# TEACHING PERCENTAGES THROUGH THE TABLE METHOD

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**Abstract**

Two-way tables can be used to present data from a word problem in a concise form, which helps to analyse and solve the problem. In this paper we apply two-way tables to word problems that involve percentages. As research shows, the topic of percentages is one of the difficult topics in high school mathematics. The suggested approach is intended to help students to overcome the difficulties in learning percentages. This method includes creating a two-way table with numerical information from a percentage problem, and using it to derive a proportion and a simple equation for the unknown value of interest. This method can be applied to most types of percentage problems.

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**INTRODUCTION**

Research on teaching high school mathematics has identified some topics causing particular learning difficulties. Ratio and proportion were included in the list of such topics in the well known study by Hart (1982). Students also have difficulties with the related concept of percentage, as shown in studies by Parker and Leinhardt (1995), Moss and Case (1999), Moss and Caswell (2004), Dole (2000), and White and Mitchelmore (2005). These studies investigate different strategies in teaching percentages. White and Mitchelmore (2005) suggested a method that "... put more emphasis on why percentages are used than on how they are calculated." They also concluded: "Calculation skills are another area requiring attention..." (White & Mitchelmore, 2005).

In this paper we suggest an approach that helps students to improve calculation skills in percentage problems with different contexts. In this approach the quantitative information from a problem is presented in a two-way table, then the table is used to derive an equation and calculate the value in question. This method is algorithmic and quite universal, it works for most standard problems about percentages.

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**DESCRIPTION OF THE METHOD**

We suggest using two-way tables to solve typical problems about percentages. Such a table has two parallel columns: one contains amounts with units such as kg, cm, $, and the other contains corresponding percentages. All the information given in the problem is entered in the table together with $x$ denoting the unknown value of interest. Next, we highlight an *important square*, that is the square with four non-blank cells, which contains $x$. The following steps complete the solution.

Step 1. Transfer the important square into a proportion.

Step 2. Cross-multiply and get an equation.

Step 3. Solve the equation for $x$.

The following example demonstrates this procedure.

**Example 1.** A jar contains 4 kg of salt solution with concentration of 3%. How much salt is in the jar?

**Solution**: We create a two-way table, which shows the amounts of salt and solution in kilograms and percentages:

| | kg | % |
|---|---|---|
| Salt | $x$ | 3 |
| Solution | 4 | 100 |

| | |
|---|---|
| $x$ | 3 |

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The information from the | 4 | 100 | question was entered in the table with $x$ for the amount of salt. In this simple case the important square contains all four numerical cells of the table:

Step 1. Transfer the important square into a proportion: $\frac{x}{4}=\frac{3}{100}$.

Step 2. Cross-multiply and get an equation: $100x = 4\times 3$.

Step 3. Solve the equation for $x$: $x=\frac{4\times 3}{100}=0.12$.

There is 0.12 kg of salt in the jar. □

Later we will apply the method to more sophisticated problems. The advantage of the method is its versatility. High school courses (see for example, Barton, 2009; Barton, 2008; Barton, 2010) usually teach percentages as several sub-topics such as:

1) calculating percentages ‘of’ quantities,

2) working out the ‘original’ quantity,

3) increasing and decreasing by percentages,

4) calculating percentage changes,

5) calculating percentage profit,

6) calculating bank interest, etc.

In the book by Immergut (2003) the chapter on percents contains such sections as percent increase and decrease, discounts, interest, profit and loss. The described

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technique of two-way tables applies to all these types of problems and does not require identifying a sub-topic or memorizing the corresponding instructions. The students only have to create a two-way table, fill it with given information, possibly perform a couple of simple additions or subtractions and follow the described Steps 1-3. This procedure is more straightforward and less mistake-prone than usual instructions in textbooks such as: "To calculate a percentage of a given quantity, multiply the quantity by the percentage. The percentage can be written as a fraction or decimal first." (Barton, 2008).

## APPLICATION OF THE METHOD TO TEACHING

Next we illustrate the method with further examples in the increasing order of difficulty. All the examples were created by the article authors but the reader can find many similar problems in mathematics textbooks. The novel part here is the solution method.

**Example 2.** Helen earned $84 and spent 35% of this money. How much money is left?

**Solution**: We create a two-way table:

| | $ | % |
|---|---|---|
| Earned | 84 | 100 |
| Spent | | 35 |
| Left | $x$ | 100 – 35 = 65 |

This is the important square (it is also highlighted in the table above):

| | |
|---|---|
| 84 | 100 |

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| | |
|---|---|
| $x$ | 65 |

Step 1. Transfer the important square into a proportion: $\frac{84}{x}=\frac{100}{65}$.

Step 2. Cross-multiply and get an equation: $100x = 84\times65$.

Step 3. Solve the equation for $x$: $x=\frac{84\times65}{100}=54.6$.

There is $54.60 left. □

**Example 3.** Ben paid for a ticket $15 plus 40% of its price. What is the price of the ticket?

**Solution**: We create a two-way table:

| | $ | % |
|---|---|---|
| Total price | $x$ | 100 |
| Part paid in dollars | 15 | 100 – 40 = 60 |
| Part paid in percents | | 40 |

The important square is highlighted in the table above.

Step 1. Transfer the important square into a proportion: $\frac{x}{15}=\frac{100}{60}$.

Step 2. Cross-multiply and get an equation: $60x = 15\times100$.

Step 3. Solve the equation for $x$: $x=\frac{15\times100}{60}=25$.

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The price of the ticket is \$25. □

**Example 4.** John lent \$1200 and after one full year he was paid back \$1560. What rate of interest did John use?

**Solution**: This is a two-way table:

| | \$ | % |
|---|---|---|
| Lent | 1200 | 100 |
| Paid back | 1560 | |
| Interest | 1560 – 1200 = 360 | $x$ |

<u>Step 1</u>. Proportion: $\frac{1200}{360} = \frac{100}{x}$.

<u>Step 2</u>. Cross-multiply: $1200x = 360 \times 100$.

<u>Step 3</u>. Solve the equation: $x = \frac{360 \times 100}{1200} = 30$.

The interest rate is 30%. □

**Example 5.** A buyer offers \$352,000 for a boat, which would infer a loss of 20% on the cost price for the boat owner. What price should the owner ask to make a gain of 20% on the cost price?

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**Solution**: This is a two-way table:

| | $ | % |
|---|---|---|
| Cost price | | 100 |
| Losing price | 352,000 | 100 – 20 = 80 |
| Gaining price | *x* | 100 + 20 = 120 |

Step 1. Proportion: $\frac{352{,}000}{x} = \frac{80}{120}$.

Step 2. Cross-multiply: $80x = 352{,}000 \times 120$.

Step 3. Solve the equation: $x = \frac{352{,}000 \times 120}{80} = 528{,}000$.

The price should be $528,000. □

**APPLICATION TO MORE DIFFICULT PROBLEMS**

The described method also works well on harder problems such as percentage puzzles in the book by Klymchuk (2001). Sometimes a problem about percentages requires a longer table or more than one table.

**Example 6.** Fresh mushrooms contain 72% of water and dried mushrooms contain 20% of water. How much dried mushrooms can you get from 20 kg of fresh ones?

**Solution**: Denote $x$ (kg) the amount of pure mushrooms in 20 kg of fresh mushrooms. It remains the same in the dried mushrooms. We create two tables.

Fresh mushrooms:

| | kg | % |
|---|---|---|

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| | | |
|---|---|---|
| Pure mushrooms | $x$ | 100 – 72 = 28 |
| Water | | 72 |
| Total | 20 | 100 |

Proportion: $\frac{x}{20} = \frac{28}{100}$; cross-multiply: $100x = 20\times28$; $x = \frac{20\times 28}{100} = 5.6$.

Dried mushrooms:

| | kg | % |
|---|---|---|
| Pure mushrooms | $x = 5.6$ | 100 – 20 = 80 |
| Water | | 20 |
| Total | $y$ | 100 |

Proportion: $\frac{5.6}{y} = \frac{80}{100}$; cross-multiply: $80y = 5.6\times100$; $y = \frac{5.6\times 100}{80} = 7$.

You can get 7 kg of dried mushrooms. □

**Example 7.** A boutique owner increased the price of a dress by 60%. However no one wanted to buy the over-priced dress, so the owner reduced its price back to the initial amount. By what percent was the new price decreased?

**Solution**: Denote $a$ the initial price in dollars. We create two tables.

Price increase:

| | $ | % |
|---|---|---|
| Initial price | $a$ | 100 |
| Increase | | 60 |
| New price | $x$ | 100 + 60 = 160 |

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Proportion: $\frac{a}{x}=\frac{100}{160}$; cross-multiply: $100x = 160a$; $x=\frac{160a}{100}=1.6a$.

Price decrease:

| | \$ | % |
|---|---|---|
| New price | $x = 1.6a$ | 100 |
| Decrease | $1.6a - a = 0.6a$ | $y$ |
| Decreased price = initial price | $a$ | |

Proportion: $\frac{1.6a}{0.6a}=\frac{100}{y}$; cross-multiply: $1.6y = 0.6\times100$; $y=\frac{0.6\times100}{1.6}=37.5$.

The new price was decreased by 37.5%. □

**Alternative solution**: This problem can be solved with one big table combining the previous two tables:

| | \$ | % | \$ | % |
|---|---|---|---|---|
| Initial price | $a$ | 100 | | |
| Increase | | 60 | | |
| New price | $x$ | 100 + 60 = 160 | $x$ | 100 |
| Decrease | | | $x-a$ | $y$ |
| Decreased price = initial price | | | $a$ | |

The dollar amount $x$ for the increased price is the same throughout the table while its percent value is not. We can create two proportions from this table, then find an expression for $x$ and the value of $y$.

$$\frac{a}{x}=\frac{100}{160},\qquad x=\frac{160a}{100}=1.6a.$$

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$$\frac{x}{x-a}=\frac{100}{y}, \quad \frac{1.6a}{1.6a-a}=\frac{100}{y}, \quad \frac{1.6a}{0.6a}=\frac{100}{y}, \quad y=\frac{0.6\times100}{1.6}=37.5.$$

The new price was decreased by 37.5%. □

**Example 8.** In March the average fruit price increased by 20%. In December of the same year, the price decreased by 10%. What was the net percentage increase in December from the initial price?

**Solution**: Denote $a$ the initial price in dollars. This is a two-way table:

| | $ | % | $ | % | $ | % |
|---|---|---|---|---|---|---|
| Initial price | $a$ | 100 | | | $a$ | 100 |
| Increase | | 20 | | | | |
| Increased price | $x$ | 100 + 20 = 120 | $x$ | 100 | | |
| Decrease | | | | 10 | | |
| Decreased price | | | $y$ | 100 – 10 = 90 | $y$ | $z$ |

We create three proportions from this table, then find expressions for $x, y$ and the value of $z$.

$$\frac{a}{x}=\frac{100}{120}, \quad x=\frac{120a}{100}=1.2a.$$

$$\frac{x}{y}=\frac{100}{90}, \quad y=\frac{90x}{100}=\frac{90\times1.2a}{100}=1.08a.$$

$$\frac{a}{y}=\frac{100}{z}, \quad z=\frac{100y}{a}=\frac{100\times1.08a}{a}=108. \quad 108-100=8.$$

The increase was 8%. □

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**Example 9.** The marked price on a book was supposed to provide 25% profit on the cost price. During a sale the price was discounted by 10% to \$45. What is the book's cost?

**Solution**: This is a two-way table:

| | \$ | % | \$ | % |
|---|---|---|---|---|
| Cost | $x$ | 100 | | |
| Intended profit | | 25 | | |
| Marked price | $y$ | 100 + 25 = 125 | $y$ | 100 |
| Reduction | | | | 10 |
| Selling price | | | 45 | 100 – 10 = 90 |

We create two proportions from this table, then find the values of $y$ and $x$:

$$\frac{y}{45}=\frac{100}{90}, \qquad y=\frac{45\times 100}{90}=50.$$

$$\frac{x}{y}=\frac{100}{125}, \qquad \frac{x}{50}=\frac{100}{125}, \qquad x=\frac{50\times 100}{125}=40.$$

The cost is \$40. □

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**Example 10.** During a sale a book was sold with 25% reduction and provided 5% profit. What percentage profit was intended on the book before the reduction?

**Solution**: This is a two-way table:

| | \$ | % | \$ | % |
|---|---|---|---|---|
| Cost | $a$ | 100 | | |
| Profit | | 5 | | |
| Selling price | $x$ | 100 + 5 = 105 | $x$ | 100 – 25 = 75 |
| Reduction | | | | 25 |
| Marked price | $y$ | | $y$ | 100 |
| Intended profit | $y - a$ | $z$ | | |

We create three proportions from this table, then find expressions for $x, y$ and the value of $z$.

From the 1st and 2nd highlighted blue rows: $\frac{a}{x}=\frac{100}{105}, \quad x=\frac{105a}{100}=1.05a.$

From the yellow important square: $\frac{x}{y}=\frac{75}{100}, \quad y=\frac{100x}{75}=\frac{100\times 1.05a}{75}=1.4a.$

From the 1st and 3rd highlighted blue rows:

$$\frac{a}{y-a}=\frac{100}{z}, \quad z=\frac{100\times(y-a)}{a}=\frac{100\times(1.4a-a)}{a}=40.$$

The intended profit was 40%. □

**Example 11.** Two coins of weights 30 g and 20 g, respectively, consist of gold, silver and copper. The first coin contains 50% of gold and the second coin contains 25% of copper. The percentage of silver in the second coin is twice the percentage of silver in

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the first coin. When the two coins are alloyed together, the alloy will contain 28% of silver. Find the amount of gold in the alloy.

**Solution**: Denote *a* the percentage of silver in the first coin. Then this percentage in the second coin equals 2*a*. We find both percentages using the following table:

| | Coin 1 | | Coin 2 | | Total: Alloy = Coin 1 + Coin 2 | |
|---|---|---|---|---|---|---|
| | g | % | g | % | g | % |
| Copper | | | | 25 | | |
| Gold | | 50 | | | | |
| Silver | $x$ | $a$ | $y$ | $2a$ | $x+y$ | 28 |
| Total | 30 | 100 | 20 | 100 | 30 + 20 = 50 | 100 |

Using the highlighted important squares we create three proportions and find expressions for $x$, $y$ and the value of $a$:

$$\frac{x}{30}=\frac{a}{100}, \qquad x=\frac{30a}{100}=0.3a.$$

$$\frac{y}{20}=\frac{2a}{100}, \qquad y=\frac{20\times 2a}{100}=0.4a. \text{ So } x+y=0.7a.$$

$$\frac{x+y}{50}=\frac{28}{100}, \qquad \frac{0.7a}{50}=\frac{28}{100}, \qquad a=\frac{50\times 28}{0.7\times 100}=20.$$

We are interested in the amounts of gold in Coin 1 and Coin 2, so we denote them $X$ and $Y$ respectively. Next we re-write the table (only the columns for Coin 1 and Coin 2) replacing $a$ by 20:

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| | Coin 1 | | Coin 2 | |
|---|---|---|---|---|
| | g | % | g | % |
| Copper | | | | 25 |
| Gold | $X$ | 50 | $Y$ | 100 – 25 – 40 = 35 |
| Silver | | 20 | | 40 |
| Total | 30 | 100 | 20 | 100 |

We create two proportions and calculate the values of $X$ and $Y$:

$$\frac{X}{30} = \frac{50}{100}, \qquad X = \frac{30 \times 50}{100} = 15.$$

$$\frac{Y}{20} = \frac{35}{100}, \qquad Y = \frac{20 \times 35}{100} = 7.$$

So the amount of gold in the alloy equals $X + Y = 15 + 7 = 22$ (g). □

**DISCUSSION**

When we use a table, we present information from a word problem in a more transparent and logical form. Using such a table, it is easy to complete the solution following a few simple steps. Filling a table and following the algorithmic steps is easier than solving each type of percentage problems as a unique question (e.g. deciding when to divide and when to multiply) or using lengthy instructions from textbooks. The main challenge for the students applying this method is filling the table properly; it usually gets easier when the students learn to assign 100% correctly, e.g. to a total amount or an old price. In our practice the students understand the idea of the method after 2-3 examples and apply it confidently. The authors use the described approach in university bridging courses.

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Formal statistical analysis of the effectiveness of this approach was not done yet but our case studies show that applying this method helps the students to improve their problem-solving skills, which in turn helps them to improve their general understanding of percentages. Mastering the described tables with percentages makes it easier for the students to study tables with conditional probabilities in probability courses, since there are similarities between the two types of tables.

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