THE ROLE OF ATTENTION IN ASSESSMENT

Sergiy Klymchuk

Auckland University of Technology, New Zealand

sergiy.klymchuk@aut.ac.nz

Cheng Chun Chor Litwin

Hong Kong Institute of Education, Hong Kong

cccheng@ied.edu.hk

Abstract

Attention to details is one of the most important specific features of a mathematical way of thinking. When evaluating mathematical questions it is important to teach students to pay attention globally and locally, consider conditions, properties and relationships, since all of these aspects are as important to develop as mathematical techniques. To develop such skills in their students school teachers should have those skills themselves. In this study, we investigate the ability of school mathematics teachers to pay attention to details and use their mathematical knowledge. We are confident that the vast majority of teachers have excellent knowledge of mathematical techniques. Hence the question is whether this kind of knowledge might structure their attention in such a way that the emphasis on procedures deviates their attention from the essential details. Two groups of teachers from New Zealand and Hong Kong were given a mini-test containing seven simple mathematics questions. All questions in the test were provocative in the sense that they looked like routine questions but in fact they had some catch. The results from the test were startling – the vast majority of the teachers did not notice any catch and gave incorrect answers to most questions in the test. After the test the teachers were given a short questionnaire to reflect on their performance on the test. The teachers' responses to the questionnaire are presented and analysed in the paper using theories on attention. Iimplementations of the results of the study in assessment and professional development are discussed.

Keywords: attention, assessment, knowledge, professional development

INTRODUCTION AND THEORETICAL FRAMEWORKS

In recent years there has been substantial research on teachers' mathematical knowledge and assessment of teachers' mathematical knowledge (Rowland & Ruthven, 2011; Ball, Thames & Phelps, 2008; Hill, Ball & Schilling, 2008; Hill et al, 2007;). In this paper, an attempt is made to investigate the role of attention in *using* existing mathematical knowledge by school mathematics teachers while doing simple but not routine mathematical tasks. The following theories on attention are employed as theoretical frameworks: the late selection theory of selective attention (Deutsche & Deutsch, 1963) based on the idea that all information is routinely processed and selection of response depends on the level of alertness; Kahneman's (1973) model of divided attention based on the idea of mental efforts and the level of arousal or state of alertness; and the feature-integration theory of attention (Treisman & Gelade, 1980) based on the idea that putting different features into a coherent object demands focused attention. Other theoretical considerations are based on research by Mason (2000, 2002, 2004). Mason has proposed that when we look at a mathematics question the focus of our attention may vary depending on whether we are *looking at* the symbols or *looking through* them. The idea is that we need to structure our attention, to know what we are aware of, and Mason describes a number of elements that we may focus attention on, including: the whole, the details, the relationships between the parts, the properties of the whole or the parts and deductions (2004).

We are confident that the vast majority of teachers have excellent mathematics knowledge of *knowing-that* (factual) and *knowing-how* (techniques and skills) as described by Mason and Spence (1999). Most of the formulas, rules and theorems however are not always applicable but have certain conditions and constraints. Often assessment questions focused on techniques are selected in such a way that the conditions/constraints of the relevant formula or rule are met. So the students might develop a habit of applying formulas or rules without checking the conditions/constraints.

But in real-life problems not all functions and equations behave so nicely and ignoring conditions and constraints might lead to significant and costly errors. One of the goals in teaching mathematics is developing and enhancing students' mathematical way of thinking while helping them to learn a variety of concepts, techniques and procedures. In particular, the mathematical way of thinking is concerned to a large extent with the analytical thinking so that an individual analyses any situation, doesn't take anything for granted and always looks for evidence, proof and justification which are the essence of mathematics. We should encourage students to pay attention to every detail, for example - conditions, constraints, locality, properties and relationships. The ability to pay attention or 'discipline of noticing' as described by Mason (2002) is equally important to develop as mathematical techniques. It needs to be a natural part of their mathematical culture. Students can see that the ability to analyse carefully a mathematics question enhances their skills to analyse critically other situations outside mathematics. To develop such skills in their students teachers should possess those skills themselves. In this paper, we are not testing teachers' knowledge of mathematical techniques, procedures and algorithms but their skills of paying attention and analyzing the question before applying a relevant formula or technique, that is the ability to 'question the question'. We argue that attention plays a crucial role in doing non-routine mathematical tasks. As Mason and Spenser (1999) propose "*knowing-to* act in the moment depends on the structure of attention in the moment, depends on what one is aware of" (p.135).

THE STUDY

The study was conducted in 2011 with two groups of teachers - one in New Zealand and the other in Hong Kong. The New Zealand (the first) group consisted of 14 experienced upper secondary school mathematics teachers who attended a workshop during a one day conference as part of their voluntary professional development. The Hong Kong (the second) group consisted of 26 secondary school mathematics teachers who attended a 5-week full-time compulsory training course. It was not a comparative study but a parallel case study. A combination of two non-probability sampling methods – convenience and judgment – was used to select the participants. Namely, it was convenient for the authors to invite the participants for the study from the groups of teachers they had easy access to. In the authors' judgement the selected participants were good representatives of the populations of mathematics teachers' in both countries. Both groups were given the mini-test containing 7 simple mathematical questions. All questions in the test were provocative in the sense that they looked like routine questions but in fact they had some catch. In some cases it was an extraneous root of an equation because of the restricted domain, in others the rule was inapplicable because the conditions were not met. The teachers had 15 minutes for the test and they were informed before the test a hint that some questions are a bit provocative. The questions from the test are below.

The mini-test

1. Find the area of the right-angled triangle if its hypotenuse is 10cm and the height dropped on the hypotenuse is 6cm.
2. Find the domain of the function $y = f(g(x))$ if $f(x) = x^2 + 1,\ \ g(x) = \sqrt{x-2}$.
3. Solve the equation $\ln(x^2 + 17x - 18) - \ln(x^2 + 5x - 6) = 0$.
4. Prove the identity $\sin x = \sqrt{(1 - \cos^2 x)}$.
5. Show that the equation $\frac{x^2 + \sqrt{x+1}}{x-1} = 0$ has a solution on the interval [0, 2].
6. Find the derivative of the function $y = \ln(2\sin(3x) - 4)$.
7. Find the integral $\int_{-1}^{1} \frac{1}{x}\,dx$.

The results and discussion of the mini-test

The results of the test are presented in Table 1.

Table 1: Correct answers to the test questions

	Q1.	Q2.	Q3.	Q4.	Q5.	Q6.	Q7.
Correct answers group 1	0%	7%	21%	7%	0%	8%	0%
Correct answers group 2	23%	12%	27%	19%	12%	15%	12%

After the test there was a detailed discussion of solutions of every question from the test.

Question 1. The correct answer is: there is no area as the triangle doesn't exist. By the Thales' theorem the hypotenuse in a right-angled triangle is a diameter of its semicircle so in this case the height cannot be bigger than 5cm.

In the first group there were no correct answers out of 14 with 12 teachers giving either $30cm^2$ or $24cm^2$. In the second group there were 6 correct answers out of 26 with some teachers arriving to the correct answer after checking their initial incorrect answer of $30cm^2$ or $24cm^2$ and rejecting it. Most teachers applied the familiar formula $A = \frac{ah}{2}$ ignoring the important piece of information about the right angle.

Question 2. The correct answer is: $x \geq 2$. The composite function $y = f(g(x))$ is defined whenever both $g(x)$ and $f(g(x))$ are defined.

In the first group there was just one correct answer out of 14. In the second group there were 3 correct answers out of 26. Most teachers did not pay attention to the definition of the domain of a composite function.

Question 3. The correct answer is: no solutions ($x = 1$ is outside of the domain of both log functions which can be easily checked by substitution).

In the first group there were 3 correct answers out of 14. In the second group there were 7 correct answers out of 26. Most teachers ignored the restricted domain.

Question 4. The correct answer is: the 'identity' is not true. Squaring both sides doesn't prove it because this operation is irreversible. It is not an identity but an equation with infinitely many solutions $x \in [2\pi n, \pi(2n+1)]$.

In the first group there was just one correct answer out of 14. In the second group there were 5 correct answers out of 26. Most teachers assumed that it was provable because of the wording of the question and did not pay attention that squaring is an irreversible operation.

Question 5. The correct answer is: there are no solutions. The equation $x^2 + \sqrt{x} + 1 = 0$ has no solutions (the left-hand side is always positive). If one can try to apply the Intermediate Value Theorem it is not applicable because the function $f(x) = \frac{x^2 + \sqrt{x} + 1}{x - 1}$

is not continuous on [0, 2].

In the first group there were no correct answers out of 14. In the second group there were 3 correct answers out of 26. Most teachers misused the Intermediate Value Theorem – they checked only that $f(0) < 0$ and $f(2) > 0$ and did not check the continuity condition.

Question 6. The correct answer is: the derivative doesn't exist because the function doesn't exist as the argument of the log function is always negative.

In the first group there was just one correct answer out of 13 (one teacher did not attempt the question). In the second group there were 4 correct answers out of 26. Most teachers failed to check the domain of the function and applied the familiar Chain Rule.

Question 7. The expected answer is: it is not a definite integral because the function $y = \frac{1}{x}$ is not continuous on [-1, 1]. For this reason the Newton-Leibniz formula is not applicable. It is beyond the secondary school curriculum (it is an improper integral and in this particular case it is not defined).

In the first group there were no correct answers out of 14. In the second group there were 3 correct answers out of 26. Many teachers failed to check the continuity condition of the Newton-Leibniz formula and applied it. Some used graphs to produce incorrect solutions.

The questionnaire and teachers' responses

After the discussion of the solutions the teachers were given a short questionnaire to reflect on their performance in the test. The response rate in both groups was 100%.

The questionnaire is below.

Question 1. What are your feelings after you have learnt about the correct solutions to the test questions?

As most of the teachers gave incorrect answers to the vast majority of the questions it was interesting to notice that their feelings were polarized. In the first group 50% of the teachers expressed self-criticism – "disappointed', "without thinking", "embarrassing", "didn't apply critical at all", "felt stupid –

oversimplified the questions" and the other 50% were happy to learn lessons from the test – "enlightened, a very good test and ensure reflection upon teaching practice", "attention to the words and wider picture", "feel OK", "like it – I should have known…', "fun, I love anything that knocks me out of academic boredom", "a bit more enlightened", "glad that I have the opportunity to see and think through these problems".

In the second group the majority of the teachers were embarrassed and uncomfortable about their performance on the test.

Question 2. What are the reasons for not solving all test questions correctly?

In the first group all 14 teachers gave comments on the lack of attention and careful thinking. The common responses are as follows:

Not thinking carefully about whether my solution method was appropriate to that particular problem; I did not think critically; not paying attention, impulsive reaction; I did not rely on my understanding but jumped straight to applying the rule; lack of knowledge and 'testing' things and being programmed to look for 'set' answers'; possibly that's how I was taught back home (South Korea), learnt lots of techniques (some difficult) but not to question the questions; not looking at all conditions; not thinking carefully and not reading the questions carefully; applying skills but not applying knowledge; not thinking about the structure of the expressions, considering its conditions; I knew there was something more to check but did not check thoroughly enough.

In the second group the teachers reported that the main reasons for making mistakes were carelessness and the expectation that each test questions had an answer (often a certain number).

There were no comments in both groups about lack of time to finish the test so we assume it was not a reason for poor performance. From our observation the vast majority of the teachers in both groups finished the test within 10 minutes.

Question 3. Would you make any changes in your teaching practice after doing the mini-test? If so – which changes? If not – why?

In the first group all 14 teachers reported that they would make changes in their teaching practice after doing the test. The common responses are as follows:

Introduce tricks like this to class to make them think; keep encouraging and creating environment where a deep conceptual knowledge is cultivated; encourage and reward checking of answers; more emphasis on the validity of solutions; teach them to examine the question thoroughly; give students more questions that will force them to think about the conditions surrounding the questions; I would encourage students to think through questions carefully; students need to understand, observe and consider answer to ensure they make sense and think before you solve; give students questions to challenge their knowledge; I try to make my students think more about restricted domains, check solutions and not trust graphical calculators; encourage kids to think about their solutions in light of the original question; give them problems occasionally that will 'trip' them up if they have not gone back and re-assessed their solutions; more emphasis on the nature of problem solving; stop answering impulsively, think before respond; I will expose students to such questions to get them to think more deeply about the conditions.

One teacher however, along with his/her positive response, made the following comment regarding the changes: "unless it is an element of the assessment I might not have time".

In the second group 13 out of 26 teachers reflected that they would change their teaching practice. Some of them reported that they would take the test back to their school and use it as a teaching material. The other 13 teachers reported that they would not change their teaching practice as such test questions are not common.

Some differences between the two groups in answering the questionnaire were probably due to cultural differences between New Zealand and Hong Kong.

However, addressing the effect of the cultural differences was not an intention of this paper.

ANALYSIS AND CONCLUSIONS

Teachers' performance on the test and their responses to the questionnaire demonstrated that the majority of the teachers had serious lack of attention and careful thinking that led them to fail most of the questions in the test. According to Mason and Spence (1999) those teachers, in spite of good *knowing-that* and *knowing-how* should enhance their *knowing-to* act skills that help them to perform better: "active, practical knowledge, knowledge that enables people to act creatively rather than merely react to stimuli with trained or habituated behaviour involves *knowing-to* act, in the moment" (p.136). Knowing-to act in many cases is a multistep activity and each step needs attention. We are absolutely confident that the participants of the study had the relevant knowledge (e.g. they knew the domain of the log function, the range of the sine function, the conditions of the Intermediate Value Theorem, the definition of a definite integral, and so on). So the question was about their ability to *use* their knowledge on the test. Theories of attention developed by psychologists might be helpful in analysing the relationship between knowledge and attention. Deutsch and Deutsch (1963) argue that "however alert or responsive we may be, there is a limit to the number of things to which we can attend at any one time" (p.80). Kahneman's (1973) model of divided attention (when attention is divided between two or more concurrent tasks) suggests that attention can be flexibly allocated between tasks based on processing priority. Treisman & Gelade (1980) went further claiming that "without focused attention, features cannot be related to each other" (p.98). In solving mathematical questions attention is required to each step and often the priority of allocation of attention to different steps is very important. In many cases attention is needed to the 'analysis of the question' step (e.g. checking conditions of the rule, domain of the functions, type of the equation, locality of the statement, and so on) before switching attention to the next steps – procedure, verification, etc. Ignoring the 'analysis of the question' or 'question the question' step can lead to incorrect solutions especially in non-routine questions as the study shows. In some cases, however, the order of steps can be changed. For example, to solve Question 3 of the test one way is to find the common domain of both log functions first by solving a system of two quadratic inequalities, perform calculations using the log rules and then check whether the solution belongs to the common domain. An easier way however, is just notice that we are dealing with the restricted domain without finding it (can be time consuming), perform calculations using the log rules and then verify the resulting solution by substitution into the original equation. Feedback of the participants of the study show that one of the main reasons for poor performance on the test was not the priority of the steps but ignoring some of the crucial steps, in most cases the ‘analysis of the question’ step. The majority of the participants reported that they would definitely make changes in their teaching practice after the test by putting more emphasis on the analysis of the question before applying a certain formula or theorem. Those participants, who reported that they would not change their teaching practice because the test questions were uncommon, probably tend to ‘teach to the test’. Including the type of questions from the mini-test into the assessment would encourage those teachers to pay more attention to details and analysis and enhance such skills in their students. After all, many situations in real life don’t have a single ‘correct’ answer like routine questions from the traditional assessment in mathematics. We believe that solving non-routine, non-standard questions would prepare students for the real world better. Enhancing their own and their students’ ‘discipline of noticing’ (Mason, 2002) by paying attention to details can also be a useful addition to teachers’ professional development.

References

Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59 (5), 389-407.

Deutsch, J. A. & Deutsch, D. (1963). Attention: some theoretical considerations. *Psychological Review*, *70*, 80–90.

Hill, H., Ball, D. L., & Schilling, S. (2008). Unpacking “pedagogical content knowledge”: Conceptualizing and measuring teachers’ topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39 (4), 372-400.

Hill, H. C., Sleep, L., Lewis, J. M., & Ball, D. L. (2007). Assessing teachers' mathematical knowledge: What knowledge matters and what evidence counts? In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 111-155). Charlotte, NC: Information Age Publishing.

Kahneman, D. (1973). *Attention and Effort*. Englewood Cliffs, NJ: Prentice-Hall.

Mason, J. (2004). Doing ≠ construing and doing + discussing ≠ learning: The importance of the structure of attention. Presented as a regular lecture at the *10th International Congress on Mathematics Education (ICME-10)*, Copenhagen, Denmark.

Mason, J. (2000). Asking mathematical questions mathematically. *International Journal of Mathematical Education in Science and Technology 31*(1), 97–111.

Mason, J. (2002). *Researching your own practice: the discipline of noticing*. UK: Routledge.

Mason, J. & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing-to act in the moment. *Educational Studies in Mathematics 38*, 135–161.

Rowland, T. & Ruthven, K. (Eds.) (2011). *Mathematical Knowledge in Teaching*. Springer.

Treisman, A. & Gelade, G. (1980). A feature-integration theory of attention. *Cognitive Psychology, 12*, 97–136.