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Editorial.

This new V 7 No 2 issue of the Mathematics Teaching-Research Journal on line restarts our operation after a year of a slowdown connected with the sabbatical of one of the editors. We offer this time two sets of articles connected to each other. We have two papers on the increase of educational STEM capacity. One is from Nkechi Agwu (BMCC,CUNY) describing her unusual project of connecting mathematics to the symbolism and its meaning of the African tribes. Agwu, similarly as Vrunda Prabhu in the case of India, has built an interesting STEM educational bridge between NYC and Nigeria. The second article in this collection comes from Texas A&M University in Laredo, from where we get the report by Goonatilake et al on the capacity effectiveness of the mentoring scheme in pre-service teacher program. Mentoring has recently awakened interest of the profession as the necessary motivational element for encountering challenging while easing teacher/students entry into profession. Mathematics Department at Hostos CC has introduced mentoring of the peer leaders in some of their classes with a significant degree of impact upon passing rates in the developmental classes of mathematics.

The next collection consists of two papers both focused on the constructivism on Mathematics; one – from the point of view of mathematics research, another from the point of view mathematics teaching. Barbara Lawrence, also from BMCC describes to us the constructivized mathematics arguing that its finitness is an asset for understanding central ideas of calculus. It's interesting to compare this view with the ideas brought by Krishnan and Tran, the authors of Contextualized Examples in Constructivist Mathematics Pedagogy from Kingsborough CC for whom every day's reality based problems constitute the essence of the pedagogy. Finiteness and concreteness are the common factors in these two different approaches based on common vision of mental constructions.



**An Expository Article on the Carnegie African Diaspora Fellowship Project
(CADFP)**

***Culture and Women's Stories:
A Framework for Capacity Building in Science,
Technology, Engineering and Mathematics (STEM)
Related Fields***

By

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Yoruba Talking Drums ~ Carrying the Message
An Expository Report on the Carnegie African Diaspora Fellowship Project
(CADFP)
Culture and Women's Stories: A Framework for Capacity Building in Science,
Technology, Engineering and Mathematics (STEM) Related Fields
By
Nkechi Agwu, Mojisola Edema, Olayinka Ogunsuyi, Olabukunola Williams, Stella
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Abstract

This expository article is a summary report of the curriculum development phase (phase I), of the Carnegie African Diaspora Fellowship Project (CADFP), *Culture and Women's Stories: A Framework for Capacity Building in Science, Technology, Engineering and Mathematics (STEM) Related Fields*. This project was one of the round one 31 selected projects for funding under a new funding initiative of the Carnegie Foundation, its African Diaspora Fellowship (ADF) Program. The Carnegie Fellow, Dr. Nkechi Agwu, a Professor of Mathematics at the Borough of Manhattan Community College (BMCC), City University of New York (CUNY), was one of the 33 selected fellows to be funded under the Foundation's ADF Program to partner and collaborate with colleagues at the Federal University of Technology, Akure (FUTA), under the aegis of its Center for Gender Issues in Science and Technology (CEGIST) with leadership on the FUTA end

by Dr. Mojisola Edema, Director of CEGIST, Dr. Olayinka Ogunsuyi, Assistant Director of CEGIST and Ms. Olabuunola Willaims, Coordinator of the Nigerian Women in Agricultural Research and Development (NiWARD) Program at CEGIST. This article highlights the goals, background, rationale, and significance of this project. It provides a summary of project activities and four samples of mathematical curricular activities developed, one from each of the following categories: work of rural women, traditional medicine and food, heritage of traditional rulers and Nigerian women leaders in STEM.

Introduction

The Carnegie Foundation's African Diaspora Fellowship (ADF) Program was initiated to foster collaboration and partnership, between African born faculty in the United States and Canada and their counterparts in institutions in Africa, in research, curriculum development and/or mentoring of graduate students, see <http://www.iie.org/Programs/Carnegie-African-Diaspora-Fellows-Program>. The Program is an out-growth of the research study of Paul Zeleza, *Engagements between African Diaspora Academics in the U.S. and Canada and African Institutions of Higher Education: Perspectives from North American and Africa* (Carnegie Corporation of New York, February 2013). It offers a novel partnership between four parties: Carnegie Corporation of New York, providing funding, the International Institute for Education providing logistical support, Quinnipiac University, providing administrative support, and an Advisory Council comprised of leading African academics and university administrators in North America and Africa providing strategic direction.

This project, *Culture and Women's Stories: A Framework for Capacity Building in Science, Technology, Engineering and Mathematics (STEM) Related Fields*, was one of the round one 31 selected projects for funding under the new ADF funding initiative of the Carnegie Foundation. The Carnegie Fellow, Dr. Nkechi Agwu, a Professor of Mathematics at the Borough of Manhattan Community College (BMCC), City University of New York (CUNY), was one of the 33 selected fellows to be funded under the Foundation's ADF Program to partner and collaborate with colleagues at the Federal University of Technology, Akure (FUTA), under the aegis of its Center for Gender Issues in Science and Technology (CEGIST) with leadership on the FUTA end by Dr. Mojisola Edema, Director of CEGIST, Dr. Olayinka Ogunsuyi, Assistant Director of CEGIST and Ms. Olabuunola Willaims, Coordinator of the Nigerian Women in Agricultural Research and Development (NiWARD) Program at CEGIST.

The goal of this project is to develop culturally based and gender sensitive curricular materials; engage in teaching research on their use, and prepare Nigerian educators in using them to teach mathematics and related disciplines at the primary; secondary and tertiary level, with the aim of fostering student innovation and creativity that is linked to

the science and technology of their culture; nurturing and mentoring girls to consider STEM related careers and providing poor and rural communities with inexpensive curricular resources that are easily obtainable from the local community. Kindly review the following press releases about this project at:

<http://nigeriamasterweb.com/Masterweb/breakingnews-8714-carnegie-african-diaspora-fellowships-support-31-projects-africa-dr-nkechi-agwu>; or

<http://www.africasti.com/profile/diaspora-fellowships-to-support-projects-in-africa>.

Specifically with regard to the host institution FUTA, this project promotes the goals of CEGIST by giving visibility through curricular activities to Nigerian women in STEM and their work, including rural women's indigenous scientific knowledge. Kindly see <http://cegist.futa.edu.ng/page.php?pageid=79>. It also facilitates the work of the General Studies Department in fostering students' knowledge about Nigerian culture, history, and indigenous knowledge related to science and technology. It facilitates the work of the Mathematical Sciences Department and other STEM departments at FUTA (e.g., the Department of Transport Management Technology whose faculty participated actively), by engaging students in learning activities in these disciplines that relate to their culture, natural environment and the work of Nigerian women leaders in STEM and rural women's indigenous scientific knowledge.

Specifically with regard to the Carnegie Fellow's home institution, BMCC, CUNY, this project promotes the goals of CUNY by enhancing the mathematics curriculum in MAT 200 – Discrete Mathematics, MAT 150 – Introduction to Statistics, MAT 100 – Fundamentals of Mathematics I, MAT 110 – Fundamentals of Mathematics I and MAT 300 – Introduction to Geometry, particularly writing intensive sections of these courses in a multicultural and gender sensitive manner, that makes visible the ethno-mathematics of a region in which the curriculum is almost silent (Africa south of the Sahara) and the work of African women in STEM.

Background

This project, *Culture and Women's Stories: A Framework for Capacity Building in Science, Technology, Engineering and Mathematics (STEM) Related Fields*, extends the ethno-mathematics research activities of the Carnegie Fellow, Dr. Nkechi Agwu.

Specifically, it extends her research activities on the cultural symbolism and mathematical traditions of the Igbo people of Nigeria which were funded by the New York City Literacy Assistance Center and Professional Staff Congress (PSC) of the CUNY. Kindly see

<https://www.rfcuny.org/rfwebsite/research/AwardedPersons.aspx?programID=PSCREG&sequence=32&desc=Mathematics>. It also extends Dr. Agwu's research activities on the cultural symbolism and mathematical traditions of the Ndebele people of Southern Africa

which was inspired by collaborative partnership with the African Views Organization within their African Cultural Exchange (ACE) Program for children at P.S. 107 in Brooklyn, New York, and the David's Builders Children's STEM Program of Vineyard International Christians Ministries in Bronx, New York.

These research activities of Dr. Agwu led to the development of African culturally based and gender sensitive curricular materials for teaching writing intensive (WI) graduation requirement courses in Discrete Mathematics and Statistics at BMCC, CUNY. These WI mathematics courses were developed under the aegis of the Writing Across the Curriculum (WAC) Program at BMCC, CUNY. For more information about the curricular activities developed by Dr. Agwu based on the African cultural milieu and the African tradition of story-telling about the lives of African women in mathematics in these WI courses, kindly review: <http://www.bmcc.cuny.edu/news/news.jsp?id=11273>.

Dr. Agwu had previously field-tested these materials in Nigeria in 2013 at a three-day mathematics education professional development session for teachers at Graceland Primary and Secondary Schools of the Anglican Diocese in Gusau, Zamfara State, Nigeria. This field-testing was an important jump-start to the work that was done at FUTA as it provided direction for how to design the activities to be engaged in at FUTA or any other incubation sites in Nigeria.

The partnership and collaboration for this project was inspired and facilitated by Dr. Stella Williams, Chair of the Board of CEGIST, FUTA, Retired Professor of Agricultural Economics, Faculty of Agriculture, Obafemi Awolowo University, Ile-Ife, and Executive Member of Pan-African Strategic Policy and Research Group (PANAFSTRAG). Networking connections for the project were initiated when Dr. Nkechi Agwu gave a presentation, *She Touched Me – Nurturing African Girls for a Career in STEM*, see <http://www.youtube.com/watch?v=CdWar62RrwE>, at the emergency planetary forum on *The African Women and STEM*, co-sponsored by the Drammeh Institute and PANAFSTRAG USA, at the parallel forum to the 58th session of the United Nations (UN) Conference for the Status of Women, sharing her curriculum development activities for teaching WI courses in Discrete Mathematics and Statistics at BMCC, CUNY. Dr. Williams was also a presenter at this forum.

This forum jump started the conversation between Dr. Williams, Dr. Agwu and the collaborators and key administrators at FUTA to submit an application for the CADFP with the purpose of bringing Dr. Agwu to collaborate and partner with colleagues at FUTA to extend her ethno-mathematics research and curricular activities, with CEGIST, FUTA serving as an initial incubation site for moving it in an integral way into the Nigerian educational system. As it turns out, CEGIST, FUTA, is an excellent incubation site for this project due to the following facts. CEGIST is actively involved in a Nigerian Women in Agricultural Research and Development (NiWARD) Program. The NiWARD

Program facilitates leadership and professional development of Nigerian women in STEM. Kindly see <http://www.niward.org>. FUTA is a science and technology focused institution and has staff primary and secondary schools. Kindly see <http://www.futa.edu.ng/futacms/index.php>. Located in Akure and its environs are some key cultural and heritage sites. All these coupled together makes FUTA an ideal location to conduct ethno-mathematics research on Nigerian culture and Nigerian women in STEM.

Additionally, the affiliation of Dr. Williams with PANAFSTRAG was a bonus. PANAFSTRAG was already engaged in programs to foster STEM education in Lagos State and its environs. This led to partnership with PANAFSTRAG under the dynamic leadership and coordination of its Program Coordinator, Mrs. Arinola Bello, to extend the project work at FUTA beyond Ondo State to Lagos State through a plenary and workshop professional development session with teachers affiliated with schools in Lagos State and its environs, within PANAFSTRAG's Math, Science and Technology to Innovate Indigenous Knowledge System (MSTIIKS) Teachers Program. Kindly see <http://www.panafstrag.org/pages/sections.php?sid=76>.

Rationale & Significance

Mathematics is the science of understanding patterns that exist around us and solving problems in our daily life. It is the foundation or gateway to knowledge of understanding the sciences around us and in our daily life management skills. It is for these reasons that the field of mathematics and the work of mathematicians are extremely important to society starting with the individual, as well as, the family and society in general.

When we look deeply with insight at any object we will see many patterns and symbols that can be interpreted within the context of some mathematical concepts. Let's say for instance a traditional Igbo *Akwete* cloth, which is creatively woven and is a common profession of rural Igbo women from the Akwete area near Aba in Abia state, see <http://www.vanguardngr.com/2012/07/akwete-cloth-an-igbo-textile-art>. The picture given below of an Igbo rural woman weaving an Akwete cloth clearly illustrates that this woman has indigenous knowledge of concepts related to geometry, symmetry and basic principles of coloring. How else could she be able to weave the beautifully half woven designed Akwete cloth that we see in this picture? Paulus Gerdes (1999) in *Geometry From Africa, Mathematical and Educational Explorations*, provides an in-depth examination of the indigenous geometric knowledge evident in the traditional textiles, artifacts and hair styles produced by African women south of the Sahara.

Figure I: An Igbo rural woman weaving a traditional Akwete cloth



<http://media-cache-ak0.pinimg.com/736x/cc/f5/39/cf5395077762e13b1cac0594c6d6fb3.jpg>

How can we help students from rural communities where this is the everyday work of their mothers and grandmothers make relevant connections between the abstract mathematical concepts they are learning about in the classroom and the work of their mothers and grandmothers, so that they can expand upon the indigenous knowledge of their mothers and grandmothers to become better skilled, more creative and highly innovative at the traditional enterprise of their mothers and grandmothers, thereby taking it to higher and ground-breaking heights?

Modeling is an extremely important aspect of problem solving in terms of its empirical attributes. Therefore, it is imperative that we as educators develop with adequate support from governmental and non-governmental institutions, programs and innovative curricula that will teach students the basics of mathematical modeling. There are many areas of government endeavor and private enterprise that we could use the tools of mathematical modeling to understand.

One area of private enterprise that can be enhanced using the tools of mathematical modeling is in the production of *Black Soap*, which has a traditional medicinal value for eradicating skin ailments, see <http://www.wikihow.com/Make-Black-Soap>. This is a traditional enterprise of rural Yoruba women. When we critically evaluate and assess the procedures that they use in creating Black Soap, we can create mathematical models to provide information on how to optimize the efficiency of production, minimize waste, improve the quality of the soap produced and production techniques, minimize cost and hours of labor, and maximize profit. Based on measurements of the ingredients for

producing Black Soap, we can develop mathematics curricular activities to reinforce students' understanding of ratios, rates and proportions.

Figure II: A rural Yoruba woman making Black Soap



<http://greenerchiro.com/African%20Pictures/Making%20black%20soap2.jpg>

How can we develop students' financial literacy skills through the work of rural Yoruba women's traditional enterprise of the production of Black Soap? How can we get them to become deeply interested in the production of Black Soap and see it as an extremely profitable business venture that is worthwhile pursuing?

According to the 2009 demographic data on Nigeria, females are close to 50% of the Nigerian population. Yet they are severely under-represented in Science, Technology, Engineering and Mathematics (STEM) related fields - less than 1% of the doctorate degree holders in these fields, with similar statistics in the higher echelons of policy and decision-making in STEM. Taking into context that STEM fields are an area of employment growth and high income, if this trend continues, it will present serious problems for the nation in terms of growth, development and decision-making policy issues. However, these statistics will not improve without a conscious effort of social re-engineering in the way we formally and informally educate our youths both girls and boys to see their place and the roles they are meant to play in the society.

We need to educate our girls at an early age that they should not be left behind and that they are quite capable of being the face of leadership even in non-traditional scientific fields, without it impacting negatively on their ability to do any of the other wonderful

things that we do as nurturers of the family. We need to show our girls and boys that a rural Igbo woman who weaves Akwete has a well-grounded basic intuitive understanding of geometry. We need to show them that a rural Yoruba woman who creates Black Soap has a well-grounded basic understanding of the science (mathematics and chemistry) behind its creation. We also need to educate our boys at an early age that girls are equally capable of leading and should not always be relegated to background supporting roles where they do carry out activities but are not seen or heard from much in public, by making visible to them the history of Nigerian women, such as the women of NiWARD, who have excelled and are leaders in STEM. This way, we will see more women heading elected position and they are appointed into teaching, research, policy and decision making offices in STEM.

West African Examinations Council (WAEC) statistical data show that students' performance in mathematics has been declining over the years and that the performance is dismal right now in the 21st century when Africa is in the sport-light of global development actions. Also, students sitting qualification exams into the tertiary institutions in Nigeria have recorded an average of 70% failure in mathematics over the past few years. This performance is partly due to the general fear of mathematics perpetuated by the type of curriculum that does not effectively engage, activate and inspire their interest in mathematics in creative and innovative ways. The fear of mathematics is one that is shared by many people around the world. As mathematicians, we are partly responsible for this. We teach the subject at lower levels from pre-k to high school and university using pedagogical approaches that divorce it from our everyday life. Our focal emphasis is more on the learning of formulae, algorithms, techniques and applications of theorems rather than on knowledge construction and on fostering critical and logical or innovative thinking. We hardly bring into the classroom the things that are around us that are exemplars of the mathematical concepts that we are trying to get our students to learn so that they can see it live and understand how it even relates to their everyday life or why they should even bother to learn it.

Let us take for example the *Okwe* or *Ayo* game, commonly known as *Mancala*. This game is popularly played in Nigeria and around the world and is a community social networking tool. *Okwe* or *Ayo* does not even require supplies to own as it can be created using two rows of six holes in the ground with 48 pebbles or stones. Yet, our mathematics teachers at the primary, secondary and university level hardly use this highly strategic game as a teaching manipulative to reinforce arithmetic concepts of addition, subtraction, multiplication and division, functional concepts of 1-1 correspondence, algebra concepts of symmetry and rotation, probability and game theory concepts of tree diagrams and decision-making. Given below is a picture that illustrates the community social networking aspect of *Okwe* or *Ayo*.

Figure III: A rural Yoruba village gathering for an Ayo game competition

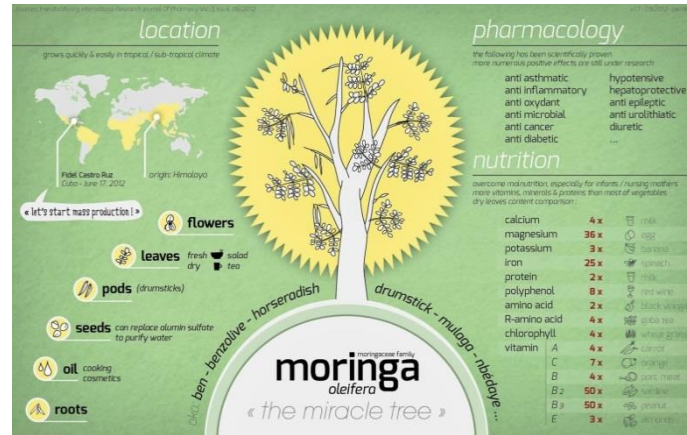


http://www.awale.info/wp-content/uploads/ayo_olopon_01.jpg

When students learn mathematics using concrete tools that are easily accessible to and around them, they begin to gain a deeper understanding of the abstract concepts they are learning and why the formulae, algorithms, techniques and applications of theorems make sense. Innovative pedagogical approaches that use cultural items, including plants that are readily and cheaply available in our environment are needed to help our children at an early age develop an understanding and liking for mathematics. We need to begin to integrate in a concrete way into the formal educational curriculum our cultural heritage that our children are gradually losing and being denied access to the knowledge of such simple everyday products in our environment due to external forces of globalization.

Let's take a look at the traditional medicinal plant *Moringa spp.* (the miracle plant). This plant is found all over Nigeria and in many parts of the world as is indicated in the picture below. It could easily be used as a teaching manipulative to facilitate students' understanding of mathematical concepts related to sequences, series and spanning trees, while simultaneously fostering students' knowledge of its pharmacological and nutritional value. Yet discussions using as our readily available traditional plants as teaching tools hardly take place in our mathematics classrooms even in our poor and rural communities that cannot afford to purchase expensive resources for teaching purposes.

Figure IV: An illustration of a *Moringa spp.* Tree



<http://3.bp.blogspot.com/-YwgnJ81zR4U/UusMtzPrjPI/AAAAAAAAACg/loTCpvU1wmo/s1600/Moringa.jpg>

All people and continents have made significant contributions to STEM, but the contributions of Africans and Africa remain highly unacknowledged in the historical literature on STEM partly due to our past oral traditions of passing down knowledge instead of written materials and documentations. This notwithstanding, efforts must be made to document their contributions to enrich the curriculum in a multicultural and interdisciplinary way, by providing a large repertoire of exemplars borne out of the African socio-cultural milieu, for use in teaching STEM related disciplines.

Our traditional Hausa architecture and traditional Oba's palaces, our Yoruba talking drums (see cover page picture), our traditional Igbo calendar are all tools that we can use for teaching a variety of mathematical concepts. Yet, how many of our children today can look at these symbols of cultural heritage and our cultural artifacts and connect them to the mathematical concepts they learn about in the classroom? How many of our children are even aware of the indigenous science and technology that went into creating these artifacts and heritage symbols?

When we begin to proudly use what our ancestors and fore-fathers gave to us in the formal education of our children, we will also develop in them a high self-esteem for their cultural identity and give them the tools to make their formal education an action of pride and once they have reached higher levels of growth they will become relevant to the needs of their local communities. In so doing, we now bring to the fore-front our cultural heritage and indigenous scientific knowledge in a manner that will promote history and social studies, that will facilitate creativity and innovation; foster international educational exchange and tourism, that will lead to cultural renaissance, that will eventually promote private enterprise of cultural relics and related vocational skills training and employment.

Project Activities in Nigeria

The project activities in Nigeria comprised primarily of the following:

- (i) Curriculum development plenaries and workshops;
- (ii) Study tours of environment sites where traditional plants and flowers could be found and sites of cultural heritage which included interviews with key site curators and personnel;
- (iii) Closed session participant interviews with two Nigerian women leaders in STEM that included observations of samples of their work.

Study Tours of Cultural Heritage and Environment Sites

Study tours were made on days when no workshops or plenaries were taking place to the following environmental sites and sites of cultural heritage during the period of August 2014:

- (i) Palace of the Deji (Oba) of Akure;
- (ii) FUTA Zoo and Botanical Gardens;
- (iii) Smoking Hills Forest in Ilara-Morkin in Ondo State;
- (iv) Ikogosi Warm Springs Forest Reservation in Ekiti State and
- (v) Benin City National Museum and other heritage sites and markets (Igun-Eronmwon Quarters) for cultural artifacts in Benin City in Edo State.

These study tours were for the purpose of researching, examining, observing, taking pictures and collecting or purchasing cultural and traditional items (e.g., Shakere musical instrument, Okwe or Ayo game, Aso-oke cloth), plants and flowers in the local community (e.g., Moringa spp., Eucalyptus) that could be used for developing curricular activities for teaching mathematics from a Nigerian socio-cultural context. A critical observation of the traditional artifacts, plants and flowers which were used by Dr. Agwu to frame the discussion in the plenaries and workshops, indicate they can help to reinforce students' arithmetic skills and their understanding of mathematical concepts related to sequences and series, coloring and counting principles, geometry of plane and solid figures, graph theory and cryptography.

At the Palace of the Deji of Akure, Princess Adetutu Adesida Ojei, interviews were conducted with the High Chief of the Regent, Chief Adebayo who is the SAO of Akure Kingdom, one of the Crown Princes, Prince (Engineer) Adesina and a few other key personnel. They provided a guided tour and a brief historical background related to the

Palace, its past and present monarchs and the history of Akure Kingdom, which included an interesting story on the engineering indigenous knowledge of a past monarch in building a wide dredge that was at least 9 foot deep all around the city with only two heavily guarded bridges from the north and south to enter the city, thereby protecting the city from potential invasion. Unfortunately, the dredge has been mostly covered up due to modernization of the surrounding environment. So, it could not be examined. However, mathematics curriculum on area, perimeter, and volume for a dredge around the Kingdom can be developed with a geographical survey map of the city to tell the story of this great engineering fit. Kindly see below a sample map that could be used for this purpose and also to develop vertex-edge graph activities related to the network system of major roads from Akure Kingdom to its environs.

Figure V: A Map of Akure



http://techloy.com/wp-content/uploads/2013/07/Akure_Google_Map.png

At the Palace of the Deji of Akure, Princess Adetutu Adesida Ojei, several pictures were taken of the splendid geometrical illustrations and symbolism in the palace architecture and cultural artifacts that are used by the Deji. The concept of infinity or a divergent sequence was evident in the real sense in two water holes carved in concrete located in a specific courtyard. Our tour guides indicated that these water holes can never be filled up with water even though they have no drainage area out of them. Below is a picture of these water holes for which curricular activities related to circumference, perimeter, area, volume, pi, sequences and series can be developed.

Figure VI: The “infinity” water holes in the palace of the Deji of Akure



The findings from the guided tour of the Palace of the Deji of Akure are currently being used to develop mathematics curriculum for teaching a variety of concepts in geometry, number theory and graph theory. Some highly mathematically significant items on Akure Kingdom purchased by Dr. Agwu where the genealogy Tree Diagram and Path of the Dejis of Akure which can be used for teaching concepts in graph theory, genealogy and about contact tracing in epidemiology.

Participant Interviews With Nigerian Women in STEM

Closed session participant interviews were conducted by the Carnegie Fellow, Dr. Nkechi Agwu with two Nigerian women leaders in STEM actively engaged in community and rural development. These participant interview sessions served the dual purpose of professional development sessions in which the interviewees were prepared on how their work can be used to teach concepts in mathematics and how to prepare mathematics educators to use it in their teaching. Given that the time frame of availability of Dr. Agwu was limited and that many things had to be accomplished to meet the goals of the project during the Carnegie Foundation funded period, the two women chosen to be interviewed were taken from the pool the project collaborators. They were Dr. Mojisola Edema and Dr. Olayinka Ogunsuyi. Both women were interviewed about their life, in particular their research, teaching, scholarship and community development activities and samples of their research publications and other teaching materials were reviewed for discovery of the mathematical concepts inherent for use in developing curriculum that will give visibility to their lives and work as Nigerian women leaders in STEM.

Dr. Edema is the CEGIST Director, a FUTA Associate Professor of Food Microbiology and a AWARD Fellow (2009) and Mentor (2013), who is actively involved in Ondo State on a World Health Organization project related to HIV/AIDS education and outreach. Dr. Edema's AWARD biography had been interpreted prior by students in Dr. Agwu's

summer II 2014, MAT 100 – Fundamentals of Mathematics I class at BMCC, CUNY, into creations of Ndebele doll robotic representations of her illustrating vertex-edge graphs taken from her NiWARD story. Dr. Edema's participant interview and samples of work led to curricular materials that can be used for teaching statistics in the area of reading and interpreting graphs, charts and correlations. For a full CEGIST staff profile of Dr. Edema kindly see <http://cegist.futa.edu.ng/profile.php?staffid=694>.

Dr. Olayinka Ogunsuyi is the CEGIST Assistant Director and a FUTA Assistant Professor of Analytical Chemistry who is actively involved in the community helping rural women enhance and improve their technology in making Black Soap and mentoring students in this regard. She has just won an AWARD Fellowship that will be announced in 2015, but for this interview she was not yet a NiWARD member, so students in Dr. Agwu's class MAT 100 class had not created Ndebele Doll representations illustrating the vertex-edge graphs in her story. Dr. Ogunsuyi's participant interview and samples of work led to curricular materials that can be used for teaching about mathematical modeling and robotics design in the area of geothermal and wind energy, and reinforcing concepts on optimization, ratios, proportions, percent and solving polynomial equations. For a full CEGIST staff profile of Dr. Ogunsuyi kindly see <http://cegist.futa.edu.ng/profile.php?staffid=728>.

Curriculum Development Plenaries and Workshops

Curriculum development plenaries and workshops were held at FUTA in Ondo State and PANAFSTRAG headquarters in Lagos State in August 2014. In all, there were two plenaries and four workshop sessions at FUTA, and a combined plenary and workshop session at PANAFSTRAG. The plenaries and workshops were well attended. Participants in the question-answer discussion portions indicated an overall general satisfaction and motivation to use the materials developed and/or this pedagogical approach in the classes they teach and/or disseminate information about this project to others who could benefit from it.

A majority of the participants in the four workshops were from the Departments of Mathematical Sciences and Transport Management Technology. There were a few from the Departments of Computer Science, Agriculture, Architecture and General Studies, and from the FUTA Staff Primary and Secondary Schools, showing that at least six departments at FUTA, excluding the Staff Schools were positively impacted.

There are at least four FUTA faculty on ground who have demonstrated that they can take leadership in continuing and moving forward the mission of this project at FUTA and beyond, Dr. Edema and Dr. Ogunsuyi, Director and Assistant Director of CEGIST, respectively, who gave lectures about this at the two plenaries and Dr. Ayodeji and Dr.

Aderibigbe, faculty of the Mathematical Sciences Department and General Studies Department, respectively, who co-led the fourth workshop with Dr. Agwu.

The first plenary at FUTA was an internal one primarily for the FUTA community. It was held on August 21, 2014 and attended by at least 50 faculty and staff from a variety of departments at FUTA, some representatives of the FUTA Staff Primary and Secondary Schools, a representative from the Akure office of the Nigerian Mathematical Center, and two representatives from the Ondo State Ministry of Education, Akure. The main lectures were given by Dr. Nkechi Agwu, the Carnegie Fellow, and Dr. Olayinka Ogunsuyi, the Assistant Director of CEGIST, with remarks from the Nigerian Mathematical Center brought on behalf of the Director, Professor A. R. T. Solarin by the Akure representative, Mr. Oloda Smart. Dr. Agwu and Dr. Ogunsuyi presented the project from different perspectives. Dr. Agwu's lecture focused on the BMCC, CUNY arena - what she had been doing using the African socio-cultural milieu and the stories of African women in STEM to teach courses in mathematics. Dr. Ogunsuyi's lecture focused on her research and teaching activities in geothermal and wind energy and how these could be used to develop curriculum for teaching about mathematical modeling, robotics design, ratios, proportions, percent, optimization, and solving polynomial equations.

The second plenary at FUTA was extended to the external community in Ondo State and beyond. It was held on August 26, 2014. It was attended by at least 50 persons. The main lectures were given by the Carnegie Fellow, Dr. Nkechi Agwu and the Director of CEGIST, Dr. Mojisola Edema, with remarks from the Vice Chancellor's office brought on behalf of the Vice Chancellor, Professor Adebisi Daramola by the Deputy Vice Chancellor of Academics, Professor Adebayo Fasakin, and remarks from the Nigerian Mathematical Center brought on behalf of the Director, Professor A. R. T. Solarin by the Akure representative, Mr. Oloda Smart. Each of the lectures focused on the project from different perspectives. Dr. Edema's lecture focused on the broader CEGIST FUTA perspective and Dr. Agwu's lecture focused on the broader national perspective, with both presenter's also using a similar approach as in the first plenary where Dr. Agwu discussed the genesis of the project and its incubation at BMCC, CUNY, while Dr. Edema also discussed her research and teaching activities and how they could be used for teaching concepts on reading and interpreting graphs, charts and correlations and preparing educators to use stories from the work of Nigerian women in STEM to teach mathematics. For a report of the second plenary, kindly see <https://www.futa.edu.ng/futacms/fileuploads/Mathematics%20on%20at%20the%20City%20University%20of%20New%20York110.pdf.pdf>.

The four workshop sessions at FUTA were held on August 27, 2014. The purpose of the workshops were to provide in-depth professional development to colleagues in the FUTA community who were interested in developing and/or teaching with curricular based on the Nigerian socio-cultural milieu and the stories of Nigerian women in STEM. The

workshops were as follows: Workshop I – Cipher Systems and Cryptography, Workshop II – Vertex-Edge Graphs and Maps, Workshop III – Number Theory, Probability, Combinatorics, Symmetry and Geometry, and Workshop IV – Problem-solving and Mathematical Modeling. Workshop I – III were led by the Carnegie Fellow, Dr. Agwu, and each workshop had at least 12 participants. The closing Workshop IV was co-led by Dr. Agwu and two FUTA faculty members, Dr. Afolabi Ayodeji of the Mathematics Department and Dr. M. O. Aderibigbe of the General Studies Department. It had at least 20 participants, including the CEGIST Director, Dr. Edema who gave closing remarks and moderated the question-answer session and other CEGIST collaborators and staff.

In Workshop I, participants discussed traditional Yoruba ciphers such as Aroko which is used to announce the coronation of an Oba, the talking drum and so on. They worked on activities showing how an Aso-oke and a beaded necklace, items whose production fall under traditional women's work, could be used to send coded messages. They learned Hue-Cryptography which is a cipher developed by one of the CSTEP mentees, Michael Villalona, and Dr. Agwu. It can be used to produce an Aso-oke that carries a coded message.

In Workshop II, participants discussed the Tree Diagram and Path of the Dejis' of Akure and how activities of this nature could be used to teach students about genealogy, contact tracing in epidemiology and get students interested in documenting their family trees and learning about their family genetic history for early detection and containment of genetically transmitted diseases such as diabetics, sickle cell anemia and so on. Recently, Nigeria did a marvelous job with containment of the spread of the *Ebola* virus in part, primarily due to contact tracing. Participants engaged in discussions of how this type of genetic mapping project could be tied to the doll project as a new-born baby whereby in now includes aspects of financial literacy in terms of costs of caring for a new-born and aspects of health and nutrition. Participants also discussed traditional architecture, such as that of the palace of the Oba of Benin and Deji of Akure and what maps and associated vertex-edge graphs of traditional palaces show about indigenous knowledge on warfare, safety and security. They worked on activities related to coloring of the floor plan of the palace of the High Chief Iyase the Younger of Benin, determining the chromatic number, creating the associated vertex-edge graph and interpreting what it shows about warfare techniques and safety and security. They also engaged in activities of story-telling about their lives or the lives of others through the creation of Ndebele dolls with vertex-edge graphs representations. This served as another avenue of blending in culture as some participants created dolls representing traditional rulers who they admired or who inspired them.

In Workshop III, participants examined traditional games and artifacts for indications of how they could be used to reinforce students understanding of concepts in number theory, probability, combinatorics, symmetry and geometry. Specifically, they engaged in

activities related to the Okwe or Ayo game and traditional plants and flowers. Activities related to the game included using the game board for story-telling about the lives of Nigerian women in science or rural women's knowledge of indigenous science and technology, teaching students to create and interpret decision tree diagrams of the game at a certain point, looking at symmetry patterns in the game board. Activities related to traditional plants and flowers included formulating the sequences and series represented in the pattern of the branches, leaves and/or petals.

In Workshop IV, participants engaged in problem-solving and mathematical modeling involving robotics design. The activities from the lives of Dr. Edema in reading and interpreting graphs, charts and correlations and Dr. Ogunsuyi in mathematical modeling for the feasibility of geothermal energy or optimization of the energy production for wind turbines were discussed. Each participant was provided with a variety of supplies and resources for robotics design to create a model that they could use in teaching students about some concept in their discipline and to discuss their model, the basis of their choice model and how they would use it for teaching purposes with the entire group.

The last combined plenary and workshop session was held on August 29 at the PANAFSTRAG Headquarters in Lagos. It was attended by at least 15 participants from different schools in Lagos State and neighboring states. The main lectures were given by the Carnegie Fellow, Dr. Nkechi Agwu, Executive Member of PANAFSTRAG and Chair of the Board of CEGIST, Dr. Stella Williams, and the Chairman of the Mathematical Association of Nigeria (MAN) in Lagos, Mr. Razaq Jimoh. Mrs. Arinola Bello, PANAFSTRAG's Program Coordinator gave opening and closing remarks and also introduced the various presenters.

The focus of Dr. Agwu's lecture was to share the curricular activities that had been expanded upon at FUTA (kindly refer to the activities discussed for the earlier plenaries and workshops). Her presentation also brought those activities closer home, Lagos State, by using a few illustrations from the Lagos cultural milieu. She showed how the beads and crown in the traditional wear of the Oba of Lagos, Oba Rilwan Akiolu, his sword, and the symbols of traditional plants and flowers on his ceremonial chair could be used to reinforce students understanding of sequences, series, paths, cycles, wheels, and other mathematical concepts in geometry, graph theory and number theory. Kindly see the picture below of the Oba of Lagos, Oba Rilwan Akiolu, that was used for demonstration purposes.

Figure VII: The Oba of Lagos in traditional attire seated on his ceremonial chair



http://25.media.tumblr.com/tumblr_lu1687PUHj1qcv93ko1_500.jpg

Mr. Jimoh's lecture focused on the big picture related to students' dismal performance in mathematics and the importance of the MAN in professional development of teachers. An interesting outcome of Mr. Jimoh's lecture was teaching participants the traditional efficient finger method used by the Yoruba's for two digit multiplication.

Dr. William's lecture focused on teacher education and the significance of this type of pedagogical approach for teaching that builds upon our indigenous knowledge systems. For more information, kindly review the accompanying report of this session. As a consequence of this session, the project had a wider reach beyond FUTA and Ondo State to schools in the South Western Region of Nigeria. For a copy of the PANAFSTRAG report of this session, kindly see <http://www.panafstrag.org/pages/sections.php?sid=78>.

Samples of Curricular Activities Developed

Provided below are four brief samples of curricular activities developed: Activity I - Akwete Cloth Activity, Activity II - Moringa spp. Activity, Activity III - Oba of Lagos Activity and Activity IV - NiWARD Women Ndebele Doll Activity. Activity I blends cultural wear and rural women's work. Activity II focuses on the natural environment from the perspective of traditional medicine and food. Activity III focuses on traditional rulers. Activity IV focuses on stories of Nigerian women leaders in STEM.

Activity I: Akwete Cloth Activity

Figure I is a picture of a rural Igbo woman weaving a traditional Akwete cloth. In the partially completed woven cloth:

1. Identify the geometric shapes displayed?
2. Identify the different types of symmetry evident?
3. Identify the number patterns that you see?
4. Find a pattern that reflects an Arithmetic Sequence, state the first term and the common difference?

Activity II: Moringa spp. Activity

Figure IV provides an illustration of a Moringa spp. tree. Based on a detailed observation of the patterns in the branches and leaves of this tree:

1. Construct a tree diagram that reflects the pattern in the branches.
2. Construct a sequence that reflects the pattern in the leaves.
3. Construct the associated series for the sequence that you constructed.

Activity III: Oba of Lagos Activity

Figure VII is a picture of the Oba of Lagos in traditional attire seated on his ceremonial chair. Study the picture.

1. Identify a vertex-edge graph that is a wheel, and determine its number of vertices, number of edges and Chromatic number.
2. Identify a vertex-edge graph that is an even cycle and determine its Chromatic number.

Activity IV: NiWARD Women Ndebele Doll Activity

Watch the African Views Organization ACE youtube video on how to make an Ndebele doll at <http://www.youtube.com/watch?v=HamUbtroHcA>. Using the directions given for how to construct Ndebele dolls:

1. Construct an Ndebele doll to represent a NiWARD woman whose biography you have read. Prepare to bring in your doll to present it to the class;
2. Decorate your doll to reflect two vertex-edge graphs you see in the NiWARD woman's story and identify the standard name of the two vertex-edge graphs if possible;
3. Color the two vertex-edge graphs for their Chromatic number, state their Chromatic number, and explain why you cannot color them with a number of colors less than what you claim is the Chromatic number;
4. Use counting techniques and principles to count and state the number of vertices and edges of the two vertex-edge graphs on your doll and create a

- table showing the graph, its name, its Chromatic number, and the number of its vertices and edges.
5. Develop an accompanying power-point presentation on the life of this NiWARD woman that presents the two vertex-edge graphs you represented on your doll, their chromatic number, the number of vertices and edges in each graph, discusses why you selected those two vertex-edge graphs over the other types that are evident in the biography of the woman, and discusses what aspects of the woman's life inspires you.

Conclusion

Overall, phase I of this project was successful in achieving the desired outcomes, particularly in terms of local, national and international impact and raising awareness. During the period of July - September, 2014, the project received a lot of local, national and international media publicity, due to its uniqueness, significance and creativity in addressing areas of need, viz., gender and class equity in STEM education and cultural renaissance. Nigerian Women on the Move (NWOM) Magazine did an extensive coverage of this project on their Facebook page on August 14, 27, 29 and 30, 2014, due to the alignment of this project in moving forward the mission of NWOM and its parent body Elect Her Appoint Her (EHAH). Kindly see <https://www.facebook.com/Nigerianwomenonthemove>. The national newspaper, Nigerian News Direct, featured the project and the Carnegie Fellow, Dr. Nkechi Agwu in their September 1, 2014, edition. The external plenary session on August 26 was covered by Radio Nigeria. CUNY featured this project and the Carnegie Fellow, Dr. Nkechi Agwu in its September 30, 2014, *Newswire* article, *Making Math More Meaningful*, kindly see <http://www1.cuny.edu/mu/forum/2014/09/30/making-math-more-meaningful/>.

As a testimony of the success of this project in meeting the goals of the Carnegie Foundation with moving forward its mission for African Higher Education and brilliantly giving local, national and international visibility to CADFP, the Carnegie Fellow, Dr. Nkechi Agwu was one of the five fellows selected to present at the Carnegie African Diaspora Fellows Roundtable at the 57th annual African Studies Association Conference in Indianapolis, Indiana, November 22, 2014. The roundtable presentations by the five selected fellows sharing their CADFP insights, experiences and lessons learned, moderated by Dr. Paul Zeleza, Chair of the Advisory Council and Vice President for Academic Affairs and Professor of History at Quinnipiac University.

The Carnegie Fellow, Dr. Nkechi Agwu, her fellow colleagues at CEGIST, FUTA, led by Dr. Mojisola Edema, Dr. Olayinka Ogunsuyi, Ms. Olabukunola Williams and her fellow collaborators at PANAFSTRAG, led by Dr. Stella Williams and Mrs. Arinola Bello, are

continuing to plant, grow, harvest and disseminate the seeds and fruits of this project throughout the primary, secondary and tertiary educational system in Nigeria and the United States. PANAFSTRAG through its MSTIIS Program has taken a leadership role in Nigeria holding brainstorming meetings and workshops with a diverse group of educational stakeholders on various avenues to foster mathematics, science and technology for the innovation of indigenous knowledge systems. Other stakeholder organizations in Nigeria committed to supporting the continuation of this project are the Nigerian Mathematical Center, Nigerian Women on the Move Magazine, and Graceland Primary and Secondary Schools of the Anglican Diocese in Gusau, Zamfara State. It is hoped that this expository article will help to pave the way for replicating this project in other countries in Africa and sourcing for future funding to continue the work on this project to have a wider reach beyond the south western region of Nigeria. Work is in-progress in developing a manuscript for publication of the curricular activities developed so that mathematics educators interested in this type of pedagogy have readily available activities that they can utilize for teaching and learning of mathematics.

References

1. *Carnegie African Diaspora Fellowship Program*, <http://www.iie.org/Programs/Carnegie-African-Diaspora-Fellows-Program>, Institute for International Education, Washington D.C.
2. Paul Zeleza, *Engagements between African Diaspora Academics in the U.S. and Canada and African Institutions of Higher Education: Perspectives from North American and Africa*, Carnegie Corporation of New York, February 2013.
3. *Breaking News 8/7/14: Carnegie African Diaspora Fellowships to Support 31 Projects in Africa; Dr. Nkechi Agwu Selected to work with FUTA, Akure, Nigeria*, <http://nigeriamasterweb.com/Masterweb/breakingnews-8714-carnegie-african-diaspora-fellowships-support-31-projects-africa-dr-nkechi-agwu>, Nigeria Master Web, July 8, 2014.
4. *Diaspora Fellowships to support projects in Africa*, <http://www.africasti.com/profile/diaspora-fellowships-to-support-projects-in-africa>, AfricaSTI, July 10, 2014.
5. *Center for Gender Issues in Science and Technology (CEGIST)*, <http://cegist.futa.edu.ng/page.php?pageid=79>, Federal University of Technology, Akure.
6. *PSC-CUNY Regular Competition Awards: Mathematics*, <https://www.rfcuny.org/rfwebsite/research/AwardedPersons.aspx?programID=PSCREG&sequence=32&desc=Mathematics>, Research Foundation of the City University of New York, New York.

7. *Making Math More Meaningful*, <http://www.bmcc.cuny.edu/news/news.jsp?id=11273>, Borough of Manhattan Community College, City University of New York, New York, July 22, 2014.
8. *Dr. Nkechi Agwu - Nurturing African Girls for a Career in STEM*, <http://www.youtube.com/watch?v=CdWar62RrwE>, Drammeh Institute, New York, September 23, 2014.
9. *NiWARD*, <http://www.niward.org>, Federal University of Technology, Akure.
10. *Federal University of Technology, Akure*, <http://www.futa.edu.ng/futacms/index.php>, Federal University of Technology, Akure.
11. *Pan-African Strategic Policy and Research Group, 2014-2016 On-going Projects*, <http://www.panafstrag.org/pages/sections.php?sid=76>, Pan-African Strategic Policy and Research Group, Lagos.
12. McPhilips Nwachukwu & Appolos Oziogu Ibebabuchi, *Akwete Cloth: An Igbo Textile Art*, <http://www.vanguardngr.com/2012/07/akwete-cloth-an-igbo-textile-art>, Vanguard Newspaper, July 26, 2012.
13. Paulus Gerdes, *Geometry From Africa, Mathematical and Educational Explorations*, Mathematical Association of America, Washington D.C., 1999.
14. *Africa: An Akwete Igbo Weaver at her Loom*, <http://media-cache-ak0.pinimg.com/736x/cc/f5/39/ccf5395077762e13b1cac0594c6d6fb3.jpg>, Photo by Lisa Aronson in *Art and Life in Africa* at <http://africa.uima.uiowa.edu/about/>, University of Iowa Museum of Art, Iowa, 2014.
15. *How to Make Black Soap*, <http://www.wikihow.com/Make-Black-Soap>, WikiHow, Palo Alto, California.
16. *Information on African Black Soap*, <http://greenerchiro.com/African%20Pictures/Making%20black%20soap2.jpg>, Greener Chiropractic, Broken Bow, Oklahoma.
17. "Ayo Olopon" *Games From South-Western Nigeria*, http://www.awale.info/wp-content/uploads/ay_o_olopon_01.jpg, Awale.Info Online Magazine at <http://www.awale.info/los-juegos-ayo-olopon-en-nigeria-en-el-siglo-xxi/?lang=en>, September 13, 2009.
18. *Health Benefits of Moringa Plant (Moringa Oleifera)*, <http://3.bp.blogspot.com/-YwgnJ81zR4U/UusMtzPrjPI/AAAAAAAAACg/loTCpvU1wmo/s1600/Moringa.jpg>, at <http://yemiagbula.blogspot.com/2014/01/health-benefits-of-moringa-plant.html>.
19. *Map of Akure*, http://techloy.com/wp-content/uploads/2013/07/Akure_Google_Map.png, Google Maps, 2013.
20. *Edema Mojisola (Basic Profile)*, <http://cegist.futa.edu.ng/profile.php?staffid=694>, Federal University of Technology, Akure.
21. *Ogunsuyi Olayinka (Basic Profile)*, <http://cegist.futa.edu.ng/profile.php?staffid=728>, Federal University of Technology, Akure.
22. *Mathematics Permeates all Human Endeavors*, <https://www.futa.edu.ng/futacms/fileuploads/Mathematics%20don%20at%20the%20>

- [City%20University%20of%20New%20York110.pdf.pdf](#), *FUTA News*, August 26, 2014.
23. *Imperial Ruler and King of Lagos, His Royal Majesty, Oba Rilwan Akiolu*, http://25.media.tumblr.com/tumblr_lu1687PUHj1qcv93ko1_500.jpg at <http://www.niajarchives.com>.
24. *Pan-African Strategic Policy and Research Group Report of Teachers Interactive Session August 2014*, <http://www.panafstrag.org/pages/sections.php?sid=78>, [Pan-African Strategic Policy and Research Group, Lagos](#).
25. ACE for Kids – Making Ndebele Dolls, <http://www.youtube.com/watch?v=HamUbtroHcA>, African Views, New York, June 3, 2011.
26. *Nigerian Women on the Move Magazine Facebook Page*, <https://www.facebook.com/Nigerianwomenonthemove>, [Nigerian Women on the Move Magazine, Lagos, August 14, 27, 29, and 30, 2014](#).
27. *Making Math More Meaningful*, <http://www1.cuny.edu/mu/forum/2014/09/30/making-math-more-meaningful/>, *CUNY Newswire*, City University of New York, New York, September 30, 2014.

A Glimpse into the Effectiveness of Mentoring and Enrichment Activities for Scholarship Recipients in a Teacher Preparation Program

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Abstract:

The extent and nature of mentoring determines the effectiveness and success of any educational program [1]. A multi-faceted mentoring scheme has been designed for the TAMIU Robert Noyce Mathematics Teacher Scholarship Program (TAMIU-NMTSP) at Texas A&M International University (TAMIU), Laredo, Texas. The TAMIU-NMTSP provides funding for up to \$10,000 per year for students who are planning on becoming secondary mathematics teachers. Students who receive the scholarship will graduate from TAMIU with a mathematics degree and 7-12 mathematics certification. In exchange for receiving the TAMIU-NMTSP Scholarship, the recipients agree to teach as highly-qualified, full-time teachers in a high-need subject area for at least four years at a school serving low-income students. As part of the scholarship students receive additional mentoring from university faculty and other professionals in the field. This paper will outline this mentoring scheme as well as examine its preliminary effectiveness.

Introduction

There is a critical need to increase participation in STEM fields by underrepresented minorities, as they were over a quarter of the national population (29%) in 2006 but only 9% of college-educated STEM occupations [2]. This National Research Council report further details this alarming trend, showing that in 2007, underrepresented minorities made up of 39% of K-12, 33% of college age population, 26% of undergraduates, 18% of science and engineering (S&E) bachelor's degrees, 18% in graduate school - but only 15% of S&E master's degrees and a miniscule 5% of S&E doctorates. Retention of minorities in STEM programs is increased when there are intensive mentoring programs in place [3 & 4]. Independent School Districts (ISDs) have not consciously adopted and worked toward the broad goals for STEM education in terms of setting intermediate goals for student success, increased enrollment in STEM courses, test scores, high school

graduation rates, college or career readiness, and advancing to postsecondary institutions. In the k-12 setting, students need to have highly qualified STEM teachers who are able to provide quality mentoring. Teacher preparation programs and undergraduate mathematics programs play a significant role in ensuring highly qualified teachers are in the k-12 classroom. The TAMIU Robert Noyce Mathematics Teacher Scholarship Program (TAMIU-NMTSP) at Texas A&M International University (TAMIU), Laredo, Texas provides funding up to \$10,000 per year for students to graduate as certified secondary mathematics educators who will in return agree to teach as highly-qualified, full-time teachers in a high-need subject area for at least four years at a school serving low-income students. This paper seeks to examine the preliminary effectiveness of the mentoring component of the TAMIU-NMTSP.

Literature Review

The benefits of mentoring have emerged as an important finding in the literature from the structured analysis of more than 300 research-based articles across the areas of education, business, and medicine [5]. This analysis finds that mentoring has enormous potential to bring about learning, personal growth, and development for professionals [6]. The characteristics of any mentoring program require developing a strong mentor-mentee relationship in an educational program, as well as activities associated with building a strong foundation of skills and an assessment method to see if the program objectives are achieved. The following examples illustrate that desired outcomes can be developed when highly regarded and experienced teachers collaborate to design and implement mentoring programs in their schools. Feiman-Nemser et al. (1992) examined how the contexts of mentoring shape the perspectives and practices of mentors [7]. A similar TAMIU NSF-Funded CSEMS Scholarship Program includes dedication to mentoring and academic atmosphere, selection of mentors, characteristics of effective mentoring relationships, and mentor training/professional learning. The benefits of mentoring are also seen through student accounts of what helps them most from a strong and effective mentoring program. Additionally, the benefits of mentoring as seen from the student perception are very useful tools in developing the "best" practices and standards in dealing with mentoring. For example, the programs with qualified staff working along with experts and sufficient staffing so they are not stretched between too many duties and other job responsibilities. Jones (2001) undertook a comparison and evaluation of procedures in mentoring with reference to the cultural and structural framework determined in each setting. Protégés of mentors participating in the mentoring program could more effectively organize and manage instruction at the beginning of the year and establish for more functional classroom routines [8]. Another component of mentoring includes carefully constructed activities for the students to participate in throughout the year [9]. The literature also describes the "Situational Mentoring Framework" which must be in place for developing a successful mentoring program. This set-up follows a systemic approach in addressing four major components of the mentoring process to achieve overwhelming results: 1) mentor selection, 2) mentor and novice teacher

preparation, 3) support team, and 4) accountability through assessment [10]. In addition, another study examines the impact of intensive mentoring as an inductive program component aimed at improving teacher quality in ways that link teaching to student engagement, and the effects of best teaching practice. This helps beginning teachers develop balanced instruction as the fiscal situation in school districts created challenges to studying the long-term effects on quality and retention [11].

There is no argument that the face-to-face interactions between advisors/mentors and mentees are a key component in the development of successful self-directed learning habits [12 & 13]. Recognizing the differences between students' learning strategies and seeking to create an environment that embraces elements of critical and reflective thinking, self-direction, autonomy, creativity, and practice for both the mentor and mentee candidates is what effective mentors must do [13 & 14]. One necessary aspect of mentoring teacher candidate students is the preparation for the realities of dealing with teaching standards and statewide assessments. Despite the development of standards-based reforms and statewide assessments, new teachers were found to have received little or no guidance about what to teach or how to teach it [15]. Left to their own devices, they struggle day to day to prepare much needed content and materials. As such, there is an urgent need to reconsider the curricula and support provided to new teachers beyond what they receive from ISDs [15]. Throughout their own perceptions and from dealing with the school cultures, students' views reveal the interplay between contextual, cultural and biographical factors affects their teaching practices [16]. The relative stability of professional identities which teachers develop in the early years of teaching make them stronger, thus the kinds of teachers they will become and their effectiveness can be strongly influenced by early teaching experiences [16]. Therefore, providing as much mentoring support as possible for new teachers, both through face-to-face interaction is critical to their success.

TAMIU-NMTSP Mentoring Program

The TAMIU-NMTSP is a multi-faceted system in preparing undergraduate mathematics educators for their profession. As first year mathematics teachers are dealing with the inherent difficulties of making the transition from college student to becoming an educator [17 & 18] increasing the mentoring resources to help them adjust and deal with mathematics and teaching issues is very beneficial [19 & 20]. Therefore, there are several levels of mentoring integrated into this system.

First, to ensure that beginning mathematics teachers have the level of support they need to effectively deliver mathematics instruction each scholar is assigned a mentor from the College of Arts and Sciences as well as from the College of Education. Each scholar has the opportunity seek out his mentors in order to receive feedback on issues of pedagogy as well as assistance on issues and concerns that are content-specific. The mentors meet

with their scholars at least three times a semester during the program of study. During these meetings, they are encourage to advise students on academic needs, field experiences, balancing of workloads as well as other topics. The goal is for the scholar to develop a long-term relationship with the mentor. The second level of support is through the form of scholarship. The students are encouraged to attend and present at national conferences, professional development workshops, and obtain journal memberships. This component of mentoring provides the scholars with authentic learning experiences as well as provides possible networking opportunities for the scholars. The third level of support is through authentic learning experiences. Accordingly, four scholars and an administrator in the program had an opportunity to present at the Western Regional Noyce Conference (WRNC) in San Francisco, CA in November 2014. This provided an enormous opportunity for them to see what other enrichment activities programs undertake to achieve these programs goals. The scholars are required to attend the TAMIU-Summer Mathematics Boot Camp (TAMIU-SMBC) and the TAMIU-Mathematics Internship Program (TAMIU-MIP). The TAMIU-SMBC, a week-long program, has a wide range of mentoring experiences from experts in the field with regard to content and technology and experience in secondary education teaching. There is a session specially designed to exchange ideas about classroom experience and a panel discussion is focused on “What do Mathematicians do?” The TAMIU-MIP is a summer internship experience tailored towards learning how to effective delivery of classroom instructions and developing classroom management skills. Both programs provide ample opportunities for prospective student teachers to develop their educational skills under the supervision and advising of mentors specializing in mathematics education. The final component of the TAMIU-NMTSP mentoring program will occur upon the scholar’s graduation. Each scholar will be mentored by the classroom teacher who will guide them to success in their teaching profession. In addition, their TAMIU mentor will continue to mentor the scholar, now alumni, and will visit their classroom twice a semester to monitor the teacher’s instruction, classroom management, and discipline techniques to provide constructive and supportive feedback. Resource sharing and cohort support between Noyce scholars and alumni will be added to the proposed mentoring relationship for the new teachers during their first year. A detailed written reflection on participants’ experiences during the beginners' first year of teaching in the classrooms is needed when the participants step into classrooms after the program of study. Using their responses as data, the nature and value of the buddy-mentoring relationship can be analyzed in relation to the needs, concerns, and professional development of the new teachers as they progress throughout the year [21]. Table 1 provides the expected number of students per cohort, a schedule of the Noyce activities for each type of student and the mentoring activities planned.

Table 1. Schedule of Completion of Program Components and Student Cohorts

Academic Student	Year: number,	Summer (Conference Travel	Fall and Spring Semesters for juniors and seniors
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type, cohort group	is for juniors or seniors)	(all activities will happen each semester, except the integrated course which will be spring semester only)
Year 1: 2013-2014 9 rising Juniors (1)	TAMIU-SMBC† (juniors) Conference Travel	Mentoring, Professional Development Workshops, Journal membership; TExES Review; TExES Exam (seniors); integrated mathematics and education course
Year 2: 2014-2015 9 graduating Seniors (1) 9 rising Juniors (2)	TAMIU-SMBC† (juniors) TAMIU-MIP± (seniors) Conference Travel	Mentoring, Professional Development Workshops, Journal membership; TExES Review; TExES Exam (seniors); integrated mathematics and education course
Year 3: 2015-2016 9 graduating Seniors (2) 9 rising Juniors (3)	TAMIU-SMBC† (juniors) TAMIU-MIP± (seniors) Conference Travel	Mentoring, Professional Development Workshops, Journal membership; TExES Review; TExES Exam (seniors); integrated mathematics and education course
Year 4: 2016-2017 9 graduating Seniors (3) 9 rising Juniors (4)	TAMIU-SMBC† (juniors) TAMIU-MIP± (seniors) Conference Travel	Mentoring, Professional Development Workshops, Journal membership; TExES Review; TExES Exam (seniors); integrated mathematics and education course
Year 5: 2017-2018 9 graduating Seniors (4)	TAMIU-MIP± Conference Travel	Mentoring, Professional Development Workshops, Journal membership; TExES Review; TExES Exam (seniors); integrated mathematics and education course

† TAMIU-SMBC: TAMIU-Summer Mathematics Boot Camp

± TAMIU-MIP: TAMIU-Mathematics Internship Program

Data, Analysis, and Results

Feedback received from both mentors and mentees of the first cohort of scholars were collected and analyzed for all ten Noyce scholars currently in the program. Figure 1 depicts the scholars' general assessment of the TAMIU Noyce mentoring program. Almost everyone expressed the opinion of this program to be either very successful or successful.

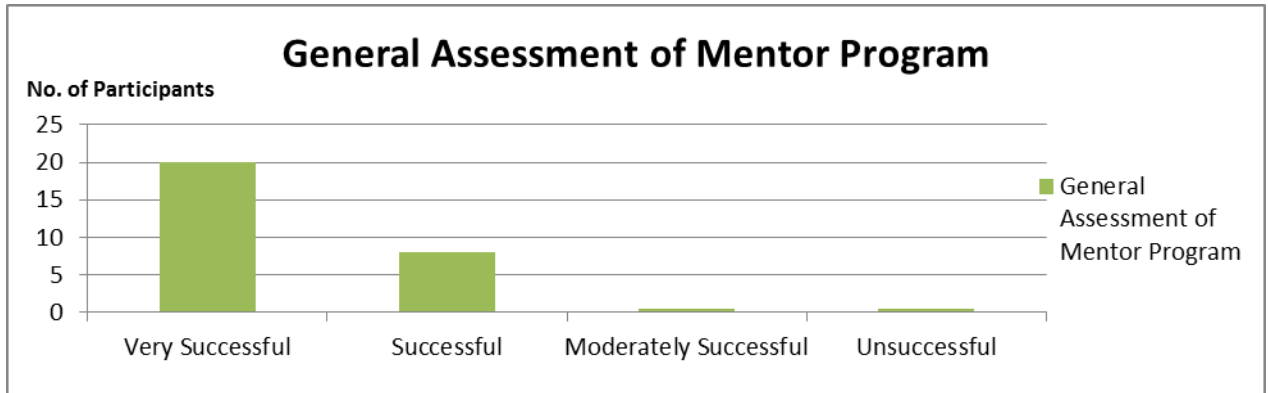


Figure 1. General Assessment of Mentor Program

Everyone, as anticipated, expressed that they have received adequate feedback from these mentor-mentee meetings according to Figure 2. Furthermore, it can be noted from Figure 3 that the satisfaction towards the mentor-mentee match is overwhelming as responded by the participants.

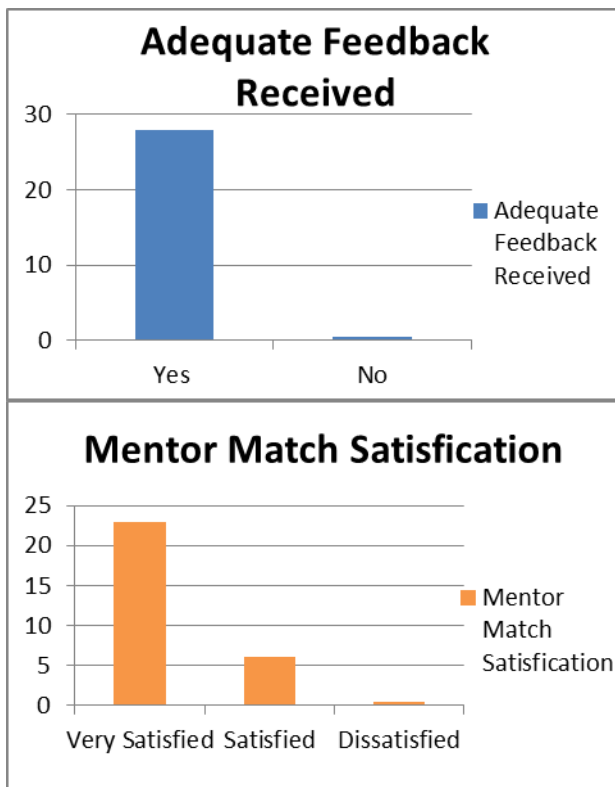


Figure 2 & 3. Adequacy of Feedback from Mentor-Mentee Meetings & Satisfaction towards Mentor and Mentee Match

Assessment on the program components in the mentor-mentee survey by Noyce scholars has been aggregated. The summary of these components appears in Figure 4. Accordingly, the Noyce scholars expressed that the extent of these components is just about right as previously expected.

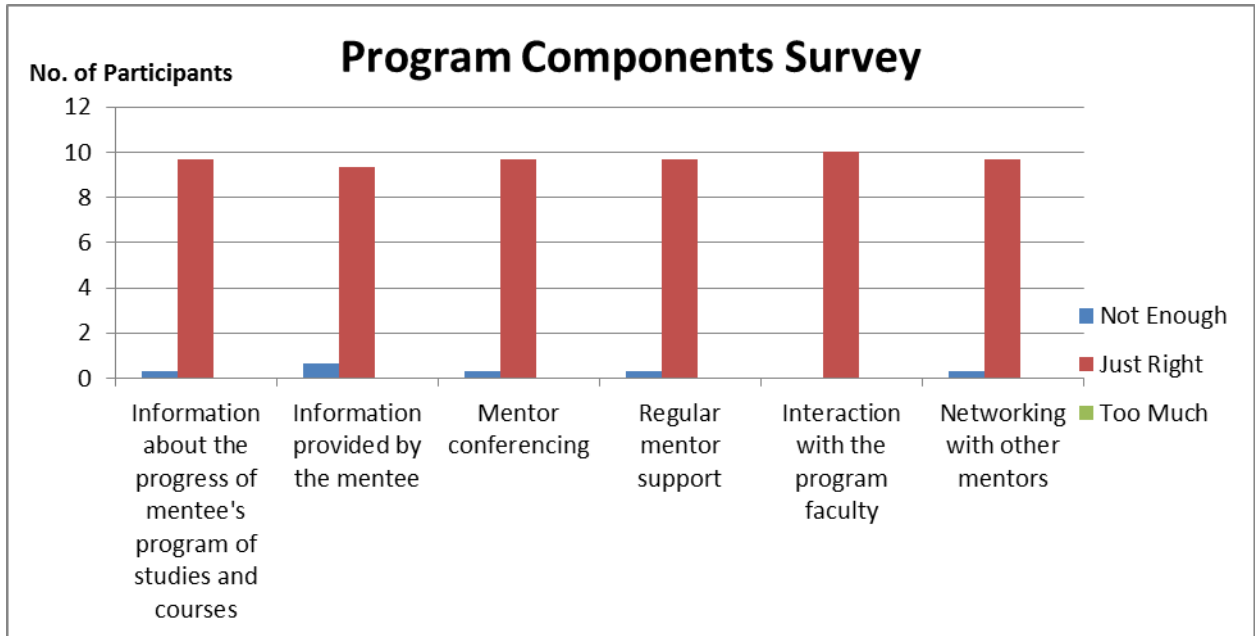


Figure 4. Assessment on Program Components Found in the Survey

Conclusions

This is an ongoing effort and it provides a glimpse of implementation in an early stage. Complete implementation, its success, and its usefulness are yet to be examined in the determination of the effectiveness of the program. However, at this initial stage of the program, all indications are that the mentoring program is solid and will have a positive impact on the preparation of the teacher candidates. As the scholarship program is underway, further analysis is being planned.

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References

- [1] How to Build a Successful Mentoring Program Using the Elements of Effective Practice: A Step-By-Step Tool Kit for Program Managers (2005). MENTOR/National Mentoring Partnership, 2005
- [2] National Research Council. (2011a). A Framework for K-12 Science Education: Practices, Crosscutting Concepts, and Core Ideas. Committee on Conceptual Framework for the New K-12 Science Education Standards, Board on Science Education. Washington, DC: The National Academies Press.
- [3] Schultz, PW; Hernandez, PR; Woodcock, A; Estrada, M; Chance, RC; Aguilar, M; Serpe, RT. (2011). Patching the Pipeline: Reducing Educational Disparities in the Sciences Through Minority Training Programs. *Educational Evaluation and Policy Analysis*, March 2011, Vol. 33, No. 1, pp. 95–114. DOI: 10.3102/0162373710392371
- [4] National Research Council. (2011b). Expanding Underrepresented Minority Participation: America's Science and Technology Talent at the Crossroads. Committee on Underrepresented Groups and the Expansion of the Science and Engineering Workforce Pipeline. Washington, DC: The National Academies Press.
- [5] Ehrich, LC; Hansford, B; Tennent, L. (2004). Formal Mentoring Program in Education and Other Professions: A Review of the Literature. *Educational Administration Quarterly*. Volume 40, Issue 4, p. 518-540.
- [6] Carter, M; Francis, R. (2001). Mentoring and Beginning Teachers' Workplace Learning. *Asia-Pacific Journal of Teacher Education*. Volume 29, Issue 3, 1 p. 249-262(14).
- [7] Feiman-Nemser, S; Parker, MB. (1992). Mentoring in Context: A Comparison of Two U.S. Programs for Beginning Teachers. *National Center for Research on Teacher Learning*.
- [8] Jones, M. (2001). Mentors' Perceptions of Their Roles in Schoolbased Teacher Training in England and Germany. *Journal of Education for Teaching: International research and pedagogy*. Volume 27, Issue 1, p. 75-94.
- [9] Evertson, CM; Smithey, MW. (2000). Mentoring Effects on Protégés' Classroom Practice: An Experimental Field Study. *The Journal of Educational Research*. Volume 93, Issue 5, p. 294-304.
- [10] Kajs, LT. (2002). Framework for Designing a Mentoring Program for Novice Teachers. *Mentoring & Tutoring: Partnership in Learning*. Volume 10, Issue 1, p. 57-69.
- [11] Stanulis, RN; Floden, RE. (2009). Intensive Mentoring as a Way to Help Beginning Teachers Develop Balanced Instruction. *Journal of Teacher Education*. Volume 60, Issue 2, p. 112-122.

- [12] Mynard, J; Navarro, D. (2010). Dialogue in self-access learning. In A. M. Stoke (Ed.), *The Japan Association for Language Teaching 2009 Conference Proceedings*. Tokyo, Japan.
- [13] Galbraith, MW. (2003). Mentoring Toward Self-Directedness. *Adult Learning*, September 2003, 14: 9-11.
- [14] Landefeld, T. (2010). Faculty Advising and Mentoring, Mentoring and Diversity, Mentoring in Academia and Industry. Vol. 4, p. 79-84.
- [15] Kauffman, D; Moore Johnson, S; Kardos, SM; Liu E; Peske, HG. (2002). Lost at Sea: New Teachers' Experiences with Curriculum and Assessment. *Teachers College Record*. Volume 104, Number 2, p. 273-300
- [16] Flores, MA; Dayb, C. (2006). Contexts which shape and reshape new teachers' identities: A multi-perspective study. *Teaching and Teacher Education*. Volume 22, Issue 2, p. 219-232.
- [17] Haggarty, L; Postlethwaite, K. (2012). An exploration of changes in thinking in the transition from student teacher to newly qualified teacher. *Research Papers in Education*. Volume 27, Issue 2, p 241-262.
- [18] Wood, MB; Jilk, LM; Paine, LW. (2012). Moving Beyond Sinking or Swimming: Reconceptualizing the Needs of Beginning Mathematics Teachers. *Teachers College Record*. Volume 114, Issue 8, Article Number 080303.
- [19] Jones, N; Youngs, P. (2012). Attitudes and Affect: Daily Emotions and Their Association with the Commitment and Burnout of Beginning Teachers. *Teachers College Record*. Volume 114, Issue 2, Article Number: 020301.
- [20] Hobson, AJ; Ashby, P. (2012). Reality aftershock and how to avert it: second-year teachers' experiences of support for their professional development. *Cambridge Journal of Education*. Volume 42, Issue 2, p 177-196.
- [21] Ballantyne, R; Hansford, B; Packer, J. (1995). Mentoring beginning teachers: a qualitative analysis of process and outcomes. *Educational Review*. Volume 47, Issue 3, p. 297-307.

Constructivized Calculus: A Subset of Constructive Mathematics

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Abstract

The purpose of this paper is to demonstrate the importance of Constructive Mathematics in today's college mathematics curriculum. In the spirit of the philosophies of LEJ Brouwer and Errett Bishop, a history of constructive mathematics will be presented. Constructive mathematics gives numerical meaning, and quantifies abstract concepts. The main goal of this paper is to identify how constructive calculus, which is based on constructive mathematics, can serve as a tool for engineers, scientists, computer scientists, economists, business majors, and applied mathematicians. Classical or traditional calculus contains many 'existence theorems' which states that a quantity exists but these theorems do not indicate how to find this quantity. The constructive version of the 'existence theorems' describes how to find the quantity and as a result how it can be used for practical purposes.

Introduction to Constructive Mathematics

Constructive mathematics finds its roots in the intuitionist philosophy of Leopold Kronecker and L.E.J. Brouwer. Starting in 1907, Brouwer strongly criticized classical mathematics about its idealism and lacking in numerical meaning. Fifty years later Errett Bishop resurrected the intuitionist philosophy of Brouwer but referred to as the constructivist movement. According to Bishop (1970)

"...It appears then that there are certain mathematical statements that are merely evocative, that make assertions without empirical validity. There are also mathematical statements of immediate empirical validity, which say that certain performable operations will produce certain observable results...Mathematics is a mixture of the real and ideal, sometime one, sometimes the other, often so presented that it is hard to tell which is which..."

Constructive mathematics seeks reliable results of activities that lead to computational manipulations. Therefore the constructivist's role included eliminating the idealism which has come to define the very existence of the traditional mathematics. In order to do this, many

definitions and concepts must be reformulated starting with the existing classical mathematical definitions and concepts. This process is diametrically opposed to starting from void and creating or developing an entirely new branch of mathematics. For example, constructivized calculus relies on the classical calculus. To present a formal system under the intuitionist/constructivist philosophy, a series of finite steps are needed

to derive a numerical result. Although it is necessary to use a finitary process, it is not sufficient. Proofs of theorems must also be presented constructively.

Comparison Between Classical and Constructive Mathematics

Comparisons between classical and constructivized mathematics have focused on the ideal versus the real, with idea of quantifying in constructive mathematics as opposed to merely accepting the existence in classical mathematics. Bishop (1968) gives an elegant difference

“...Constructive existence is much more restrictive than the ideal existence of classical mathematics. The only way to show that an object exists is to give a finite routine for finding it, whereas in classical mathematics other methods can be used...Theorem after theorem of classical mathematics depends in an essential way on the limited principle of omniscience, and therefore not constructively valid...”

An example of an existence theorem in integral calculus is the Mean Value Theorem for Integrals which states:

If f is a continuous on the closed interval $[a, b]$, then there exists a number c in the closed interval $[a, b]$ such that $\int_a^b f(x)dx = f(c)(b - a)$.

The theorem lacks the steps it takes to find the c . Instructors and teachers of calculus must teach students the geometric interpretation of the theorem. In addition, it is necessary to teach students how to find the value of c using the definite integral of $f(x)$; this requires using the functional value to find the independent variable now represented by c .

The constructive version of the Mean Value Theorem for Integrals states:

Let $f(x)$ be continuous on $[a, b] \subset \mathbb{R}$, then for $\forall \varepsilon > 0$, one can find a $x \in \mathbb{R}$ such that $a < x < b$ subject to $|f(x)| < \varepsilon$ and

$$\left| \int_a^b f(t)dt - f(x)(b - a) \right| < \varepsilon \text{ or } \left| f(x)(b - a) - \int_a^b f(t)dt \right| \leq \varepsilon$$

On the contrary, the constructive version of the Mean Value Theorem for Integrals requires constructing a rectangle that has the same length as the interval $[a, b]$ but a height that depends on finding a c_i 's which represents the midpoints of intervals determined by the interval halving process such that the height is equal to $f(c_i)$. Other examples of existence theorems in classical calculus are the Intermediate Value Theorem for Derivatives and Rolle's Theorem.

Importance of a Constructivized Calculus Curriculum

The traditional calculus curriculum in the liberal arts programs emphasizes the two basic operations of differentiation and integration using algebraic and transcendental functions. The calculus definitions, concepts, theorems and principles required to solve these problems include sequences, series, countably infinite sets, continuous functions, least upper bound, convergence, Rolle's theorem, the law of the mean, the intermediate value theorem and the fundamental theorems of calculus, which are idealistic and abstract in nature, lacking a direct process to describe the numerical value. The proofs of the theorems often require a step by step procedure to verify the hypothesis of the conditional

statement but whether there is always a correspondence to the set of real numbers or even the set of integers, has been determined to not be the case. The Constructive proofs must have a finite number of steps whereas the classical proofs do have this restriction. In addition, the Law of Excluded Middle and proof by contradiction cannot be used. These are common techniques in classical calculus. This weakness of the traditional calculus curriculum could be strengthened by implementing a constructivized calculus curriculum.

Even non-mathematics majors enrolled in calculus courses are expected to apply these concepts in various fields. For example, one of the problems of calculus determines the activity of a body in motion whereas other problems involve finding areas, volumes, arc lengths, center of masses, instantaneous speeds and optics. When a problem involves finding the largest/smallest (maximum/minimum) of a function in the form of a mathematical model in the fields social and natural sciences, calculus techniques are used. Students are more concerned with how to find quantities and less concerned with the theoretical basis of the techniques.

The goal of the college mathematics is to provide students with the mathematical skills needed for their chosen fields. These skills include but are not limited to developing quantitative skills, providing conditions to enhance the ability to think clearly, using technology and improving problem solving skills. The general education goals are to foster the development of students' communication skills, learning skills and motivate them to move to higher levels of independent learning. These goals are based on the recommendations from the Mathematical Association of America (MAA), the National Council of Teachers of Mathematics (NCTM) and Society for Industrial and Applied Mathematics (SIAM).

Several majors require students to take calculus courses as part of their programs of study. In this role, the Mathematics Department provides this service. According to MAA, the specific skills that other departments need from the calculus sequence are to serve the needs of these

disciplines through the use of computational techniques. These majors also need the mathematics to help support recent developments in their fields and enhance the use of technology in order to make the appropriate decisions. The MAA report also states,

‘...The increased technological sophistication of the world we live in has created a need for a more mathematically and technologically literate citizenry. Mathematics courses, textbooks, and curricula changed dramatically, during the twentieth century often in response to the physics and engineering of the time. Further changes are needed in the twenty-first century to respond to the needs of the expanding set of disciplines for which mathematics now plays an increasingly important role...’

The report also describes other ways that Mathematics Departments have and can support programs in other disciplines.

‘...At some institutions, mathematics courses to support the programs of partner disciplines are taught within those disciplines... When this is the case, mathematical sciences departments can approach these departments to signal a willingness to be involved in the further development of the course. Such an overture may be especially

welcome when the partner departments are having difficulty staffing the courses. Often the reason the courses were originally developed because mathematicians were not interested or were not teaching the topics in a way or at a level that was appropriate for the students in the partner discipline...'

In most calculus classes, students who need to take the course are majoring in engineering, business, computer science and mathematics (STEM). According to the U.S. National Center for Education Statistics, of the students who received degrees in STEM majors except mathematics ($\approx 22\%$) far outnumber STEM students who received degrees in mathematics ($\approx 0.11\%$). The traditional calculus courses are more appropriate for goals of the mathematics major and only marginally meet the needs of the remaining STEM students. According to the ICMI (2008) study,

'...It would be desirable that service mathematics courses enabled students to acquire a range of essential knowledge, skills, and modes of mathematical thought; each professional activity demands a particular mathematical literacy so mathematics courses must include applications, examples, modeling processes, etc. which motivate students...'

Some of the concepts, theorems, and their proofs in classical calculus lack numerical meaning and are non-computational. Therefore classical calculus is neither as effective nor as efficient a tool as it should be for the business, engineering science or computer science major. In computer science, constructive mathematics is extremely helpful for writing computer programs for use in the real world or to assist in research. It can also be used to determine the center of mass of objects. Constructive mathematics is also of interest to symbolic logicians. Bridger's (2007) point clarifies this position,

'...Not every student in Real Analysis is a math major, and in many schools, only a small percentage of math majors intend to do graduate work in mathematics. A modern course is populated by a wide range of students. Some are headed for careers in secondary education, while there is often a large contingent from the physical sciences and an even larger group from computer science. These students are in the course because they need or want more than a cookbook calculus course. Some need to know more about computability and calculability of floating-point numbers, hence more about the actual nature of the reals. They also need to know about continuity because they need to know about approximations; some need to know about convergence and improper integrals because they need to know about computing special functions and transforms...'

In practice, calculus instructors' solutions to teaching calculus to so many non-mathematics majors is to adapt the curriculum by omitting certain topics or emphasizing procedures and methods while de-emphasizing theorems and their proofs. While this could work in some situations, the rigor of classical calculus suffers. To maintain the rigor of classical calculus while addressing the needs of non-mathematics majors, the constructivized version of calculus can replace it. Constructivized calculus is more general and rigorous than classical calculus. Errett Bishop, the pioneer of constructive analysis, was a liberal constructivist; his approach to constructivization was to only

substitute those statements in traditional mathematics which were non-constructive (i.e. existence theorems and proofs which utilized the Law of the Excluded Middle) with a constructive version. Alsina (2007) discusses the role of mathematics at the collegiate level,

‘...The central importance of mathematics in our technologically complex world is undeniable, and the possibilities of new applications are almost endless. But at the undergraduate level, little of this excitement is being conveyed to our students. Currently, attention is being focused on reforming calculus, the traditional gateway course into the undergraduate curriculum. No one is questioning the importance and beauty of continuous mathematics. However, reformed or not, calculus is one branch (and a highly technical one) of a very rich subject. We know the breadth and richness of our subject; how, then, do we expect the students who are starting their study to gain insights...’

Some students can gain an insight through constructive calculus because it will meet the needs of the technologically complex world for which we are preparing them.

The traditional calculus course can be revised to include the constructivized versions of the definitions of functions and limits, the concepts of continuity, the derivative and the definite integral, and Rolle’s, Intermediate Value, Mean Value Theorems and the Fundamental Theorem of Calculus. Students of calculus should be exposed to the constructive calculus curriculum. The students might face the challenges of understanding the differences between the two versions of calculus, particularly if they took previously took a calculus course. This is best summarized by Taschner (1998)

‘...Although severe differences between the traditional form of mathematics and the constructive access to analysis exist...It is merely to show the students how to switch back to the traditional beaten track...’

The traditional calculus student can use the same approach when shifting back and forth between concepts and theorems which are constructivized to other concepts and theorems which do not require constructivization.

Conclusion

Students majoring in engineering, economics, business, computer science and even physics can benefit greatly from a constructive calculus course. The concepts and theorems of a constructivized calculus would prepare the non-mathematics STEM students to apply these ideas in their respective areas of expertise. Another benefit of a constructive access to calculus is that students gain a feeling of what is really possible in mathematics – and what melts into the shadow of purely formal reasoning (Taschner, 1998). Most importantly, students also gain a deeper consideration of appropriate definitions. The following statement can summarize the significance of a constructivized calculus curriculum (Fletcher, 1998)

‘...It is an advantage of constructivism that it stimulates us to consider alternative mathematical treatments...constructivism permits alternative ways of handling infinity and infinitesimals...’

Several branches of mathematics have been researched and studied to be presented in the constructivist version such as algebra, topology, and analysis. Mathematics has always been a tool for other practical areas such as engineering, economics, business, computer science, biology, and physics, to name a few. The question of using constructive mathematics as a basis has been discussed recently in the works of several mathematicians such as Bridges (1999), Seidenberg (1974), Richman (1982), and Fletcher (2002). Because constructive mathematics emphasizes finding numerical meaning in a finite number of steps it would have a significant role in computers and technology, which defines the twenty-first century.

References

1. Alsina, C.
(2007). Less chalk, less words, less symbols, more objects, more context, more action. In W. Blum(Ed.). *Modelling and applications in mathematics education* (pp. 35-44). Doi:10.1007/978-0-387-29822-21
2. Bishop, E.
(1970). *Mathematics as a numerical language, intuitionism and proof theory*. Amsterdam: North Holland
3. Bishop, E.,
& Bridges, D.S. (1985). *Constructive analysis*. New York: Springer-Verlag.
4. Bishop, E.
(1968). *Foundations of constructive analysis*. New York: Springer-Verlag
5. Bridger, M.
(2007). *Real analysis: A constructive approach*. New York: John Wiley & Sons, Inc.
6. Bridges,
D.S. (1989). A general constructive intermediate value theorem. *Mathematical Logic Quarterly*, 35(5), 433-35.
7. Bridges, D.
S. (1999). Can constructive mathematics be applied in physics? *Journal of Philosophical Logic*, 28(5), 439-449.
8. Bridges,
D.S., & Vita, L.S. (2006). *Techniques of constructive analysis*. New York: Springer Science + Business Media.
9. Brouwer,
L.E.J. (1999). Intuitionism and formalism. *Bulletin of the American Mathematical Society*, 37(1), 55-64.

10. Cruz-Filipe, L. (2003). *A constructive formalism of the fundamental theorem of calculus*. Berlin: Department of Computer Science, University of Nijmegen, The Netherlands, Center for Logic and Computation, Springer-Verlag.
11. Dresden, H., & Loeb, I. (2011). Constructive reverse investigations into differential equations. *Journal of Logic & Analysis*, 3(8), 8-10.
12. Encyclopedia Britannica. Philosophy of mathematics. Retrieved from <http://www.britannica.com/EBchecked/topic/39237/philosophy-of-mathematics>.
13. Fletcher, P.(2002). A constructivist perspective on physics. *Philosophica Mathematica*, 10(1), 26-42.
14. International Commission on Mathematics Instruction ICMI-2008. Retrieved from www.icmi2008.org.
15. Larson, R. Hostetler, R., & Edwards, B. (1994). *Calculus with analytic geometry-alternate edition*. Massachusetts: D. C. Heath and Co.
16. Richman, F.(1982). Meaning and information in constructive mathematics. *American Mathematical Monthly*, 89(6), 385-388.
17. Stark, E.I., (1978). Applications of a mean value theorem for integrals to series summation. *Ammerican Mathematical Monthly*, 85(6), 481-483.
18. Taschner,R. (1998). Constructive mathematics for beginners. *Lecture Series: Wissenswertes Aus Der Mathematik*. Retrieved from info.tuwien.ac.at/Goldstein/wisenswertes/s98/taschner.txt.
19. Townsley, L. (2005). *Why do we use theorem in calculus?* Retrieved from <http://apcentral.collegeboard.com>.
20. Tucker, T.W. (1997). Rethinking rigor in calculus: the role of the mean value theorem. *American Mathematical Monthly*, 104(3), 238.



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21. United States Census Bureau, Statistical Abstract of the United States, Degrees and Awards Earned below Bachelor's by Field. Retrieved from www.census.gov/prod/www/abs/statab.html.
22. Van Dalen, D. (2011). *Historical introduction and fundamental notions: Brouwer's Cambridge lectures on intuitionism*. Cambridge: Cambridge University Press.

Contextualized Examples in Constructivist Mathematics Pedagogy

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Abstract

We argue that one can adopt a constructivist approach to teaching basic mathematics by using contextualized examples arising in everyday life. Mathematical ideas are hidden in many everyday situations and using these situations to make the ideas more tangible aids in their grasp and assimilation. We argue a correlation between linguistic and mathematical ability and suggest that using contextualized examples can aid in implementation of the Writing Across the Curriculum (WAC) program within CUNY.

1. Introduction

Constructivist pedagogical practices have been of interest as a scholarly subject for the past century and a half. The epistemological premise of constructivist pedagogy is that knowledge and meaning is accrued through interaction with the world through individual experiences and ideas. We see this play out in children who learn concepts heuristically and attempt to test their learned ideas in the context of their experiences. The Montessori approach to elementary education, for instance, prioritizes this natural learning over direct instruction. This approach presumes the existence of various innate psychological “human tendencies” such as order, exploration, self-perfection, abstraction etc. and attempts to provide a framework where there is efficient actualization of these innate characteristics [1].

It is obviously of interest to see how constructivist pedagogies influence the learning of the social sciences, since the subject matter in these disciplines has a significantly more subjective flair, than, say, the physical sciences. It is reasonable to imagine that the circumstances of one’s childhood and one’s idiosyncratic bent of mind will have a significant influence in how concepts in the subjects of, for example, history, economics, sociology or philosophy are received and internalized. This aspect then also affords an opportunity to investigate to what extent, if any, the basic concepts of these subjects are independent of individual perceptions. We are thankfully in the position of investigating the constructivist approach in what is perhaps the most objective of all the scholarly disciplines, mathematics. Without going into deeper philosophical considerations about the ontology of mathematical objects, it is safe to say that insofar as elementary understanding of numbers, proportions, shapes, relations etc. is concerned,

ontological differences in individual perceptions are not crucial where basic conceptual and algorithmic aspects are concerned. In mathematics, then, one can efficaciously apply the constructivist approach for a grasping of the basic concepts. Here we show how contextualized examples using concepts arising in everyday life, and using everyday language, one can achieve efficient and comprehensive conceptual understanding of basic mathematical ideas.

In his article “Reconstructing Mathematics Pedagogy from a Constructivist Perspective” [6], Martin Simon investigates the instructor’s role in facilitating the trajectory of students’ mathematics thinking and learning. In it he refers to the French mathematics pedagogue Guy Brousseau’s ideas on the theory of “didactic situations”. Simon says: “Brousseau asserts that part of the role of the teacher is to take the non-contextualized mathematical ideas that are to be taught and embed them in a context for student investigation. Such a context should be personally meaningful to the students, allowing them to solve problems in that context, the solution of which might be a specific instantiation of the idea to be learned.” We heartily approve of this sentiment. From experience in the classroom, we often hear the refrain “What’s the purpose of all of this?” If mathematical ideas can be contextualized appropriately and their relevance to everyday life highlighted, students are inclined to expend interest and energy in learning the subject. As we argue in this article, contextualized problems arising in everyday life provide an appropriate framework to instill a sense of the utility of mathematics.

2. Language as Primary Source and the WAC Program at CUNY

Mathematics is curiously situated both within and without natural language. One needs natural language to speak about mathematics, but one obviously does not need mathematics to investigate natural language. Every mathematical symbol used has a corresponding word in natural language that is used to refer to it. In this sense, one recalls the oft-quoted sentiment that mathematics itself is a language. In fact, it’s a supremely efficient one: imagine what the size of a calculus textbook would be if one were to go through the entire text and replace every symbol with the corresponding word in natural language! This aspect of the interplay between mathematics and natural language finds its earliest expression in elementary mathematical concepts such as numbers, fractions, proportions etc. In these early situations, the natural language is used to talk about and fathom the physical world, and in so doing, mathematical ideas begin to seamlessly percolate into the psyche. Thus, in learning the words for natural numbers, one realizes that the natural numbers continue forever, without end. By the time one has arrived at calculus or group theory, of course, one has internalized the rules of formation and expression in the pure mathematical language, and one does “pure mathematics”. Indeed, many concepts of higher mathematics are pure abstractions, devoid of counterparts in the physical world. But in the beginning stages of mathematical learning, one does not have the facility to consider mathematics as a purely formal language. It is in this area that constructivist pedagogy can have the greatest impact in facilitating conceptual understanding, to be followed eventually by algorithmic fluency.

Our students come from very diverse backgrounds. Often times we instructors cannot take for granted even a consistent vocabulary set to be used for instruction, given the vast differences in cultural backgrounds of our students. This is a well-known issue in education. The famous oarsman-regatta question in the SAT comes to mind [2]. Many years ago an analogy question in the SAT asked students to find the pair of terms which was most similar in their relationship as the pair of words runner-marathon. The correct answer, oarsman-regatta, would typically be beyond the experiential sphere of those who did not grow up in a wealthy environment where rowing would be a popular sport. Such language issues are of greater relevance in the humanities. In mathematics, however, we do not need to presuppose awareness of specific cultural paradigms in order for a grasping of the concepts. Everyday examples devoid of cultural specificity can be advantageously used in aiding mathematics instruction. We give below a variety of examples to show how this can be done.

The issue is not so much linguistic comprehension, but that of comprehension in a particular language. Thus, it is conceivable, and even expected, that students who grew up in non-English speaking cultures would have trouble with English idioms, for example. But it is reasonable to assume that all students understand the concepts relating to everyday situations arising all over the world without regard to country or culture. It is these kind of examples we focus on, and it is in this sense that we consider language as a “primary source”. In other words, “language” does not refer to any particular language, but a certain linguistic facility, which could be in any language that one uses to think in and comprehend the world. Mathematical concepts are hidden in most everyday situations. The concepts do not have to be always numerical or geometrical: logic plays a role in many situations.

The Writing Across the Curriculum (WAC) program in CUNY is motivated by the considerations above. Graduating from CUNY with a college degree now requires a student to have taken and passed at least one WAC course. These are courses in which there is a substantial writing component. Instructors teaching the WAC courses have to be certified by going through a training process where they learn how to incorporate writing assignments into their courses. In the current instance, the first author is WAC certified, and routinely teaches WAC courses in mathematics. The writing assignments get the students to engage with mathematical symbols and ideas and help them realize that once the symbols are rendered in ordinary English, their meaning is not as arcane as the symbolism appears. This engagement with the natural language that is going on “behind the scenes” in mathematics helps the students to get a better appreciation for the use of, and need for, mathematical symbolism. The examples appearing here can all be converted to writing assignments for a WAC course.

In the context of linguistic vs. mathematical ability, some readers might be interested in a recent research report that suggests that the correlation between reading and mathematics ability at age twelve has a substantial genetic component [3].

3. Contextualized Examples

We assume that the basic concepts up to addition and subtraction and the corresponding algorithms are understood. It is perhaps impossible to articulate in natural language how we grasp these basic concepts. The idea of number is perhaps “hard-wired” into our brains. Philosophers generally take cognizance of the natural numbers as a starting point for meta-mathematical investigations. Philosophers such as Kant hold that numbers are *a priori* concepts. In this context, even as adults we find it problematic to explain our blind faith in the elementary algorithm for the addition of two numbers. Thus, we promptly agree to the statement that $36 + 72 = 108$, but no one would seriously claim to have made a mental construction of grasping 36 (different) objects, then mentally juxtaposing them with another 72 (different) objects, and mentally verifying that this new aggregate now actually has 108 objects. On the contrary, we use highly sophisticated concepts such as induction on a meta-mathematical level to convince ourselves that the algorithm for addition always yields a correct result if the basic steps are carried out without error. The basic steps, corresponding to the rules for adding single digit numbers, can easily be verified using actual physical objects.

Beginning mathematics students typically start to have difficulties when we move to multiplication, division, proportions, fractions etc. since typically the approach to teaching these subjects is to stress the algorithm, rather than give a sense of the origin and utility of these concepts in everyday life. We outline here a variety of examples which treat elementary concepts and ground them in everyday experience. This approach can be carried out in regards to most elementary mathematical concepts, with a little thought. Once one gets to more advanced mathematical concepts, this approach is unnecessary (though still useful) since by then our brains have “magically” learned to appreciate the form and function of pure mathematics. We attempt to address the beginning student, such as perhaps someone in the remedial mathematics courses at CUNY, and we hope the constructivist approach will make that student cultivate a friendly disposition to the subject.

We start with a simple and elegant problem.

Two pipes A and B supply water to a swimming pool. Pipe A by itself can fill the pool in 4 hours, and pipe B by itself in 6 hours. How long will it take for the pool to be filled if both pipes are kept open?

The beginning student faces a variety of difficulties with this problem. It is not immediate how this problem can be approached. The first step is to grasp the problem from a real-world perspective. If the real-world implication of the problem is grasped properly, then one would realize that the final answer, whatever it maybe, has to be less than 4 hours. For someone who has not grasped the problem at all, “both pipes” suggest addition, with a resulting answer of 10 hours. The constructivist approach in class would be to first start a discussion on how a solution can be attempted. As ideas are suggested and their error or correctness is discussed, the problem becomes more familiar and grounded in experience. Visualization of a pool with two pipes gushing water can be

suggested if one wants to incorporate a “contemplative practices” aspect to the instruction. At some point, one idea will either be discovered by the class to be key, or else revealed by the instructor as key: *what part of the pool is filled by pipe A by itself in one hour?* The answer, $1/4^{\text{th}}$, will already cause difficulty to some students. Once this is carefully grasped, the solution is in sight. Pipe B by itself fills $1/6^{\text{th}}$ of the pool in one hour, and therefore together $(1/4 + 1/6) = 5/12^{\text{th}}$ of the pool is filled in one hour when both pipes are open. From here, the idea the whole pool is filled in $12/5$ hours, or 2 hours and 24 minutes, is yet another idea that is not straightforward, but can be linked to the earlier idea of how to calculate what part of the pool was individually filled by each of the pipes, and shown to be a reverse process. It can be emphasized that this is a problem that can actually arise in real life if one is fortunate enough to live in a house with a pool, and that it requires mathematics for its solution.

In the foregoing problem, fractions played an important role. The idea of the Least Common Denominator can similarly be shown to have a real life significance as in the following problem:

You have one candy bar. How will you give someone a total of a half plus a third of the candy bar?

This problem can first be motivated by simpler examples, say involving a half and a third of an hour. Thus, if one task takes half an hour to complete, and another task takes a third of an hour, it is clear that the two tasks together take fifty minutes to complete. When it comes to a candy bar, a half is obtained by dividing the bar into two equal parts and choosing one part, and a third is obtained by dividing the bar into three equal parts and choosing one part. So far so good, but if there is only one bar, how could one simultaneously divide it into two and three equal parts? The idea that a half of the bar can equally be obtained by dividing the bar into *six* equal parts and taking *three* of them, and similarly for a third of the bar, at once shows the power of the Least Common Denominator. Just by one appropriate division of the bar into a certain number, one can now simultaneously obtain a half and a third of the bar. Of course, the denominator does not have to be “least”: any common denominator will do, and this leads to further exploration and verification of the traditional algorithms for adding fractions, reducing fractions etc.

Multiplication and division show up naturally in percentage problems, and are an excellent way to solidify these concepts. To start with, the etymology of the word “percent” shows that we are considering quantities *per hundred*. A regular shape such as a square or a circle gives a good visual analogy. Thus, to calculate thirty percent of a pizza pie, one would divide the pie into a hundred equal parts, and then take thirty of those parts. To calculate say 40% of 25, one is essentially dividing 25 into 100 parts, and then taking 40 of those parts. That is, one is doing the computation $(25/100)*40$. But typically the computation is introduced as $(0.40)*25$, which is really the same thing, but conceptually less illuminating. If 30% of a quantity is 60, this means that 30 parts of that quantity make up 60, and hence that 1%, or 1 part, is 2, and hence the full quantity which

is 100%, or 100 parts, is 200. The interplay between multiplication and division is clear in these simple examples. But the following problem tests the patience of many students:

After a 20% discount, the price of a shirt is \$40. What is the original price of the shirt?

The student who is inclined to jump to computations before having grasped the problem will doubtless calculate 20% of 40 to get 8, and add this answer to 40 to get the answer of \$48, a wrong answer. In this problem, visualization is key. One has to visualize that there is a starting quantity and that 20% of that starting quantity is removed, so that 80% of that quantity remains. If this is grasped, then as previously, 80% equals 40, whereby 1% equals 0.5 and hence the original quantity, which is 100%, equals 50, giving the correct answer of \$50. This problem also shows the pitfall of formulas. While formulas are very handy, understanding and correctly substituting for the variables in the formula is key. One common formula taught in doing percentage problems states: is/of = %/100. A careless application of the formula, using numbers provided in the problem also gives a wrong answer. The constructivist approach favors an intuitive solution to problems by grounding them in everyday experience rather than attempting to apply formulas right off the bat. Once the problem is understood in the proper perspective, one discerns a natural path between what is stated and what is asked for.

The following problem treats proportions.

Nine people stranded in a desert have enough water to last them for eighteen days. Three of them are abducted by aliens in UFOs. How many days will the water last the remaining six people? Assume that every person consumes water at the same rate.

For students familiar with setting up proportion problems as the appropriate fractions separated by the equals-to sign and cross-multiplying to find the answer, the process will yield a wrong answer unless they observe that the problem is not one of direct proportion, but one of inverse proportion. Engagement with the problem in a real-world scenario (except for the aliens in the UFO part) suggests that the water should last longer, since now there are lesser number of people. But this problem can also be attempted without setting up equations. The content of the last sentence of the problem can be used to introduce a variable denoting the rate of consumption per person per day. If this variable is called “w”, then the total amount of water present, prior to abduction, is $9 \cdot w \cdot 18$. If now there are only six people remaining, then they consume $6 \cdot w$ amount of water per day, and hence the water would last them $(9 \cdot w \cdot 18) / (6 \cdot w) = 27$ days. As in the previous problem, engagement with the problem can suggest intuitive answers without having to invoke potentially tricky formulas.

Mathematics also deals with logical principles. Some of these principles are often used implicitly in solving problems, without one’s being aware of their usage. Some principles are treated as axioms, in that the principle is seen to be evidently true when the concepts involved in the formulation of the principle are studied. The Pigeonhole Principle is one such. Thus, in a class consisting of say forty students, it has to be the case that some two of the students will share the same month of birth. Discussion as to why

this has to be true then leads to the question of how many students would have to be present so that some two of them necessarily share the same day and month of birth. This illustrates a distinctly *mathematical* or *analytical* way of thinking.

To illustrate with another example, when discussing exponents, one can introduce the idea of ancestors and how the number of ancestors for each person grows exponentially when considering both parents of each ancestor. After calculating some initial exponents, it is quickly seen that these numbers start to grow very quickly. A question for discussion at this point would be to investigate if these numbers can keep on growing, yielding more and more ancestors. Interesting discussions ensue, ultimately resulting in the realization that this unbounded growth cannot continue, leading to a simple “proof” that all humans must have evolved from a set of common ancestors. On the same topic, another problem is that a (very lucky) person finds one penny on day one, two pennies on day two, four pennies on day three, eight pennies on day four, sixteen pennies on day five, thirty-two pennies on day six, and so on; how long would it take for this person hit \$1 Million? Before doing any actual calculations, if the class is asked for their estimate of the time, it usually is a large number, since \$1 Million is a large amount. But the actual answer (28 days) is somewhat shocking, and illustrates how quickly exponents grow. A rather incomprehensible and fascinating fact in this context is this: a Googol is 10 raised to the power of 100, and a Googolplex is 10 raised to the power of a Googol; while a Googol can obviously be written down, a Googolplex cannot even be written down: there is not enough space in the universe even to write down this many number of zeroes! [4].

The grounding of examples in reality does not have to be restricted to elementary mathematics. Concepts arising in middle and high school mathematics are also shown to arise naturally in real life. Thus, interest calculations, mortgages and annuities, and population growth are naturally modeled by arithmetic and geometric series. Set operations such as power sets, subsets and the like can be exemplified using distribution of a basket of candy amongst recipients. The harmonic series provides a good example of how our intuition can lead us astray when it comes to very small quantities. A classic problem in pre-calculus states:

Suppose you start from the foot of a mountain at 12 noon and climb to the top of the mountain following a certain path. You camp at the top overnight and start back from the top of the mountain the next day at 12 noon, and follow the same path down. Show that there will be some particular time at which you would have been at the same point on the path on both your journey up and on your journey down.

This is not intuitively clear immediately, but once the graphs of distance vs. time are drawn for the journeys up and down, and shown to intersect, the intuition becomes clearer as to why the stated conclusion must be true. After all, intuition is also not a fixed, but grows more sophisticated with more learning. A similar problem in calculus states that on any great circle around the world, for any scalar quantity which varies continuously such as temperature, pressure, elevation, or carbon-di-oxide concentration,

there will be two antipodal points that share the same value for the variable [5]. Intuitively this is far from obvious. As a final example, a simple problem such as determining the shape of a fence, given a fixed amount of rope, so as to maximize the area included *requires* calculus for its solution.

4. Conclusion

There is no silver bullet when it comes to constructivist techniques for mathematics instruction. Bauersfeld [7] states: “The fundamentally constructive nature of human cognition and the processual emergence of themes, regularities, and norms for mathematizing across social interaction, to bring the [psychological] and the social together, make it impossible to end up with a simple prescriptive summary for teaching. There is no way towards an operationalization of the social constructivist perspective without destroying the perspective.” That said, instructors develop an intuition for what works and what doesn’t, from experience in the classroom. In our experience, grounding problems in everyday life has the best effect when it comes to retaining the attention and interest of students.

There is a subtle interplay in mathematics between form and content, and, curiously, this is mirrored in the issues surrounding the “math wars” [8]. It is quite impossible to ground the calculation of, say 45.67×9.876 , in “everyday experience”. Here one must learn the multiplication algorithm and practice it many times to do similar computations without error. At the same time, simpler examples of the same kind can be grounded in everyday life. Thus, form and content feed off of each other: algorithmic fluency and conceptual understanding develop hand-in-hand. In the beginning stages, however, the conceptual understanding is somewhat harder to instill. This is where contextualized examples grounded in everyday life can be of the greatest efficacy.

References

1. The Human Tendencies and Montessori Education, Mario Montessori, AMI/USA.
2. http://en.wikipedia.org/wiki/SAT#Cultural_bias
3. The correlation between reading and mathematics ability at age twelve has a substantial genetic component, Oliver S. P. Davis et al, *Nature Communications*, 5, Article number: 4204.
4. <http://en.wikipedia.org/wiki/Googolplex>
5. http://en.wikipedia.org/wiki/Intermediate_value_theorem
6. Reconstructing Mathematics Pedagogy from a Constructivist Perspective, Martin A. Simon, *Journal for Research in Mathematics Education*, 1995, Vol 26, No. 2, 114-145.
7. Bauersfeld. H. (1995). Development and function of mathematizing as a social practice. In L. Steffe & J. Gale (Eds.), *Constructivism in education* (pp. 137- 158). Hillsdale, NJ: Lawrence Erlbaum.
8. http://en.wikipedia.org/wiki/Math_Wars