Problem Posing, Problem Solving Dynamics In the Context of Teaching- Research and Discovery method.

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ABSTRACT

Problem posing is practiced in the context the TR/NYCity methodology of Teaching-Research (Czarnocha, Prabhu 2006), which has been utilized in the mathematics classrooms for a decade. Problem solving turned out to be an essential teaching strategy for developmental mathematics classrooms of Arithmetic and Algebra, where motivation in learning, interest in mathematics, and the relevance of the subject is unclear to adult learners. Problem posing and problem solving are brought into play together so that moments of understanding occur in the context classroom inquiries and discoveries, and a pattern of these moments of understanding can lead to self-directed discovery, becoming the natural mode of learning. Facilitation of student moments of understanding as manifestations of their creative capacity emerges from classroom teaching-research practice and from its relationship with the theory of the Act of Creation (Koestler, 1964). Discovery returns to the remedial mathematics classroom, jumpstarting reform. This Teaching-Research report is based on the collaborative teaching-experiment (Czarnocha et al, 2010) supported by C^3IRG grant of CUNY.

Keywords: Discovery, enquiry, problem posing, problem solving, creativity, learning environment, teaching-research.
**Introduction: posing the general problem.** Enquiry is the path to discovery along which the central problem decomposes into a series of posed questions.

![Diagram](Image)

**Fig. 1 Inquiry method of teaching and the decomposition into posed questions/problems.**

That problem posing decomposition is the essential route for reaching discovery; its absence derails success by denying access to that discovery. Transformation of the process of inquiry into a series of smaller posed problems generated by the participants allows every student reach, to discover sought after solution. According to Silver et al (1996), Dunker, (1945) asserted that “problem solving consists of successive reformulations of an initial problem” (p.294) to solve, and that view became increasingly common among researchers studying problem solving. Moreover, (Brown and Walter, 1983) in The Art of Problem Posing, pose and answer the question: “Why, however, would anyone be interested in problem posing in the first place? A partial answer is that problem posing can help students to see a standard topic in a sharper light and enable them to acquire a deeper understanding of it as well. It can also encourage the creation of new ideas derived from any given topic—whether a part of the standard curriculum or otherwise”.(p.169)

The central problem posed in front of the mathematics teacher teaching within an urban community has dimensions that are of a global and a local scale. Both ends of the scale can generate the solution of the problem if appropriate questions are posed to reformulate it to the needed precision for the scale at hand. Such a problem is The Achievement Gap. Thus the central problem addressed in this chapter is how to bridge the Achievement Gap and the role of problem posing/problem solving dynamics in this process. Its two scales are, on the one hand, that which drives political machinery: funding initiatives at the National Science Foundation, ED and other funding agencies, and on the other hand, the situation in a community college mathematics classroom – talent, capacity for deep thinking, yet its clarity disturbed, so grades awarded are not high. The gap at both scales, is just a gap; so that the solution to the common posed problem at one end of the scale, of how to fill/bridge/eliminate the gap, can lead to a flow between the local and the national problem, in that the solution at the local scale informs the problem posed at the global national scale.

The posed problem has multiple dimensions including:
(a) student voices with the actual classroom difficulties, such as: “what is \(-3 + 5\), why is it not \(-2\)”, or” why must I take a long answer test, when the final exam is multiple choice”, or “why don’t you teach, you just make us solve problems”.

(b) Teacher’s voices with the curricular fixes that they think will/has definitely eliminated the gap in their own classroom, of say fractions; and who through that discovery/solved problem, wish to let the secret be available to all students to fix the fraction gap on a broader scale.

(c) Administration obsessed with standardized exams measuring student skills development but not their understanding.

The problems posed by the different constituents are sub-probes to the posed problem of the achievement gap and each of these sub-problems fall into mutually affecting strands. In the classroom, these fall under the categories of (Barbatis, Prabhu, Watson, 2012):

(i) Cognition

(ii) Affect

(iii) Self-Regulated Learning practices.

In this article we will illustrate our classrooms’ problem posing possibilities. Mathematics is thinking technology in which posing problems, attempting to solve them, and solving them to the extent possible with the thinking technology available, is the foundation and basis of the discipline. By repeatedly posing questions to solve the problem in its broad scope, we have discovered that creativity, i.p., mathematical creativity, can jumpstart remedial reform, confirming this way assertions of (Silver et al, 1996; Singer et al, 2011). Mathematics answers questions – “why?” and “how?” as it uses minimal building blocks on which its edifice is constructed. Thus at any level of the study of mathematics, problem posing and problem solving are inextricable pieces of the endeavor.

TR/NYC Model is the classroom investigation of students learning conducted simultaneously with teaching by the classroom teacher, whose aim is the improvement of learning in the very same classroom, and beyond (Czarnocha, Prabhu, 2006). Teaching-Research, NYC (TR/NYC) model has been in effective use in mathematics classrooms of Bronx CC and Hostos CC, the Bronx community colleges of the City University of New York, for more than a decade. The investigation of student learning and their mathematical thinking necessitates the design of questions and tasks that reveal its nature to the classroom teacher-researcher. Thus problem posing became the method for facilitation of student mathematical thinking employed by TR/NYC. This method of teaching naturally connects with the discovery method proposed originally by (Dewey, 1916) and Moore (Mahavier, 1999) . Utilization of
TR/NYCity in conjunction with the discovery method let us, teacher-researchers, to discover that repeatedly posing questions to students facilitates student creativity, and as such it can jumpstart remedial reform in our classrooms (Czarnocha et al, 2010). That realization confirmed the work of (Silver, 1997), (Singer et al, 2011) and others in the field who assert that problem posing is directly related to the facilitation of student creativity.

The Act of Creation by (Koestler, 1964) allows to extend our understanding of classroom creativity to the methodology of TR/NYCity itself. The Art of Creation asserts that bisociation – that is the moment of insight is facilitated and can take place only when two or more different frames of discourse or action are present in the activity. Since teaching-research is the integration of two significantly different professional activities, teaching and research, TR/NYCity with its constant probing questions to reveal student thinking presents itself as the natural facilitator of teacher’s creativity as well. TR Cycle below is the theoretical framework within which problem posing/problem solving dynamics as the terrain of student and teachers classroom creativity is being iterated through consecutive semesters. The process of iteration produces new knowledge about learning and problem posing/problem solving instructional materials.

Fig. 2 Teaching Research Cycle with two iterations.

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The TR cycle, iterated every semester of teaching the particular course, allows to diagnose student difficulties at any moment of learning, to design appropriate instruction and to assess its effectiveness. During each semester, student difficulties are cycled over at least twice so that the diagnosed difficulty can be addressed and its success assessed in agreement with the principles of adaptive instruction (Daro et al, 2011, p28). Over the span of several semesters, the methodology creates an increasing set of materials which are refined over succeeding cycles, and acquire characteristics of use to all students studying the mathematical topics under consideration. The learning environment itself develops into a translatable syllabus for the course from several supports for the learning in the classroom so that the applicability of the techniques at use in a teaching-research (TR-NYC) classroom, is easy and replicable for other instructors facing the same difficulties of the achievement gap in their own classroom, and who have an interest in becoming a teacher-researcher in the process of finding the solution to the larger problem in their own classroom.

In the classes of Remedial Mathematics (i.e., classes of Arithmetic and Elementary Algebra) at the community college, Teaching-Research Experiments have been carried out since 2006. In the period from 2006-2012, success began to be evidenced in 2010 following a broader teaching-research team approach described later in this section. The initiative in Remedial Mathematics, followed the successful use of the methodology in calculus classes under the NSF-ROLE#0126141 award, entitled, Introducing Indivisibles in Calculus Instruction. In Calculus classes (NSF-ROLE#0126141) when the appropriate scaffolding dynamic had been embedded in the Learning Environment, students underprepared in, to name the main difficulties, fractions on the line, logic of if-then, algebra of functions and Limit (essential for definite integral conception as the limit of the sequence of partial Riemann sums) were nonetheless able to perform at an introductory Analysis level (as distinct from the level of standard calculus course). Discovery was the ‘natural’ means of exploration in calculus classes, and enquiry leading to discovery through problem posing/problem solving dynamics was able to take place without student resistance.

In classes of Remedial Mathematics, the situation though, is markedly different. Student resistance to learning is prompted by years of not succeeding in the subject, and the general attitude is of ‘just tell me how to do it’. Discovery and enquiry are not welcome means. In the period 2006-2010, the mathematical materials continued to develop, and the learning trajectory of fraction described later also developed in this period. However, the success was not in student learning. In 2007-2008, as part of CUNY funded teaching experiment, Investigating Effectiveness of Fraction Grid, Fraction Domino in mathematics classrooms of community colleges of the Bronx, it was already discovered that a satisfactory student partnership in learning, a didactic contract (Brousseau, 1997) or in classroom language, a mutual “handshake” confirming the commitment to student learning, was essential in confirming the role of problem posing on the affect and self-regulatory learning (Akay and Boz, 2010). In 2010, following a Bronx Community College consultancy to FET colleges in South Africa, a new direction to address the
problem was found. The situation in classrooms whether in South Africa or in the Bronx, needed a simultaneous attention to student affect.

Development of Learning Environment. The relationship between cognitive and affective components of learning has recently been recognized (Araujo et al, 2003; Gomez -Chacon, 2000). According to (Goldin, 2002,) „When individuals are doing mathematics, the affective system is not merely auxiliary to cognition - it is central“ p.60. (Furinghetti and Morselli, 2004) assert, (in the context of the discussion of mathematical proof) „The cognitive pathway towards the final proof presents stops, dead ends, impasses, steps forward. The causes of these diversions reside only partially in the domain of cognition; they are also in the domain of the affect.” (p.217). There is a need, in addition to attention being paid to the cognitive pathways, to consider [and impact] the affective pathways, which are described by (DeBellis and Goldin 1997) as „the sequence of (local) states and feelings, possibly quite complex, that interact with cognitive representation (p.211).

A learning environment began to develop under iterative loops of the TR cycle, and the components of this learning environment are captured in the concept map below. The teaching-research team now constituted a counselor (also the Vice President for Student Development), a librarian and the mathematics instructor.
Fig. 3 The components of Learning Environment centered on Creative Problem Solving.

The detailed explanation of how to read the concept map with its emphasis on the improvement of classroom performance as the function of motivation, self-regulated learning and cognitive development, is contained in the Appendix 2.

In the period 2010-2012, during the process of developing the conducive learning environment, three factors emerged as anchoring the learning environment (Barbatis, Prabhu and Watson, 2012), viz., simultaneous attention to:

(i) Cognition (materials and classroom discourse well scaffold, paying attention to the development of the Zone of Proximal Development via meaningful questioning in the classroom and via instructional materials designed in accordance with Bruner’s (Bruner, 1978) theoretical position of development of concepts along the concrete, iconic and symbolic forms)

(ii) Affect (classroom discourse and independent learning guided by development of positive attitude toward mathematics through instances and moments of understanding of enjoyment of problems at hand, extended by self-directed
means of keeping up with the changing attitude toward mathematics and its learning)

(iii) Self-regulated learning practices (SRLp) (learning how to learn, usefulness of careful note-taking, daily attention to homework, to asking questions, paying attention to metacognition and independent work).

Two simultaneous developments took place during the construction of the learning environment anchored in these three aspects. The craft knowledge of the teaching-research team had a common focus of employment – the development and viewing of the mathematical material on several planes of reference (Koestler, 1964), i.e., for a problem, say \( \frac{1}{2} + \frac{1}{3} \), the counselor of the mathematician-counselor pair would keep the mathematical focus constant while alternating between concrete examples of cookies, pizzas, etc; the method exposed students to the process of generalization. This was then extended by the mathematics instructor in removing the monotony of ‘not remembering’ the rules for operations on fractions by using the rules for operations on fractions in more involved problems such as those involving rules of exponents. The novelty and intrigue of decoding the problems of exponents, made the rules for fractions ‘easier’ to remember or look up. Creativity had emerged as an organic development from the craft knowledge of the instructor, however, it was the support of Arthur Koestler’s The Act of Creation (Koestler, 1964) that provided a theoretical base in which to anchor thinking and development of creativity.

**Theory of the Act of Creation.** (Koestler, 1964) sketches the theory of the act of creation, or the creative act and coins the term bisociation to indicate it. Bisociation refers to the ‘flash of insight’ resulting from ‘perceiving reality on several planes at once’ and hence, not just associating two familiar frames, but seeing a new out of them, which had not been possible before. This moment of understanding or bisociation is facilitated in the teaching-research classroom through problem posing which lead to a pattern that changes habit to originality and mathematics is no longer the ‘old and boring stuff that needs to be done’, but is a source of enjoyment, so that even when the class period ends, students are still interested in continuing to puzzle over problems and when the enjoyment translates into performance and closing of the achievement gap a student at a time.

Koestler’s theory of creativity is based on making connections of the concept in question across three domains or shades of creativity: humor, discovery and art. Note that our Creative Learning Environment anchored in Cognition, Affect and SRL assumes overlapping and mutually conducive roles. Humor addresses affect, discovery addresses cognition and learning how to learn when refined so that it is natural, the learner can transform his discoveries to deeper levels, or art. A quick glimpse of Koestler’s theory is encapsulated in the following two concept maps. The Habit and originality concept map provides the workings of the transformation involved in the creative process. The Habit+Matrix = Discovery concept map digs deeper into this transformative process,
showing the important role of affect/humor in the creative process. Both become directly usable in the development of the Creative Learning Environment in the classroom.

Fig. 4 The role of the bisociative act in transforming the habit into originality. Mathematics Teaching-Research though the TR cycle clearly lends itself to creating a problem posing problem solving dynamic. How does it do so? In the next section we provide several actual classroom instances where problem posing has brought back discovery and enquiry on course. The following concept map links the creativity with the Problem Posing/Problem Solving Dynamics.
Problem Posing/Problem Solving Dynamics. Problem Posing Illustration 1. This particular example is from an Elementary Algebra class. The time is just after the first exam, about a month into the semester. Students have had shorter quizzes before. On the day from which this example is taken, almost the entire class, stages a rebellion. They state the instructor does not teach and they solve problems and the fact that the class is remedial means the instructor has to teach. A couple of the students explain what they mean by ‘teach’. One student states her previous instructor did a problem on the board and then they did several like it. Another student adamantly declares that she needs “rules” for how to do every problem. After the uproar subsides, the instructor guides
them through the test, throughout ensuring that the student in question is doing the problem – thinking aloud and throughout pointing out the rules or the significant places to pay attention.

**Problem 1** Compute:

(a) \( 36 + (-20) + 50 - (-17) - 10 = \)

(b) \( 2 - (-4 - 10) \)

(c) \( -18 - (-6 + 2) \)

(d) \( 2 - (-13) + (-7) - 20 \)

(e) \( 8 - 5 \times 2 + 9 = \)

(f) \( 6 \times 7(-1) - 3 \times 8(-2) \)

(m) \( 7(-4)(8) - 9 \times 6(-2) \)

(n) \( 15 - 2(-5) - (20 - 4) \div 8 \)

Each problem is solved/thought out aloud by the student selected by the instructor, and she/he reads the problem, and when there is a symbol stated, such as parenthesis, the student is asked for the meaning of the symbol (posing a problem). Once the whole problem is read aloud with meaning, the student has to determine which is the order in which to proceed and why (solving a problem), and then the student actually does the computation in question.

At the end of the class, attention is brought back to the work done, how it constituted reading comprehension, paying attention to the structure of the problem and then paying attention to the meaning of individual symbols and thinking of structure and meaning together. There was clarity, satisfaction, and a turnaround in problem solving after this session.

What did this session do in the classroom? First, it debunked the myth, that one has to memorize something in order to solve every problem. Second, it took away the authority of the teacher as knowing which the class was reluctant to give up, and finally when each person carefully read and translated/made sense of the problem in terms of symbols and structure, they saw the process of posing and solving working in unison with one of their own classmates carrying out all the thinking. Hence, for example, when the student who was doing the problem, read “parenthesis”, she was questioned, as to what the parenthesis means, and what role it had to play in the problem (posing problems). The mathematical language with its various hidden symbols, many symbols with one meaning, or one symbol with many meanings are all sources of ‘confusion’ for students and situations such as the one narrated here, provide for self-reflection, and clarification of the language.
and the meaning of the language of mathematics. It requires many posed questions along the way to clarity. Note, how affect, cognition and metacognition, all three enter the dialogic thinking that instructor and students went through together.

**Problem Posing Illustration 2.** In this example, the class is Elementary Algebra. Students have trouble determining which rule of exponents is to be applied for the given problem. There is a tendency to arbitrarily use anything without justification. The class problems are followed by a quiz, in which there is great difficulty among students in determining which rule is applicable for the problem under consideration. Again, it is a question of not being able to slow down the thinking to observe the structure of the problem and the similarity of the structure with one or more rules. Students are asked to work on the following assignment:

**Rules of Exponents**
1. \( a^n \times a^m = a^{n+m} \)
2. \( \frac{a^n}{a^m} = a^{n-m} \)
3. \( (a^n)^m = a^{nm} \)
4. \( a^0 = 1 \)
5. \( a^{-n} = 1/a^n \)

Make up your own problems using combinations below of the rules of exponents:
- Rules 1 and 2
- Rules 1 and 3
- Rules 1, 2 and 3
- Rules 1 and 4
- Rules 2 and 4
- Rules 1, 2 and 5
- Rules 1 and 5
- Rules 1, 2, 3, 4, and 5

Solve each of the problems you created.

It was noted in the work students submitted that they created problems that had one term that required the use of say Rule 1 (e.g. \( X^7 Y^8 \)) and another term that required the use of Rule 2 (e.g. \( \frac{X^5}{Y^3} \)), but there were no problems that had one term requiring the use of both rules (e.g. \( \frac{Y^5 X Y^7}{Y^{10}} \)). This gives the instructor in question a point from which to further develop the problem solving through deeper problem posing, i.e., through dialogic think aloud face to face sessions, students are asked to observe the structure of the given problem and state the similarity to all rules where similarity is observed (this led to examples of posed questions which led to making complex exercises by the teacher-
researcher). This increases student repertoire in problem solving as evidenced in the following quiz and test.

**Problem Posing Illustration 3.** In this illustration, we provide the triptych used in Statistics classes (also used in Arithmetic and Algebra, but not included here), developed through Koestler’s work of the development of creativity. A triptych in Koestler’s usage is a collection of rows as shown below where the columns indicate humor discovery and art. In order to get to the discovery of the central concept, the learner can work their way into probing the concept through some word that is known and even funny. Students are provided the triptych below with two rows completed. These completed rows are discussed in class as to whether they make sense. Students clarify their understandings in the discussion. It is then expected that students will complete all rows of the triptych and write a couple of sentences of explanation of the connections between the 3 words. When all students have submitted their triptychs, the class triptychs are placed on the electronic platform, Blackboard, and students view and reflect on each others’ work, and create a new triptych for the end of the semester again with a few sentences explaining the connections of the concept and its illustration across the row of the triptych.

**Statistics Triptych**

Trailblazer ↔ Outlier ↔ Original/ity

↔ Sampling ↔

↔ Probability ↔

↔ Confidence interval ↔

↔ Law of large numbers ↔

Lurking variable ↔ Correlation ↔ Causation

**The Algebra and Arithmetic Triptych**
These classes need greater scaffolding with the triptych and here the elements of the triptych are introduced Just-in-Time as the topic under consideration is being covered in the class. Hence for example:

Powers ↔ decimal representation ↔ polynomial is discussed during the chapter on polynomials. The following is the general strategy of facilitation of discovery and understanding from the teacher-researcher’s perspective:
Problem posing is a constant in the discovery oriented enquiry-based learning environment. Operations on integers and i.p., adding and subtracting with visualizing of the number line forms the basis for ongoing questioning and posing of a problem between students and teacher-researcher.

Algebra as the field of making sense of structure simultaneously with making sense of number provides opportunities for problem posing along the Particularity \(\leftrightarrow\) Abstraction \(\leftrightarrow\) Generality of the Arithmetic-Algebra spectrum. In Algebra classes, it is harder to ease use of scaffolding, and problem posing occurs solely on the side of the teaching-research team in wondering about the way to include triptychs in the Learning Environment mix. In the process, the triptych rows evolve into ‘simpler’ usable forms.

Fig. 6 Algebra Triptychs
Discussion and Results. The results discussed below were obtained after 3 TR cycles for this work. They will be incorporated into the next TR cycle based on the described ideas and practice. We have discussed how our cyclical involvement in TR-NYC model of teaching research to solve the problem of our classrooms – students’ understanding and mastery of mathematics led us to pose to ourselves a general question: what are the necessary components of student success in mathematics? Our answer to this problem investigated in the teaching experiment Jumpstart to Reform directed our attention to student creativity as the motivating factor for their advancement in learning. In turn, our facilitation of student creativity is scaffolded by a series of posed problems/questions designed either by the teacher or students of the classroom. The quantitative results (Appendix 1) of the teaching experiment Problem Solving in Remedial Mathematics – Jumpstarting the Reform supported by C^3IRG 7 awarded to the team in 2010 confirm the impact of the approach for the improvement of student problem solving capacity. We point out that the art of posing series of problems scaffolding student understanding strongly depends on teachers’ judgment concerning the appropriate amount of cognitive challenge. Solving these problems in practice leads again to posing of a general question, which, in agreement with the principles of TR/NYCity leads beyond the confines of our classroom: What is a learning trajectory (LT) of, for example, fraction in my classes? We illustrate the learning trajectory for fractions that developed over the period 2006-2012, with some movement at times, none at others and a lot more when students are active learners. Problem posing has been an active element in that process within the student - teacher mutual understanding.

(a) Meaning of fraction established and revisited
(b) How to visualize fractions? Fractions Grid developed as a visual tool (Czarnocha, 2008)

Over time the following LT developed:

![Learning Trajectory for fractions](image_url)

**Fig.7 Learning Trajectory for fractions.**
(i) Equivalent fractions visualized – operation: scaling – visualize with FG and then scaling
(ii) Increasing, decreasing order arrangement- prime factorization – common denominator – FG and then reasoning; common denominators are meaningful before any other standard operations
(iii) Addition and subtraction
(iv) Multiplication
(v) Division
(vi) Transition to language – what is half of 16?...

(c) Proportional reasoning: Picture in various versions – seeing the interconnectedness of fraction in different representations: decimal, percent, pie chart
(d) Meaning of fraction revisited.

This learning trajectory will be refined through subsequent cycles of the course. Developing the LT for fractions is an illustration of how problem posing works in the context of a satisfactory handshake on the part of learners. Problems utilizing exponents is an example of active problem posing leading to its successful integration by learners. The mastery of the language of mathematics through self-directed attention to reading comprehension is an example of how the repertoire needed for problem posing and solving needs to be consistently built up.

The development of several Learning Trajectories one of which is shown here demonstrate the usability of the methodology and developed materials for a much larger audience of students who fall in the category of self-proclaimed "no good at math" "dont like math" etc. The process of development of Learning Trajectories proceeds through the elimination of learning difficulties in the collaborating classrooms. Repeated problem posing-problem solving dynamic increases learners’ repertoire of recognizing own moments of understanding and the emerging pattern of understanding. Writing as the medium utilized for learning to write and writing to learn, makes the understanding lasting, concrete and reusable by learners.

The overarching result is that a discovery-based approach to the learning of Basic Mathematics coupled with due attention to cultivation of positive affect is found to sustain development of learning “how to learn”. The learning environment so created is thus a creative learning environment in that it is capable of stimulating creative moments of understanding and extending these to pattern of understanding that transforms learners' habits of doing/learning mathematics to an enquiry oriented approach that fosters enjoyment and consequently boosts performance. Students’ didactic contract/handshake toward their own learning markedly improves in having found mathematics to be enjoyable and the success in tests boosts confidence and wanting to achieve. Fear with which the class starts the semester and the accompanying it resistance to learning are non-existent in the majority of the class and the two that continue to hold on to it are a
minority and begin taking greater interest. The emphasis on classroom creativity outlines the pathway across the Achievement Gap.

**Conclusion.** Mathematics as the creative expression of the human mind, is intrinsically questioning/wondering why and how, and through reflection/contemplation, gaining insight through careful justification on the answers to the questions posed. Problem posing and problem solving are thus the core elements of ‘doing mathematics’. In contemporary contexts of teaching and learning of mathematics, this core of mathematics, is hidden from sight, and a syllabus, learning objectives, learning outcomes, etc., are more prominent, making mathematics seem like a set of objectives and sometimes even called skills to be mastered by the student who is then considered proficient or competent in those skills. The high failure rate in mathematics starting as early as third grade (MSP-Promyse, 2007), dislike of mathematics reflected not just among students, but societally, the low number of students seeking advanced degrees in mathematics are reflective of mathematics not being appreciated for what it is – the quest of the human mind toward knowing, and wanting to know why and how.

In the particular context of community colleges of the Bronx of the City University of New York, and analogously the large percentage of high school students who need remedial/developmental mathematics courses in college, problem posing has to be directly connected and on a regular basis with the classroom curriculum. The objective is urgent: closing the achievement gap. The problem as it exists, is that absence of proficiency in mathematics (i.e., scores on placement tests) could well prevent students from college education. The question is how to change this trend? (Knott, 2010) states, “Recent developments in mathematics education research have shown that creating active classrooms, posing and solving cognitively challenging problems, promoting reflection, metacognition and facilitating broad ranging discussions, enhances students’ understanding of mathematics at all levels. The associated discourse is enabled not only by the teacher’s expertise in the content area, but also by what the teacher says, what kind of questions the teacher asks, and what kind of responses and participation the teacher expects and negotiates with the students. Teacher expectations are reflected in the social and socio-mathematical norms established in the classroom.”

(Vygotsky, 1978.) describes the Zone of Proximal Development (ZPD) as “the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or collaboration of more capable peers” P.86. In the classroom environments we encounter, where to be effective, the classroom environment requires a careful integration of simultaneous attention to cognition, affect and self-regulatory learning practices (Prabhu, Barbatis, Watson, 2012), the ZPD has to be “characterized from both cognitive and affective perspectives. From the cognitive perspective we say that material should not be too difficult or easy. From the affective perspective we say that the learner
should avoid the extremes of being bored and being confused and frustrated (p.370)” (Murray & Arroyo, 2002).

Teaching and learning in a teaching-research environment is necessarily collaborative, as our work has demonstrated. This collaborative nature creates an open community environment in the classroom, which is beneficial to the problem posing requirement. Mathematics as enquiry, as enjoyment and as development of the thinking technology do not remain terms or unfamiliar notions to learners. In the span of one semester, college readiness has to be achieved so that the regular credit bearing mathematics courses can be completed satisfactorily. Enquiry facilitating discovery, the modus operandi, now possible to take hold because of the creative learning environment, has provided learners with the keys to success in learning and understanding of mathematics.

The problem posing style of education in general whether Freire (Freire, 2000), “reading the world”, or in the style of Montessori (Montessori, 1972), in the design of the learning environment, all find use and applicability in the Remedial Mathematics classroom. Further, the Discovery method, or Moore method as learned from William Mahavier, Emory University, (Mahavier, 1999) successful in calculus classes, now finds a route into generating learners enjoying and performing well in mathematics in Remedial classes, leading the route to the closing of the achievement gap and creating readiness for higher level mathematics classes.

References


Appendix 1

Problem Solving in Remedial Mathematics: A Jumpstart to Reform.
William Baker, Bronislaw Czarnocha, Olen Dias and Vrunda Prabhu
Short Summary and report from the project

Implementation

Four professors were involved in the TR experiment at two different community colleges in an urban environment Professors: Prabhu, Dias, Baker and Czarnocha. The control groups (class sections) used a more traditional curriculum of strictly class lecture organized by topics: whole numbers, fractions, decimals, ratio and proportions and percents. The experimental group used modified discovery learning and modules focused on problem solving. Each instructor taught one experimental and one control section.

Statistical Analysis of the results

A pre-test (5 questions) was administered at the beginning of the semester to students and a post-test (8 questions) at the end to students in all sections of both the control and experimental sections, there were 4 common questions to both exams. These tests focused on students’ abstract structural (Sfard, 1991) understanding of the second strategy phase of Polya’s stages of problem solving more specifically the transition from initial reading to strategy formation. In the control group there were 46 students that completed both the pre- and post-tests while in the experimental group there were 34. The mean score of the control group on the pre test was 43% and on the post-test it was 42% the mean score of the experimental group on the pre-test was 40.1% and on the post-test it was 54.5%.

Pre & Post Results (all instructors)
Control group: (N=46)
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<td>Total Score</td>
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<td>42%</td>
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Experimental group (N=34)
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<td>Total Score</td>
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The four common or repeated exercises are listed individually for the experimental group (N=34)
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<tr>
<th>Exercise</th>
<th>Pre%</th>
<th>Post%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exercise #1) Ratio</td>
<td>33.8%</td>
<td>46.3% (p=0.1) not significant</td>
</tr>
<tr>
<td>Exercise #2) Division</td>
<td>63.2%</td>
<td>61.7% (p=0.3) not significant</td>
</tr>
<tr>
<td>Exercise #3) Operator</td>
<td>38.2%</td>
<td>61.7% (p=0.04) significant</td>
</tr>
<tr>
<td>Exercise #4) Quotient</td>
<td>58.9%</td>
<td>79.4% (p=0.05) borderline significant</td>
</tr>
</tbody>
</table>

Significance is taken to be p<0.05.
The values of the pretest for the control (43%) and experimental groups (40%) were not significantly different (p=0.78) and thus, we fail to reject the null hypothesis that the pre and post test means are. In contrast, for the experimental group (p=0.0001) we reject the null hypothesis. Thus, we conclude that, the mean scores for the pre and post tests are significantly different which demonstrates a substantial improvement in the experimental group students’ ability to think about problem solving in an algebraic or structural manner.

The difference between the control (42%) and experimental (54.7%) mean scores on the post test was significant at the 0.01 level (p< 0.001). Thus, we reject the null hypothesis that the means for these two groups are the same and instead conclude that the mean score for the experimental group was significantly higher. The difference between the control group pre-test (43%) and the experimental group (40%) was not significantly different (p=0.48). However, the mean score for the control group post-test (42%) was significantly different than the experimental group post- test (54.7%, p=0.02). That the difference between the pre test scores for the control and experimental groups were not significantly different but the post test scores were indicates that while both groups started at the same level of ability with structural problem solving the experimental group outperformed the control group by the end of the semester.

This indicates that the problem solving module approach implemented through guided discovery learning had a real impact on students’ ability to think about problem solving in a structural manner that would allow them to understand solution strategies for typical word problems involving concepts of: ratios, quotients, operator-fraction as well as percent.

Appendix 2. How to read a concept map?
First we look at the general structure of the concept map, where we find four components of learning environment: Creative Problem Solving set, student Success Manual, Making Sense of Numbers – a component of pedagogy, and the collection of instructional materials Story of Number. Each of those components have branches going up explaining the content or the method utilized by each, and they have branches going down, which shows the relation the particular component makes with Cognition, Affect and Self-regulatory learning which found themselves to be necessary student success in mathematics.

Second, we can investigate branches of each component separately paying attention to particular concepts (in boxes) and to connecting phrases (no boxes). We can proceed down or up along the branches, depending on preference. For example we see that the Creative Problem Set connects cognition with affect by facilitating enjoyment in problem solving and thus in learning mathematics, which leads to the improved performance.