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TEACHING REMEDIAL MATHEMATICS
TO
LEARNING-DISABLED COLLEGE STUDENTS

by

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Abstract

This article summarizes the author's approach to teaching a section of speech-and-language learning disabled students in her community college's remedial elementary algebra course. The approach involves using derivations rather than rules, "why" questions regarding the steps in an algorithm, and frequent assessment to inform subsequent teaching. Where possible, newspaper articles or author-developed handouts tie the subject matter to a real-world context. While constraints imposed by the syllabus necessitate primarily a lecture-based rather than a discovery format, the author's experience suggests that teaching for sense-making can facilitate both conceptual understanding and subject matter enjoyment, even for students with the weakest skill sets.

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Introduction

This article summarizes the author's experiences teaching remedial elementary algebra to a group of entering community college students assigned, based on putative speech and language learning disabilities, to a special mathematics section. The section in question formed part of a learning community intended to support students with weak academic skill sets.

The article first discusses the characteristics of the course and of this group of students. The paper next summarizes current thinking regarding characteristics of learning disabilities, including mathematics disabilities. Ensuing sections address current research on learning disabilities and the author's approach to teaching her elementary algebra course. A brief conclusion follows.

Course and Student Characteristics

The Course

The course in question is Elementary Algebra, the upper level of two remedial

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courses intended to foster success on the COMPASS, the high stakes algebra mastery test required by the City University of New York for graduation and college-level study. Because of the nature of this test, the curriculum covers an inordinate number of topics: operations with signed numbers, algebraic expressions, solving linear equations in one variable (including applications and word problems), exponential expressions, operations with and factoring polynomials, operations with rational expressions, solving quadratic equations (again including word problems), roots and radicals, graphing linear equations and finding the equations of lines, and solving systems of linear equations in two variables. Topics are covered at a gallop. As a result, the course tends to be taught primarily in a lecture format, with the students as passive note-takers (See Hinds, 2009). The curriculum and related study guides tend to emphasize rules and algorithms necessary for success on the test. The reasoning underlying the rules can get lost in the rush to cover the syllabus, thus reducing the "sense-making" necessary to mathematics performance (Shoenfeld, 1992, pp. 335, 339, 340, 344). Yet success on the test is important. Difficulty completing mathematics courses constitutes a significant contributing factor to low graduation rates, both at CUNY (Hinds, 2009; Hostos Community College, 2008) and nationwide (e.g., Biswas, 2007).

The Students

The section in question constituted part of a learning community in which students share all of their classes. The goal is to foster mutual support as the students confront the rigors of college demands (Berger, 2010; Hinds, 2009). Students assigned to

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this particular learning community, having failed upon entrance to qualify for college-level English literature and composition courses without being eligible for English-language-learner courses, are thought to suffer from speech and language issues (Gampert, 2010). Such issues, in turn, are thought to impair mathematics performance as well (e.g., Gersten, Jordan, & Flojo, 2005; Geary, 2004; 1993).

In actuality, of the 31 students initially enrolled in this class, one student, fresh from abroad, demonstrated exceptional mathematics facility combined with lack of familiarity with the English language. Of the 31 enrolled students, 25 registered for MathXL, the Pearson Publishing online homework vehicle designed to complement the course textbook (Martin-Gay, 2007) through both daily homework assignments, which allow multiple attempts to completion, and end-of-chapter quizzes and tests, for which only a single attempt per question is allowed. Discounting the 4 students whose homework scores alone totaled less than 15%, fully 14 (or two thirds) of the 21 remaining MathXL students exhibited a disparity between their homework and overall online scores, thereby suggesting either a need for extra time or overall test anxiety. For example, one student scored 99.8% on the homework yet only 32.9% overall, while two others scored 99.4% and 65.6% on the homework, compared to 48.8% and 33.3% overall. Many of these students also exhibited low academic self-esteem resulting from a learned expectation of failure (see Hagedorn, Sagher, & Siadhat, 2000). Specifically, the students lacked what is known as a "productive disposition—the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence

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and one's own efficacy" (National Research Council, 2001, p. 116; Hinds, 2009). Rather, the students considered mathematics to be intimidating and were anxious to be given step-by-step algorithms into which to substitute numbers in order to pass the test. Finally, as with many other community college students (Berger, 2010; Hinds, 2009; Rameley & Zia, 2005), many of the students in this section suffered from the burden of additional family and financial obligations, some quite severe.

Teaching Students with Learning Disabilities

Nature of the Dysfunction

Learning disabilities traditionally have been regarded as an academic deficit that is both specific rather than global, and unexpected in view of the student's overall intelligence (e.g., Kirk, 1962, Kirk & Bateman, 1962; 1963; Fuchs, D., Fuchs, L.S., Mathes, P.G., Lipsey, M.W., & Roberts, P.H., 2002; Dyslexia Working Group, 2002). In the case of mathematics disabilities, no consensus yet exists as to whether such disabilities stem from a purely numerical deficit (e.g., Butterworth, 2005) or whether they can have roots in more generic linguistic, visual-spatial, or executive function disorders (e.g., Geary, 2004). Nevertheless, difficulty with number facts (Russell & Ginsburg, 1984; Gersten, et al, 2005) and word problems (e.g., Hanich, Jordan, Kaplan, & Dick, 2001) have long been considered the salient characteristics of the dysfunction.

Suggested Teaching Approach

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Compared to the plethora of studies on reading disabilities, research on mathematics learning disabilities is somewhat scant (e.g., National Mathematics Advisory Panel, 2008a).

However, research suggests that strategies such as modeled instruction and the use of visual representations such as mathematical drawings are appropriate for both students with generic mathematics difficulties and those with mathematics learning disabilities (National Mathematics Advisory Panel, 2008b). A teacher-structured explicit learning environment often has been recommended for students with mathematics learning disabilities (for summary, see Woodward, 2004). Nevertheless, for all students, including those with mathematics disabilities (National Mathematics Advisory Panel, 2008b) teaching mathematics for understanding is thought to support development of the long-term concept retention necessary to both current comprehension and further progress (e.g., Tall, 2008; Hinds, 2009). Whereas short-term rule memorization is fragile and error prone, conceptual understanding is long-lasting (e.g., Tall, 2008; Hinds, 2009), thus permitting the recovery of an algorithm forgotten during the stress of a high-stakes test (Weber, 2002). Gaining conceptual understanding therefore supports rather than impedes the mastery of procedural learning (National Research Council, 2001). In addition, research with remedial mathematics students supports the efficacy of frequent teacher-developed assessment as a tool for promoting study skills and for facilitating subsequent classroom re-examination of areas in which students demonstrate weakness (Hagedorn, et al., 2000; Hinds, 2009).

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Recent research at CUNY with several cohorts of GED graduates supports the use of the strategies—teaching for understanding, coupled with frequent assessment--just described. Such graduates traditionally have had lower pass-rates for mathematics than for other subjects on the GED exam and, once at CUNY, a lower pass rate than other high school graduates on the COMPASS exam. Providing such graduates with a semester-long program of reading, writing, mathematics, and academic advisement, in which the algebra component was taught with fewer topics for more in-depth understanding, was found to promote dramatic improvement in such students' COMPASS algebra test pass-rates (Hinds, 2009). Algebra was taught in a real-world context, with fewer topics and frequent questioning and assessment (Hinds, 2009).

The Author's Teaching Approach

As indicated above, many of the students in this section intended for supposed speech-and-language learning disabled individuals manifested fear of failure in general and of mathematics in particular rather than speech and language issues. Nevertheless (albeit subject to the constraints resulting from the demands of syllabus), the author employed her generic approach. This approach involves using a variety of verbal and pictorial methods to explain the concepts underlying the rules, with the goal of facilitating concept understanding and retention rather than short-term rule memorization. In addition, the Socratic Method is employed to inquire why each step is taken, newspaper articles are circulated to demonstrate the current relevance of the procedures being learned, and frequent assessments are used to revise instruction based on student

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misunderstandings. Although constraints imposed by the length of the syllabus result in an approach that is primarily lecture-based, the frequent questioning and assessment strategies are similar to those employed with success in the GED experiments (Hinds, 2009).

Teaching for Conceptual Understanding

An example from the elementary algebra course is the division of a polynomial by a monomial. Consider the following problem:

$$\frac{16x^8 + 8x^4 + 2x^2}{2x^2}.$$

Based on the author's experience, students' instinctive reaction is to cancel the two terms of $2x^2$, thus producing the erroneous result of $16x^8 + 8x^4$. However, if students can learn to see division as multiplication by the reciprocal of the divisor, with the distributive property resulting in multiplication of every component of the numerator $(16x^8 + 8x^4 + 2x^2)$ by $\frac{1}{2x^2}$, then it becomes apparent that each term of the numerator must be divided by the monomial $2x^2$. This analysis produces the equation $\frac{16x^8}{2x^2} + \frac{8x^4}{2x^2} + \frac{2x^2}{2x^2}$, in turn producing $8x^6 + 4x^2 + 1$, the correct result. While a number of the weaker students persist in treating the term $\frac{2x^2}{2x^2}$ as 0 rather than as 1, the approach just described nevertheless seems to produce better results than simply stating the rule that each term of the polynomial must be divided by the monomial.

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Using the Socratic Method

Using the technique experienced in law school, the author peppers her explanations with "why" questions regarding the reasoning behind each step in a given algorithm. This approach is also useful for reinforcing the reasoning behind previously-introduced concepts, as in the use of the distributive property demonstrated above. An approach similar to this proved efficacious in the CUNY GED research described above (Hinds, 2009).

Current Events Context

In this course, as in her others, the author circulated newspaper articles that demonstrate a real-world context for many of the topics addressed in the course. For example, the current recession has provided ample examples of signed numbers, while both the collapse of the credit derivatives market and the flooding of New Orleans during Hurricane Katrina provide examples of the failure of incomplete mathematical models. Other examples are the author-developed handouts that use the boiling and freezing temperatures of water in both Fahrenheit and Celsius to demonstrate the derivation of an equation of a line from two points (Appendix A), and the use of platform shoes to demonstrate the concept of the slope of a line (Appendix B).

Frequent Assessment

In addition to the four cumulative exams required for this course, the author gives

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weekly quizzes designed to reinforce student learning and to elicit areas of student weakness. Each quiz, approximately seven to eight questions long, is intended to result in "aha" moments by pairing related examples that require different results. For example, under the rules governing exponents, anything raised to the zero power equals one, because dividing any combination of numbers and variables by itself (thus resulting in a zero power under the Exponent Quotient Rule) results in the number one. In particular, $(-7x)^0$ equals one, because $\frac{(-7x)^1}{(-7x)^1} = (-7x)^{1-1} = 1$, just as $\frac{(-7x)}{(-7x)} = 1$. Two things that are equal to the same thing (here, the number 1) must be equal to each other (here, $(-7x)^0$ and $\frac{(-7x)}{(-7x)}$). By contrast, $-7x^0 = -1 \bullet 7 \bullet x^0$, which equals $-1 \bullet 7 \bullet 1$, which equals -7. The paired problem posed in the quiz on exponents is meant to evoke this analysis.

Similar examples can be found in a quiz on operations with radicals and a quiz on factoring. For instance, a quiz on radicals might compare the division of $\frac{\sqrt{16z^3}}{\sqrt{4z}}$ (which equals $\sqrt{\left(\frac{16}{4}\right)\left(\frac{z^3}{z}\right)}$, which equals $\sqrt{4z^2}$, which equals 2z), with the division of $\sqrt{\frac{16z^3}{4z^2}}$ (which equals $\sqrt{\left(\frac{16}{4}\right)\left(\frac{z^3}{z^2}\right)}$, which equals $\sqrt{4z}$, which equals $2\sqrt{z}$). Similarly, a quiz on factoring trinomials might contrast $x^2 + 7x + 10$, which factors as $(x+2)(x+5)$, with $2x^2 + 14x + 20$, which factors as $2(x+2)(x+5)$. The results of these

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weekly quizzes are then used to orient subsequent teaching.

Conclusion

The teaching methods just described (eschewing rules in favor of derivations, asking frequent “why” questions, and using frequent assessments to inform subsequent teaching) are similar to those used with success in teaching elementary algebra to CUNY’s GED students. While students are initially perplexed by the teaching approach just described, which conflicts with their desire simply to be given rules into which to substitute numbers, the majority comes to enjoy and prosper under this approach. As stated by one student on an anonymous voluntary questionnaire on teaching methods administered by the author to an earlier section of this course, “The professor didn’t just give the problems and solved them, she explained the rules and broke it down for us to understand better ” (Cunningham, 2011). There exists no silver bullet for teaching remedial mathematics successfully, particularly to learning disabled students. However, teaching for sense-making rather than for rule memorization can facilitate both conceptual understanding and subject matter enjoyment, even for students with the weakest skill sets.

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Appendix

Various Methods for Solving a Word Problem Involving a Linear Equation in One Variable

Suppose that the sum of *3 consecutive odd ("impar") integers* is 39, and that you are asked to find *all three* integers. Here are three of many different ways of solving the problem.

Alternative I

Define Your Terms

Let x = the 1st integer
Then x + 2 = the 2nd
And x + 4 = the 3rd

Write and Solve Equation

1st + 2nd + 3rd = 39	Simplify word problem
x + x+2 + x+4 = 39	Translate equation using your defined terms
3x + 6 = 39	Combine like terms
-6 -6	Subtraction Principle of Equality
3x = 33	
3x/3 = 33/3	Division Principle of Equality
x = 11	Solve for x
x = 11 = 1st	
x+2 = 13 = 2nd	**Answer the question using your defined terms**
x+4 = 15 = 3rd	

Alternative II

Define Your Terms

Let x = 1st integer
Then x – 2 = 2nd integer
And x - 4 = 3rd integer

Write and Solve Equation

1st + 2nd + 3rd = 39	Simplify word problem
x + x-2 + x-4 = 39	Translate equation using your defined terms
3x – 6 = 39	Combine Like Terms
+ 6 + 6	Addition Principle of Equality
3x = 45	
3x/3 = 45/3	Division Principle of Equality
x = 15	Solve for x
x = 15 = 1st	**Answer the question using your defined terms**
x - 2 = 13 = 2nd	

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$x - 4 = 11 = 3^{rd}$

Check 39 PLEASE TURN OVER!

Solving Word Problems Involving Linear Equations in One Variable (continued)

Alternative III

Define Your Terms

Let $x + 1 = 1^{st}$ integer
Then $x + 3 = 2^{nd}$ integer
And $x + 5 = 3^{rd}$ integer

Write and Solve Equation

$1^{st} + 2^{nd} + 3^{rd} = 39$ Simplify word problem
$x+1 + x+3 + x+5 = 39$ Translate equation using your defined terms
$3x + 9 = 39$ Combine Like Terms
$-9 \quad -9$ Subtraction Principle of Equality

$3x = 30$

$3x/3 = 30/3$ Division Principle of Equality
$x = 10$ Solve for x
$x + 1 = 11 = 1st$ **Answer the question using your defined terms**
$x + 3 = 13 = 2^{nd}$
$x + 5 = 15 = 3^{rd}$

Check 39

These are some of the multiple ways of solving word problems. You are the artist! However, once you decide how to define the first term of the problem, all remaining terms must be defined in terms of that first term. Then simplify the word problem into a few simple words and translate it into algebra by substituting your defined terms. Solve the problem, being sure to answer the question and not just solving for the variable that you have chosen. Then check your work. Once you get the hang of it, solving word problems is a lot of fun. Go for it! And have fun!

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