regards to research in problem solving (Silver, 1997) has spurred a significant amount of research and a multitude of definitions and what creativity in discovering what is known to others means Mann (2006).

One might also point out that definitions and understandings used to measure creativity with gifted students within the mathematical classroom can be viewed as being developed to distinguish between gifted and non-gifted or ordinary students. Leikin (2009b) notes two formulations of creativity in mathematics educational research that have been used to assess an individual’s propensity for creativity in the mathematics classroom or student giftedness. The first is the ability for convergent and divergent thinking due to J. Guilford. ”Convergent thinking involves aiming for a single correct solution to a problem, whereas divergent thinking involves the creative generation of multiple answers to a problem or phenomena, and is described more frequently as flexible thinking.” Her review also notes the definition suggested E. Torrance i.e. the capacity of an individual for flexibility, fluency, novelty and elaboration. “Fluency refers to the continuity of ideas, flow of associations, and use of basic and universal knowledge. Flexibility is associated with changing ideas, approaching a problem in various ways, and producing a variety of solutions. Novelty is characterized by a unique way of thinking and unique products of a mental or artistic activity. Elaboration refers to the ability to describe, illuminate, and generalize ideas.” (p.129) Using this document

Transition from gifted to ordinary students

Yet these characteristics such as fluency with mathematical content, the ability to view and understand various different approaches to a problem solving specifically including those not modelled by the instructor can be used to identify and characterize students not in science, mathematics, engineering or related fields who are enrolled in mandatory first year math course (Blair et al. 2006)

The lack of educational research on creativity with the general population has been pointed out by Srimanen et al (2011) "The role of creativity within mathematics education with students who do not consider themselves gifted is essentially non-existent" (p.120). Clearly if such research is to be done what we (Prabhu and Czarnocha, 2014)) shall refer to as the 'democratization' of creativity one needs a definition in which to measure creativity that does not exclude ordinary students. Before reviewing Koestler theoretical framework for creativity and its relationship to the issues raised it is useful to consider on why one might be interested in creativity with students who do not consider themselves gifted in mathematics. Liljedahl (2013) studies students he classified as 'resistant', "Many of the students would describe themselves as math phobic, math-incapable or a combination of the two. They usually have negative beliefs about their abilities to do mathematics, poor attitudes about the subject, and dread the thought of having to take a mathematics course"(p.256) exposing them to creative pedagogy and asking them to write about their experiences. He reported that they experienced illumination or "Aha" moments both while listening to the instructor and while working problems out for themselves the latter being much more common and that such experiences frequently did alter their attitudes towards mathematics. One of the basic insights of Prof. V. Prabhu is that students who struggle with mathematics especially in underserved populations frequently have a negative affect towards mathematics and perhaps the only way to reach such students is through a pedagogy that guides and exposes students to their own creative potential, a social contract (Surrazy & Noventná, 2013) that encourages them to own their learning process what she would refer to as a handshake between the instructor and the student. Furthermore, the lack of such pedagogy propagates a society in which teachers such as those described by Liljedahl (2013) (before his research project) will pass on their own negative view of mathematics to children who will follow their lead and then pass on such negative views to friends and children, in such a way an entire society itself begins to accept that is cannot do mathematics and that such a situation is a natural state.

KOESTLER THEORETICAL FRAMEWORK FOR CREATIVITY

Koestler (1964) describes the mechanism of creativity in terms of an (hidden) analogy between two or more previously unrelated frames of reference: "I have coined the term *bisociation* in order to make a distinction between the routine skills of thinking on a *single plane* as it were, and the creative act, which … always operates on *more than one plane*" (Koestler, 1964, p. 36).

The terms matrix and code are defined broadly and used by Koestler with a great amount of flexibility. He writes, "I use the term *matrix* to denote any ability, habit, or skill, any pattern of ordered behaviour governed by a *code* or fixed rules" (p. 38).

> The *matrix* is the pattern before you, representing the ensemble of permissible moves. The *code* which governs the *matrix*…is the fixed invariable factor in a skill or habit; the *matrix* its variable aspect. The two words do not refer to different entities; they refer to different aspects of the same activity. (Koestler, 1964, p. 40)

Thus, for Koestler (1964), *bisociation* represents a "spontaneous flash of insight ... which connects previously unconnected matrices of experience" (p. 45). That is the"…transfer of the train of thought from one matrix to another governed by a different logic or code …" (p. 95)

The focus of the Act of Creation Theory is on the bisociative leap of insight, that is, an *Aha!* moment, or a *moment of understanding*, — a phenomenon that contains both an affective component of the 'Aha' moment and a cognitive component of the synthesis of two previous unrelated matrices of thoughts the hidden analogy as Koestler would refer to it. These components in so far as they can be observed amongst the general population suggests Koestler's framework as suitable for measuring and analyzing the creative aspect of self discovery during learning.

REFLECTIE ABSTRACTION AND BISOCATION

In order to observe bisociation within the self discovery of mathematics during the many leaps of insight some obtained when listening to the instructor or during working as a class, in a small group or individually it is useful to reflect upon the relationship between Bisociation and Koestler's framework for creativity with reflective abstraction the mechanism used by Piaget to explain learning. Like the mechanisms of bisociation it is based upon conscious reflection and abstraction of solution activity.. According to Piaget, construction of schema and cognitive change:

> …proceeds from the subject's actions and operations, according to two processes that are necessarily associated: (1) a projection onto a higher level (for example, of representations) of what is derived from the lower level (for example, an action system), and (2) reflection, which reconstructs and reorganizes, within a larger system, what is transferred by projection. (Piaget & Garcia, 1989, p. 2)

Dubinsky (1991) employs Piaget's mechanisms of reflective abstraction on actions and processes within the process/object duality of concept development. Dubinsky considers *interiorization*, *coordination*, *generalization*, *reversibility* and *encapsulation*. Interiorization involves the internalization of processes or actions, while reversibility involves reflection upon the inverse of a known process. Coordination of two processes is an integral part of reflective abstraction as defined above when after projection the learner reflects upon and begins to reconstruct and reorganize his original knowledge coordinating it with processes and structures available in the larger structure. The original projection of knowledge onto the higher plane and the reflection required during the coordination of this projected knowledge with processes of the higher plane generalizes and abstracts the learners knowledge to the higher structural level. This work of Dubinsky has laid the foundation for APOS or action-process-object-schema theory in which the learner transitions between through three types of knowledge or understanding the action level: actions, processes, and objects, which are themselves organized into schema. "An action is any physical or mental transformation of objects to obtain other objects. It occurs as a reaction to stimuli which the individual perceives as external. " (Cottrill et al 1996,p.2) In contrast to a process in which the individual is in control and can reflect upon the transformation. Processes are the result of the Interiorization of actions and at the process level an individual can employ the reflective abstraction processes of coordination and reversibility. The object level is obtained through the final encapsulation processes.

Simon et al. (2004) ascertain that such reflective abstraction, or more specifically, coordination, is brought about by comparisons of the solution activity and its effect in relation to the expected outcome:

> Thus, within each subset of the records of experience (positive or negative results), the learners mental comparison of the records allows for recognition of patterns, that is, abstraction of the relationship between activity and effect. Because both the activity and the effect are embodiments of available conceptions, the abstracted activity-effect relationship involves a coordination of conceptions … Note the activity and the effect are conception-based mental activities, our interpretation of Piaget's notion of coordination of actions (Simon et al., 2004, pp. 319-320)

This interpretation of Piaget's mechanism of coordination broadens the scope of reflection upon actions to reflection upon solution activity. In Koestler's framework for problem-solving, the experiential mental records are matrices, and coordination occurs initially between the individual's record of possibly related matrices and the given problem situation. A straightforward analogy, or association, is characterized by an isomorphic mapping, or a projection, of a matrix to the current situation, resulting in assimilation. When no such equivalent matrix is found, the solver searches for a matrix that, although not previously considered completely analogous, is suddenly seen as giving structure and meaning to the present situation. The hidden analogy comes to light. When this realization is led by intuition, Koestler refers to it as bisociation, and likens it to the process often exercised by mathematicians and scientists to exploit the structures of one discipline to give meaning, through analogy, to a problem in another.

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