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Problem Solving in Pre-Algebra and College Level Mathematics

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Abstract

Problem solving is one of the most important and challenging areas of teaching and learning mathematics. While there is considerable educational research concerning the obstacles secondary students experience in problem solving, there is less research with adult pre-algebra students. Adult students who require pre-algebra in the United States frequently enroll in remedial mathematics courses, often at community colleges. These students generally struggle with mathematical problem solving; especially formulating and following up on a reasonable plan towards a solution. In this study the ability of remedial pre-algebra students to engage in metacognitive thinking while solving a proportional reasoning exercise was measured and compared to the same data for their peers in a college level mathematics course at an urban community college. Polya's four phase problem solving heuristic was used as a guide to assess students' ability at each phase of the problem solving process. A schematic diagram of the results for the pre-algebra students is used to suggest an appropriate pedagogy for problem solving in remedial mathematics.

Literature Review

Mathematics and Problem Solving in the United States

In the late 80's and early 90's reports from the National Research Council, the National Assessment of Educational Progress and other studies indicated that the mathematical skills of secondary school students in the U.S. was lagging behind the ability of students in other industrial nations particularly in the area of problem solving

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(Bottage, 2001 and Jitendra, et. al., 1998).

American society is struggling with the notion that we cannot do mathematics. Willingham (2009) states, "…it seems as though our society has accepted the 'fact' that math is not for most of us. The problem is that this notion is a myth" (p.14).

Remedial Mathematics

Many high school graduates with weak mathematics skills enroll in remedial mathematics courses in college. At Hostos Community College in the Spring 2010 semester, about 75% of the incoming Freshman were enrolled in a remedial or developmental mathematics course. Students who fail both portions of a placement exam are placed in remedial pre-algebra and are considered 'high risk' because of low pass rates in the course and even lower completion rates through the remedial sequence (Akst, 2005).

Mathematics Reform

One approach to students' difficulty with mathematics has been to reform the way in which mathematics is taught in the classroom. Several sources castigate the current traditional curriculum which focuses on rote computational skills to obtain the correct solution instead of the process of solving the problem (e.g. Rivera, 1997, Xin, 2007 and Blair, 2006). Reform advocates favor an approach that allows students to monitor their own thinking and call for mathematics educators to act as guides through the problem solving process (e.g. Blair, 2006, Rittle-Johnson & Star, 2007).

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With so much emphasis on reforming mathematics instruction to improve students' problem solving abilities, one can legitimately ask, "Has anything changed in the mathematics classroom?" Lithner (2008) notes that even after 20 years of research and reform, the majority of students still cling to rote memorization techniques and do not apply new problem solving strategies introduced by these educational reforms.

Problem solving

Polya defined problem solving as "searching for an appropriate course of action to attain an aim that is not immediately attainable" (Bottage, 2001, p.105). Thus, one distinct feature of an application problem is uncertainty on the part of the problem solver. Fuchs et. al. (2002) further describes problem solving as a skill which,"…requires students to apply knowledge, skills and strategies to novel problems" (p.90).

Fuchs et. al. (2008) make a distinction between computational exercises and application problems very concisely; "Whereas a computation problem is already set up for solution, a word problem requires students to use the text to identify missing information, construct the number sentence, and derive the calculation problem for finding the missing information" (p.30).

The distinction between what is novel versus computational raises questions about the competency or comfort level of an individual when presented with a mathematics exercise, what is routine for one is a novel problem for another. Some researchers have noticed that students 'affect' or reaction to a problem solving situation can influence their ability to solve a problem. "In certain students' performance the emergence of affective factors was so striking that we assumed the point of view of considering together

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affective and cognitive factors and their role in the process of proving" (Furinghetti et.al., 2009, p.73). The importance of 'affect' in the role of learning mathematics is highlighted by the American Mathematics Association of Two Year Colleges (AMATYC) when they describe the difficulties many community college students have engaging with mathematics, "some students believe that mathematics is about computation and that they are to find the correct answer in five minutes of less. They believe they are to be passive in the learning process…depending upon the degree of mathematics anxiety, the student's fears can develop into "learned helplessness," the belief that one is unable to do mathematics at all" (Blair, 2006, p.21).

Theoretical Framework: Phases of problem solving

We now investigate more proficient and less proficient problem solvers using Polya's problem solving phases because this structure highlights the practice of metacognition (1973). Polya's four phases of problem solving are:

1. Understanding the Problem (orientation).
2. Devising a Plan (organization).
3. Carrying Out the Plan (execution).
4. Looking Back (verification).

Phase I

In the first phase students become oriented to the problem and ask, "what is the problem about?" Kramarski et. al. (2002) characterize low achieving students as not reading thoroughly and/or not able to discern relevant information. "Low achievers, do not see the task as a whole, and thus focus on only parts of the task…low achievers read

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rapidly the task at the expense of fully comprehending it" (p.226). Other researchers have found that low achieving students have difficulty identifying and ignoring extra information in word problems which causes them to become disoriented in the first phase of solving the problem (Jitendra et. al., 1998). Despite the importance of this phase and the struggles low achievers have with it, Fan and Zhu (2007) found that only 27.8% of the problems in nine U.S. secondary school mathematics textbooks contained solutions that modeled this phase.

Phase II

In the second phase of Polya's method, students must make a general plan and select relevant methods or appropriate heuristics for solving the problem. This second phase involves relating the information from phase I with one's previous knowledge or problem-solving schema (i.e. one's internal representation of similar experiences). Students who have an appropriate schemata or internal representation of similar experiences can synthesize this information into a strategy or plan. Xin states, "Successful problem solvers seek and find underlying structural information (e.g. problem schemata), where as unsuccessful problem solvers tend to focus on the surface features of problems making it difficult for them to transfer their learning to a wide range of structurally similar problems" (p.347).

Anecdotal experience in the classroom and educational research suggests that the traditional approach of instruction where the teacher presents a model problem for the students to replicate tends to favor high achievers. Mevarech and Kramarski note that,

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"high achievers were both better able to utilize the worked-out examples than lower achievers since the former were able to utilize their prior knowledge not only during the process of understanding the worked-out examples but also when they tried to generalize the conditions under which it can be applied" (p.466). High achievers have the ability to internalize and generalize examples done in class or examples from the textbook, whereas low achievers were not able to transfer the skills that the teacher demonstrated to new problems (e.g. Bottge, 2001, Mevarech & Kramarski, 2003, Fan & Zhu, 2007).

Phase III and IV

In the third phase of Polya's method (execution), students perform the computations required to implement the plan devised in the second phase, keeping on track to obtain the solution. Fan and Zhu (2007) found, this third phase was modeled by 100% of the problems and arguably receives the bulk of the attention in the classroom when instructors model problem solving on the board. In the fourth phase (verification) students review what they have done, check the correctness of their solutions and reflect on key ideas and processes in order to generalize or extend these processes and results. This fourth phase was modeled in 43.2% of the problems in U.S. textbooks involved in the study by Fan and Zhu (2007).

Setting

The sample included two groups of students the first, 117 adult students taking

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developmental pre-algebra (DPA) and the second, 71 students taking a college level mathematics course for Allied Health Majors (a.k.a. College Math for Allied Health or CMAH). All students were enrolled at Hostos Community College, an urban community colleges in the City University of New York (CUNY) system. The student body at Hostos Community College is predominately comprised of females (70%) and minorities (95%).

The first group of DPA students is considered the mathematically weakest group of adult students in the CUNY system, because they have been placed in this course after failing both the arithmetic and algebra components of the placement exam and are considered ‘high risk’ due to their low pass rates. The pre-algebra course contains; whole numbers, fractions, decimals, ratio and proportions, percents and an introduction to basic algebra topics.

The second group of CMAH students had passed both parts of the placement exam and been placed in a college level mathematics for nursing course. This course makes frequent use of dimensional analysis, a form of proportional reasoning to solve application problems in pharmacology. These students are motivated to enter into their chosen profession and the course has much higher completion and pass rates than the developmental mathematics courses. Although the mathematics for nursing majors course does use fractions, there is no formal review of this topic in the course.

Method

Both groups of students were given a proportional reasoning exercise with a worksheet that contained four tasks that evaluated their reasoning in Polya’s four

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problem-solving phases. The students were given the worksheet with 20-25 minutes at the end of class, they were informed that their participation was voluntary and would not impact their course grade.

> Exercise: Juan and Maria are making lemonade for lunch by mixing cups of sugar with glasses of water that are the same size. Maria who is on a diet uses one cup of sugar for every three glasses of water, Juan who likes sweet lemonade uses three cups of sugar for every eight glasses of water. If each glass contains exactly two cups then whose lemonade is going to be sweeter?

Phase 1: Understanding the Problem

The first task measured students' ability to read and understand the information presented in proportional task and how this information could relate to solving the task.

> Task #1) Circle ANY & ALL of the following that are true:
> I) The amount of sugar that Juan & Maria use is necessary to answer the problem.
> II) The amount of water that Juan and Maria use is necessary to answer the problem.
> III) The fact that the glasses used are exactly two cups is necessary to answer this problem.
> IV) The fact that Juan likes sweet lemonade and that Maria is on a diet can be used to answer this problem.
> V) None of the above can be used to determine whose lemonade is sweeter.

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Phase 2: Devise a Strategy or Plan

In the second task, students were presented with several strategies formulated in general terms and asked to identify the correct ones. This task assessed students' ability to generalize and think about strategies.

> Task #2) Circle ANY & ALL answers below that describe a strategy that can be used to solve this problem.
> I) Compare the amount of sugar that Juan used to that Maria used.
> II) Compare the ratio of sugar to water than Juan used to the ratio of sugar to water that Maria used.
> III) Find the ratio of cups of sugar to total cups of lemonade for Juan and compare this to the ratio of cups of sugar to total cups of lemonade for Maria.
> IV) Use the fact Maria is on a diet and Juan loves sweet lemonade.
> V) Find the percent of sugar in Juan's lemonade and compare this to the percent of sugar in Maria's lemonade.
> VI) Not enough information given to answer this problem.

Phase 3: Carrying out the Plan

Solving a proportion problem involves comparing two fractions hence in the third task students were presented with two fractions and asked to identify which of several concrete steps would be involved to compare them.

> Task #3) Jorge and Alba need to determine which fraction is largest: $\frac{7}{9}$ or $\frac{8}{11}$?
> Circle ANY & ALL the following that correctly describe a way to determine which fraction is largest.
> a) The second fraction is largest because 8 is larger than 7.
> b) With both fractions, divide the denominator into the numerator and then compare the size of the resulting decimals.
> c) Cross multiply and compare the product of the means 9x8 with product of the extremes 7x11.
> d) The first is largest because 9 is smaller than 11.
> e) Convert both fractions to equivalent fractions over the LCD = 99 and compare

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the resulting numerators.

Phase 4: Looking Back

The fourth task was designed to assess students' ability to generalize (internalize) the ratio or rate concepts in this proportional reasoning problem.

Task #4) Circle ANY & ALL of the following which are true:

a) The sweetness of lemonade depends only upon how much sugar was used.
b) The Sweetness of lemonade depends upon the rate of sugar to water and this can be expressed as a fraction.
c) Comparing the sweetness of two separate lemonade mixtures involves proportional reasoning between the rate of sugar to water in each mixture.
d) The sweetness of lemonade depends upon the rate of sugar to water and this rate can be expressed as a decimal.
e) To accurately compare the sweetness of separate batches of lemonade they must both use the same amount of water.
f) All of the above are true.

The worksheet was given to approximately ten sections of pre-algebra and three sections of college level mathematics course. The rubrics used to score the tasks are included in Appendix A. The courses were taught by the professors involved in this study.

Research Questions

The proportional problem-solving exercise listed was one of a set of exercises used in two earlier studies. Charalambos and Pitta-Pantazi (2007) studied 600 5th and 6th grade students' scores on a set of exercises that evaluated understanding of different fractional concept (sub-constructs): ratio, part-whole partitioning and fractional equivalence. The same exercise set was given to 95 adult pre-algebra students at urban community colleges by Baker et. al. (2009) and the results compared to the earlier study.

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The mean scores of the children and adults were found to be remarkably similar however, the correlation between the ratio exercises which included the proportional reasoning exercise in this study and the other fraction concepts were very high for the children but extremely low for the adults. This lack of correlation suggests a loose understanding of the relationship between the concepts of fractions, rates and proportional reasoning for adult DPA students. This led to the present study, to compare the proportional reasoning ability of adult DPA students with CMAH students using Polya's four phases as a theoretical framework. A schematic diagram of the results for the DPA students is used to reflect upon an appropriate pedagogy for problem solving in pre-algebra mathematics.

Research Question 1: What is the difference in the level of understanding between the DPA and CMAH students, on Polya's four phases of problem solving for this proportional reasoning exercise?

Research Question 2: What is the problem-solving schematic diagram of the DPA students in this proportional reasoning exercise and what insight does it yield for the pedagogy of problem solving in remedial mathematics?

Results

Research Question 1

Table 1-A Mean Score & Standard Deviation for the DPA-students

	task 1	task 2	task 3	task 4
Mean	2.42	1.77	2.22	1.64
St. Dev.	1.58	1.19	0.67	1.13
n	111	112	110	111

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Table 1-B Mean Score & Standard Deviation for the CMAH-students

	task 1	task 2	task 3	task 4
Mean	1.99	1.96	1.60	1.64
St. Dev.	1.84	1.08	0.80	1.02
n	70	67	68	65

Phase 1- on task 1 the mean score of DPA students (2.42) was higher than that of the CMAH students (2.00).

Phase 2- on task 2 the mean score of the CMAH students (1.96) was higher than the mean score of the DPA students (1.77) in fact this was the only phase in which the DPA students were outperformed by the CMAH students.

Phase 3-on task 3 the mean of the DPA students (2.22) was much higher than the mean score of the college students (1.60) and a 2 sided T value for this difference was statistically significant at the 0.001 level.

Phase 4-on task 4 both groups of students did poorly with a mean score of 1.64.

Research Question 2

Table 2 Correlations between the tasks for DPA students

	Task 1	Task 2	Task 3	Task 4
Task 1 P. Correlation	1	0.555**	0.107	0.254**
Sig.(2-tailed)	.	.000	0.276	0.009
N	110	108	105	105
Task 2 P. Correlation		1	0.265**	0.322**
Sig.(2-tailed)		.	0.005	0.001
N		112	108	109
Task 3 P. Correlation			1	0.147
Sig.(2-tailed)			.	0.127
N			110	109
Task 4 P. Correlation				1
Sig.(2-tailed)				.
N				111

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Pearson Correlations/Signficance (2-tailed)/total sample size
*Significant at the 0.05 level,
**Significant at the 0.01 level

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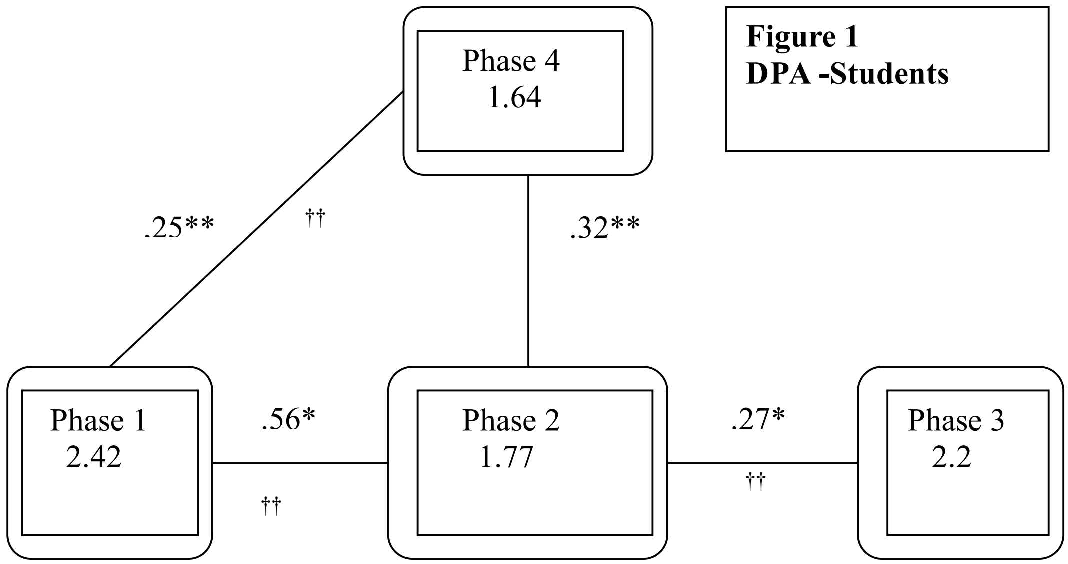
Figure 1 Schematic Diagram of Mean Scores and Correlations: table 1-A, 2



Pearson Correlations: *Significant at the 0.05 level, **Significant at the 0.01 level,
T-test to gauge the difference between the mean scores: † Significant at 0.05 level, ††Significant at the 0.01 level.

Discussion of Results

Research Question 1-Comparison of mean scores on each phase

Phase 1

Previous literature suggests that in the first phase (orientation) low proficiency students experience difficulty reading and understanding relevant information. (Kramarski et.al., 2002 and Jitendra et.al., 1998). Polya suggests that weakness in this phase is behind much of students' troubles with problem solving, "incomplete understanding of the problem, owing to lack of concentration is perhaps the most widespread deficiency in solving problems" (p.95). However, the results of the first

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research question indicate that the DPA students were more proficient at discerning relevant information than the CMAH students in the first task. This result represents a marked distinction between the previous research with children as well as Polya's comments.

One possible explanation for this phenomenon is that less proficient students, whose prior knowledge is not as organized; spend a lot of inefficient time in this phase in contrast to more proficient students who transition quickly into phase 2. In a comparative study between successful and unsuccessful problem-solving groups of college students, Stillman and Gailbraith found that, "The three successful groups spent on average 35% less time on orientation than the other groups" (p.183).

Phase 2

Some authors suggest that low achievers have a less developed schema that makes it difficult for them to generalize and transfer their prior knowledge to new situations furthermore, these students experience difficulty considering more than one strategy (e.g. Mevarech & Kramarski, 2003, Blair, 2006 and Xin, 2007). In task 2, which measured student's ability to recognize various generalized strategies, the CMAH students, had a higher mean score (1.96) than the developmental level students (1.77). This is significant because the DPA students receive more than three weeks of instruction in fractions and several weeks of instruction in proportions yet they did not perform as well as the CMAH students who receive no classroom instruction on these topics. Furthermore, for the DPA students this was the only phase they did worse than the CMAH students.

Phase 3

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On task 3 the mean score for the DPA students (2.22) was higher than the mean score (1.60) for the CMAH students furthermore, this difference was statistically different at the 0.01 level using a T-test. This indicates that knowledge of fractions is not typically retained in long term memory and needs to be frequently reviewed in advanced level mathematics courses.

Phase 4

The fourth task was designed to assess students' ability to generalize the ratio or rate concepts used in this proportional reasoning problem. The low mean score on this abstract task for both groups of students reflect the difficulty students have with abstract proportional reasoning.

Research Question 2 & Pedagogy

Figure 1 highlights the centrality of phase 2 in that it alone correlates significantly with all the other phases. As figure 1 indicates the DPA students understood the information presented in phase 1 and as demonstrated by the significant correlation between phase 1 and 2, they understood the relationship between phase 1 and forming a strategy in phase 2.

The weakness that pre-algebra student experience with phase 2 becomes a barrier to them because it prevents them from utilizing their strength or knowledge in phase 1 and transitioning this to their strength in computational proficiency in phase 3. The difficulty and importance of this transition is highlighted by Polya, "the way from understanding the problem to conceiving a plan can be long and tortuous. The main

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achievement in the solution of a problem is to conceive the idea of a plan”(p.8). Furthermore, if a faulty plan of action is taken, it is at the end of phase 3 and in phase 4 when the calculations have been completed that students may realize the plan was flawed because phase 3 does not correlate with phase 1 and phase 4 is the students weakest area, figure 1 suggests DPA students will have difficulty returning to phase 1 and starting over. This lends support to Blair’s statement that, “Developmental mathematics college students rarely plan a solution in advance, may demonstrate an inability to consistently monitor their progress, and have varying degrees of success recognizing that a solution attempt is not progressing toward the desired goal. When their initial strategy is not successful, these students have difficulty switching to an alternate strategy” (p.22).

In figure 1 the mean score of phase 1 is higher than phase 2 and there is a significant correlation between them; this implies instruction in problem-solving for DPA students should begin with their stronger knowledge of phase 1 and emphasize the connections to developing a strategy in phase 2. In particular, the R square value $(0.56)^2$ is 0.31 thus, approximately 31% of students knowledge in phase 1 will translate to strategy formation in phase 2. To assist this process, Furinghetti and Morselli (2009) suggest that, rephrasing the original question may help students build up their ability to plan, “the reformulation bridges to the phase (not always explicitly expressed) of developing a plan” (p.74).

Figure 1 reveals that, the mean score of phase 3 is higher than phase 2 and there is a significant correlation between them. This hints at support for the traditional approach, which focuses on correct modeling of problem solutions (phase 3) with the assumption

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that students will make a connection to the strategy (phase 2). However, the connection between phase 3 and phase 2 in figure 1 is given by the R square value of $(0.27)^2$ or 0.07 thus, only 7% of DPA students' knowledge of phase 3 will transfer to strategy formation in phase 2.

An alternate approach suggested by the centrality of phase 2 in figure 1 and the weakness DPA students experience with this phase is direct instruction or intervention in phase 2. In the literature direct instruction on problem-solving strategies or heuristics has had mixed results, "…despite all the enthusiasm for the approach, there was not clear evidence that the students had actually learned more as a result of their heuristics instruction or that they had learned any general problem-solving skills that transferred to novel situations" (Schoenfeld, 1987, p.41). Instruction in strategies has been especially disappointing for weaker students, "lower achievers highly benefitted under metacognitive instruction but performed very poorly when they were exposed only to strategy application." (Mevarech & Kramarski, 2003, p.466).

To students, strategy formation is often experienced as intuition rather than conscious planning, "I just took down all the information and from there it worked out. While you are doing that I suppose a plan just materializes" (Stillman & Galbraith, 1998, p.175). Polya states that the elusive nature of strategy formulation and the difficulties students have with phase 2 makes instruction in strategy formation a challenge, "the best that the teacher can do for the student is to procure for him [sic], by unobtrusive help a bright idea…to provoke such an idea"(p.9).

The main approach that Polya suggests for provoking bright ideas from students is

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through the use of analogy. The pedagogical technique of demonstrating related or analogous problems and comparing the solution techniques is a major component in reform methodology, "a central tenet of reform pedagogy in mathematics has been that students benefit from comparing, reflecting on, and discussing multiple solution methods…furthermore, teachers in high performing countries such as Japan and Hong Kong often have students produce and discuss multiple solution methods" (Rittle-Johnson & Star, 2007, p.561).

The weakness that DPA students encounter with phase 2 implies the use of analogies as an approach to problem solving may be subject to the same limitations as direct instruction in strategies with less proficient students. However, anecdotal classroom evidence reveals that DPA students frequently exhibit good intuitive knowledge of strategy choice in straight forward problems involving whole numbers. The authors conjecture that if such problems are sequenced with more difficulty problems involving fractions, decimals or multiple steps then students will be able to transfer their intuitive knowledge to strategy formation in more complicated situations.

Conclusion

Polya noted that drilling students in rote computation operations tends to destroy their interest in mathematics but challenging their curiosity leads to better overall results which in turn leads to more independent thinking. An important question that arises from the results of this study is whether or not these adult developmental students will effectively learn to engage in reflection to increase their problem solving ability and what

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is the best methodology for instruction of these students who typically struggle with problem solving.

The educational literature illustrates the difficulties that low proficiency students have with reading problem information and strategy formation. Polya's comments confirm his belief that weakness or lack of concentration in reading to understand the problem is at the heart of much of students' struggles with problem solving. The results of this study confirm that adult developmental pre-algebra (DPA) students experience weakness in strategy formation however, they challenge the notion that these students have difficulty reading to understand the problem. The developmental students scored higher than the college math students in both reading to understand (phase 1) and computational proficiency (phase 3) indicating they have good reading comprehension skills and have benefited from instruction in procedures. However the central place that strategy formation (phase 2) holds in the problem-solving schema (figure 1) and the weakness that adult developmental students exhibit in this phase characterizes it a significant barrier to their success in mathematics.

In figure 1 the mean scores of the DPA students on both the first and third phases are higher than on phase 2 furthermore, both the first and third phases correlate significantly with the second phase. Thus, DPA have the potential to use their knowledge and skills in both phase 1 and phase 3 to benefit their understanding of strategy formation in phase 2. However, the correlation values of figure 1 reveal that, DPA transfer 31% of phase 1 knowledge to phase 2 while only 7% of their phase 3 knowledge to strategy formation. This result implies that a pedagogy of problem solving for DPA students that

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focuses on phase 1 reformulating the problem as necessary will have more success with developing strategy formation skills than the traditional approach of modeling correct procedures to solve problems (phase 3).

While direct instruction in strategy formation has had mixed results in the literature, the authors recommend future research on whether DPA students can transfer their intuitive knowledge of correct strategies in straightforward problems involving whole numbers to more complex situations.

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ADDENDUM

We analyze the difference in the means for the stage 1,2,3, and 4 exercises between the DPA and CMAH groups of students using ANOVA and multivariate ANOVA techniques. It was previously determined that, the difference between the means in stage 3 for the two groups was statistically significant (0.05-level) using a two sided t-test. Furthermore, stage 3 which is linked closely to computational proficiency was the only stage in which the means of the two groups were statistically different. This significant difference and (anecdotal) classroom observation suggests that many of the CMAH students did not relate to these questions. Thus, while the DPA student tended to recognize these questions as germane to course material this was not true for many of the CMAH students. To further analyze this hypothesis each group was divided into high performing and low performing subgroups based on the following criteria. Students who did not receive at least a 2 were on the sum of stage 1 & 3 (maximum points value of 8) and at least a 4 on stages 1,3&4 (maximum point value of 12) were assigned to a low performing subgroup (stage 2 as dependent variable not used to subdivide). Thus, there were two groups with two levels of proficiency in each: N(DPA,high)=98, N(DPA,low)=19, CMAH,high) = 51, N(CMAH,low)=19. The t test for difference between the means of stage 2 for the high performing DPA and CMAH groups was 0.09 which is between 0.1 and 0.05. This gives some but not definitive support of hypothesis that, the difficulty students in the low proficiency CMAH group had relating to these exercises impacted the results obscuring the CMAH students' ability to formulate strategies within the context of proportional reasoning. Next, a multivariate ANOVA

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analysis was performed on all the groups using stage 2 & stage 3 as dependent variables with all 4 sub-groups as the independent variables.

Table 3 Mean Values DPA & CMAH and statistical significance of difference

group	DPA-high	CMAH-high	p<0.05	DPA-low	CMAH-low	p<0.05
Stage 3	2.22	1.65	yes	1.37	1.31	no
level	DPA-high	DPA-low		CMAH-high	CMAH-low	
Stage 3	2,22	1,37	yes	1.65	1.31	yes
group	DPA-high	CMAH-high		DPA-low	CMAH-low	
Stage 2	1.86	2.20	no	0.84	1.01	no
level	DPA-high	DPA-low		CMAH-high	CMAH-low	
Stage 2	1.86	0.84	yes	2.20	1.01	yes

yes =>statistically significant different at 0.05 level,
no=> not statistically significant at the 0.05 level

Note that, the performance between the levels within the groups tended to be statistically significant however those between the groups at the same level did not except for stage 3 high performing DPA&CMAH. Although not shown on the table, the CMAH mean for stage 3 of 1.65 was not statistically different than the 1.37 mean of the DPA-low nor the 1.31 mean of the CMAH students. Clearly the DPA students had an edge in computational proficiency over the CMAH students who had not been exposed to such proportional exercises. This, lends further support to the hypothesis stated earlier that proficiency with fractions is not retained by most students (even those proficient with mathematics) and should be reviewed in higher level math courses.

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The statistically significant differences between level (table 3) indicate that a multiple variable ANOVA with independent factors of two group (DPA,CMAH) and two levels (high,low) with stage 3 as dependent variable may be appropriate. In this ANOVA model the group factor was significant for $p < 0.03$, the level (high,low) factor at $p<0.01$ and the interaction between group and level was significant at the 0.08 level indicating the interaction between the groups and level may have had some impact on the results of stage 3.

In conclusion within the context of proportional reasoning, computational proficiency with fractions tended to be related to course material in that, those who had been exposed to the material recently were significantly better performers. In contrast, the ability to formulate strategies was not related so much to course material but instead to proficiency level within the course. Thus, while computational proficiency is learned within the course (but not retained) in contrast skills with strategy formation appear to be based upon an individual's proficiency with mathematics rather than the course material they studying.

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1. Understanding the Problem	1. Shows limited understanding of the problem, what information is relevant versus superficial and what you are asked to find.
	2. Shows partially developed understanding of the problem and indentifies a few specific factors that influence the approach to a problem before solving.
	3. Shows clear understanding of the problem and identifies many specific factors that influence the approach to a problem before solving
	4. Shows clear understanding of problem and identifies specific factors that influence the approach to a problem before solving. Recognizes unnecessary factors.

2. Devising a Plan-Identify Strategies	1. Select a strategy without regard to a fit
	2. Identifies a viable strategy
	3. Designs or shows understanding of one or more strategies in the context of the problem
	4. Designs or shows understanding of one or more strategies, along with either articulating the decision or ability to identify incorrect strategies.

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3. Carrying out the Plan-Generate Solutions	1 Rarely recognizes the need for multiple paths to carry out the plan minimal, thought or reasoning in carrying out the plan can recognize or state at most potential solution method.
	2 Sometimes recognizes multiple paths to carry out the plan but reasoning or thought in carrying out the plan is not well developed will typically relate to one perhaps more potential solution.
	3 Frequently recognizes the need for multiple solutions, or multiple paths to carry out the plan reasoning or thought in carrying out the plan is well developed will typically state one or more alternate and accurate (even creative) potential solution(s).
	4 Always recognizes the need for multiple paths to carry out the plan reasoning or thought in carrying out the plan is fully developed will typically state one or more alternate and accurate solution methods. Recognizes inadequate methods.
3. Carrying out the Plan-Generate Solutions	1 Rarely recognizes the need for multiple paths to carry out the plan minimal, thought or reasoning in carrying out the plan can recognize or state at most potential solution method.
	2 Sometimes recognizes multiple paths to carry out the plan but reasoning or thought in carrying out the plan is not well developed will typically relate to one perhaps more potential solution.
	3 Frequently recognizes the need for multiple solutions, or multiple paths to carry out the plan reasoning or thought in carrying out the plan is well developed will typically state one or more alternate and accurate (even creative) potential solution(s).
	4 Always recognizes the need for multiple paths to carry out the plan reasoning or thought in carrying out the plan is fully developed will typically state one or more alternate and accurate solution methods. Recognizes inadequate methods.

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4. Looking Back	1. Does not analyze or synthesize results, shows limited understanding of what was involved in solving the problem.
	2. Sometime analyzes or synthesizes results, shows some understanding of what was involved in solving the problem.
	3. Frequently analyzes or synthesizes results from more than one perspective, shows good understanding of what was involved in solving the problem.
	4 Always analyzes or synthesizes results from a wide range of perspectives, shows good understanding of what was and able to discern what was not required for solving the problem

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Appendix A

The rubrics used to score each task are presented below. A score of zero was assigned to students who had an answer completely incorrect. The results from students

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who skipped a question were omitted from the overall scores for that question only. The rubrics used to score the exercises were adapted from the work of the American Association of Colleges and Universities, AACU spring 2009 draft release of their value rubrics which was adapted by the general education committee at Hostos Community College both the original AACU and the Hostos adapted versions are available at: http://www.hostos.cuny.edu/oaa/ctl_rubrics.htm.