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Course exploring teacher perceptions of problem solving: What does it mean to "teach" problem solving to secondary mathematics learners?

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Background for the Study

The recent push for national education standards has brought with it a growing emphasis on the fundamentals we want our children to know, do and understand by the time they graduate from high school. In specific regards to mathematics, decades of debate has focused on what it means for students develop the mathematical understandings needed for life. According to the National Council of Teachers of Mathematics, the forefront leader in the national mathematics standards movement, there are five processes that are essential for learning and understanding mathematics. These standards for school mathematics describe the conceptual understandings, knowledge, and skills that should be spiraled throughout P-12 curriculum. One of those process strands, which is the focus of our study, is problem solving (National Council of Teachers of Mathematics, 2010).

According to their website, "solving problems is not only a goal of learning mathematics but also a major means of doing so" (National Council of Teachers of Mathematics, 2010). These problem solving standards call for students to be able to do four things: (1) build new mathematical knowledge through problem solving; (2) solve problems that arise in mathematics and in other contexts: (3) apply and adapt a variety of

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appropriate strategies to solve problems; and (4) monitor and reflect on the process of mathematical problem solving. They encourage teachers to create a supportive problem-solving environment that supports students' efforts in trying new things, exploring, taking risks, and working through their failures and successes. As with the other process standards (which are beyond the scope of this paper but which can be found at the NCTM website), problem solving should not be an isolated part of the curriculum, but a rich part of what it means to do and understand mathematics (National Council of Teachers of Mathematics, 2010).

Given the prominent problem solving focus called for by the NCTM Standards for School Mathematics, it is essential that practicing teachers have a clear understanding of this concept and its implications for mathematics teaching and learning. Research suggests however that a strong emphasis on problem solving has not been associated with a corresponding knowledge of its characteristics and consequences (Sweller, 1988; Chapman, 1997). According to Chapman (1997), problem solving can be defined as numerous things, including a (1) goal, (2) process, (3) basic skill, (4) mode of inquiry, (5) mathematical thinking and (6) teaching approach (Chapman, 1997). Historically, research into the nature of problem solving has shifted away from studying the conditions under which solutions are reached, and has more recently focused on the processes of problem solving (Chi, Glaser, & Rees, 1982). Chi, Glaser and Rees (1982) suggest that the concept of "problem solving" is convoluted for all of these reasons.

In seeking to update Chapman's (1997) research and investigate what if any effect the increased emphasis on mathematical problem solving has had on mathematics

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teachers' understanding of what it means to problem solve, this study was designed to uncover what (if any) diversities exists between secondary math teachers' views on problem solving and the aspects of problem solving identified by the NCTM Principles for school mathematics. With this research, we will examine implications for teaching critical thinking and problem solving, given these findings. The essential question guiding this research study is "What are secondary mathematics teachers' perceptions about problem solving within the mathematics curriculum?" Sub questions include: "What diversities exist within teacher perceptions of problem solving?" and "What implications do these findings have on mathematics teaching and learning?"

Research Design

The focus of this research study is to investigate secondary mathematics teachers' conceptions of "problem solving." Based on the work of Bruner, Goodnow, & Austin (1956), the survey follows a modified concept attainment protocol in that it initially asked participants to list out the essential attributes of the concept of problem solving and what they think it means to teach problem solving. Then participants used these essential attributes to distinguish exemplars of mathematics questions exemplifying problem solving from non-exemplars using their criterion. This method of survey data collection was designed as a means of helping the participant clarify his/her notion of this concept, while giving the researchers with the qualitative data that will be analyzed in the results portion of this paper.

The participant survey (which can be found in Appendix A) began with the

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question prompt "What does it mean to teach problem solving?" This question was designed as such because the focus of this research study is to examine the implications of discrepancies of the concept of problem solving on teaching and learning. We sought to approach this question from a teaching standpoint to avoid confusion between what it would mean for a mathematician or an expert in the field to problem solve, and to keep the focus on problem solving from the perspective of an middle or high school student. We also sought for the focus of this survey to remain on teaching and learning, as this is what our participants (who are all practicing mathematics teachers) are most comfortable with. In the spirit of the concept attainment model, this question was used as a prompt for listing the essential attributes of this concept that would be used later for exemplar classification.

The participants then received five mathematics questions taken verbatim from the January 2010 and June 2010 New York State Integrated Algebra exams. Questions were chosen from these exams because Integrated Algebra is the only mathematics exam required for graduation with a New York State Regents diploma. We sought to use this exam rather than select questions from a textbook or online because there are many textbooks and websites to choose from, but only one mandated mathematics exam. Since these were all New York State certified teachers, and New York State is one of the leading proponents of standardized exams, this seemed to be the best instrument from which to choose questions.

The Integrated Algebra exam covers a potpourri of algebra, geometry, trigonometry, probability and statistics. The questions chosen for this survey were

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intentionally chosen to cover the five mathematical topics previously listed to eliminate the potential bias of choosing a singular topic that may better lend itself better to a problem solving focus. Of the five questions selected, three of them were in a multiple-choice format with the answer choices provided, and two were short answer questions requiring written explanations.

Participants were asked to use their previous list of problem solving attributes to make a judgment as to whether each question was an exemplar or a non-exemplar of a question requiring students to problem solve. For each question it was imperative that participants provided a rationale for their decision, as these were analyzed to develop a clearer picture of the individual's notion of problem solving. The comments for each question are displayed in Appendix B and will be discussed in our research findings.

Procedure

After designing the survey to resemble a concept attainment protocol, the researchers found ten practicing secondary mathematics teachers willing to participate in this study. All of these participants were middle and high school teachers ranging in years of experience from 1 to 12 years. All of these participants were New York State certified teachers, although one was not currently teaching in New York State. Demographically, three teachers taught in Lockport, NY, three taught in Buffalo, NY, two taught in Niagara Falls, NY, one taught in Lewiston, NY and one taught in Valparaiso, IN. There were three teachers from public charter high schools, three were from public high schools and four taught in public middle schools. These teachers represented a spread of urban, suburban and rural school districts at both middle and high school levels.

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Although all of these participants completed this survey independently, five of these participants completed the survey in person with their answers written on paper, with the remaining five submitting their answers via an email attachment. Clarifying questions were handled either in person or electronically via email. Regardless of whether the survey was given in person or online, all participants were given the same directions and provided clarification support if necessary.

Results

After collecting the surveys, we analyzed our data through two related but seemingly diverse lenses. First, we looked at our written feedback for trends and patterns. Since all 10 participants are currently practicing secondary math educators, our initial goal was to simply look for commonalities among their responses, repeated phrasing or recurring notions about problem solving. Then we considered our teacher perception data as it aligned to the National Council of Teachers of Mathematics four-part definition of problem solving. Being that NCTM provides national standards for mathematics understandings, knowledge and skills from pre-kindergarten to grade 12, we chose their problem solving standards as a point of reference to judge our teachers' conceptual alignment to national guidelines. With significant implications for curricular design, instructional planning and lesson delivery (which will be described in detail at the end of this piece) we were looking to compare and contrast the data from our sample with these nationally established benchmarks.

Data Analysis

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Comparative Analysis

To begin our data analysis, we considered participant responses to the question "What does it mean to "teach" problem solving?" and their classification of Regents questions comparatively. We looked for commonalities and distinctions amongst our sample as well as trends in our data. Interestingly, participant conceptions of problem solving fell into one or more of three categories. Our participants wrote that problem solving requires that students:

(1) Apply mathematics to real world situations and everyday life
(2) Use higher level thinking and critical reasoning skills
(3) Use multiple steps to derive their answer

Though participant answers were not mutually exclusive, there were no instances where a participant did not mention one of these three categories somewhere in their responses, three out of ten citing all three.

One of the most frequently recorded conceptions of problem solving was that it must include some knowledge transfer - specifically stating that problem solving must involve "real world situations" or "application to everyday life." Similar to Perkins' (2009) writings, 4 out of 10 teachers stated that problem solving must involve the transfer of knowledge to novel circumstances or relationships, including application of mathematical formulas to new contexts. Each of these teachers stated that without a new, real life context there was no more thinking and problem solving abilities required than would be to perform a simple rote memorization task.

This understanding of problem solving was also iterated in many of the participant's

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classification of the five Regents examination questions as exemplars or non-exemplars of problem solving. Two of the 10 teachers distinguished exemplars from non-exemplars based solely on the context with which the question was presented. Three of 10 teachers stated that questions #1, #3 and #5 would aid in their assessment of a students' problem solving abilities based on the fact that the mathematics is applied to the real world and #2 and #4 as non-examples because they are not applications of mathematical skill.

The second conceptualization of problem solving reported by our sample was the notion that problem solving must be a complex, multi-faceted process with multiple decisions to be made before reaching a conclusion. 7 out of 10 of our participants stated that higher level critical thinking skills were required to problem solve and used phrases such as "make inferences," "make predictions," "justify a response," "deduce information," "generalize ideas" and "extrapolate new information" to describe this skill set. Two teachers used the words "more than 'plug and chug'" to describe what they viewed as the difference between solving problems and completing exercises.

This distinction between the complexity of thinking skills needed and problem-solving abilities was also evident in their classification of Regents questions where 4 out of 10 teachers classified question #1 as a non-exemplar of problem solving because "the picture/diagram is provided." To these individuals, having the diagram available eliminated the need to critically think about how the mathematics should be represented and therefore did not require problem-solving abilities. A similar sentiment was expressed by 9 of our 10 participants on question #4, who stated that this question did not require students to use problem-solving skills. Specific participant rationales included,

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“this question only tests to see if you have your properties of real numbers memorized,” “this problem is mere memorization,” “ this is just a test of memory” and “this only requires algebra skills” among others. Though all 10 participants classified question four as a non-example, one participant’s rationale for her classifications was due to the fact that it was not a real world example, not because a certain skill set was needed to complete it.

The final conceptualization of problem solving, which was presented by the participants in our study, was the idea that problem solving requires that students use multiple steps to derive their answer. Five out of 10 participants in our study wrote that problem solving involved “lengthy” problems and required students have the ability to distinguish between relevant and extraneous data. To these teachers, the ability to break down what a problem is asking and accomplish smaller tasks to reach a final conclusion was evidence of a student’s problem solving abilities.

In terms of the classification of Regents questions, this understanding of problem solving led three of our 10 participants to classify question #5 as an example of a problem- solving task. One teacher listed the possible ways students could a go about answering this question (“draw a line of best fit,” “use a ruler,” “work backwards”) stating that since students have “options to get to the answer” this would be a good assessment of a student’s problem solving abilities. Others simply stated that the “multiple steps” and information to “broken down” problem solving was required.

To provide further clarity to nature of our participant’s responses, the concept map below is a synthesis our findings.

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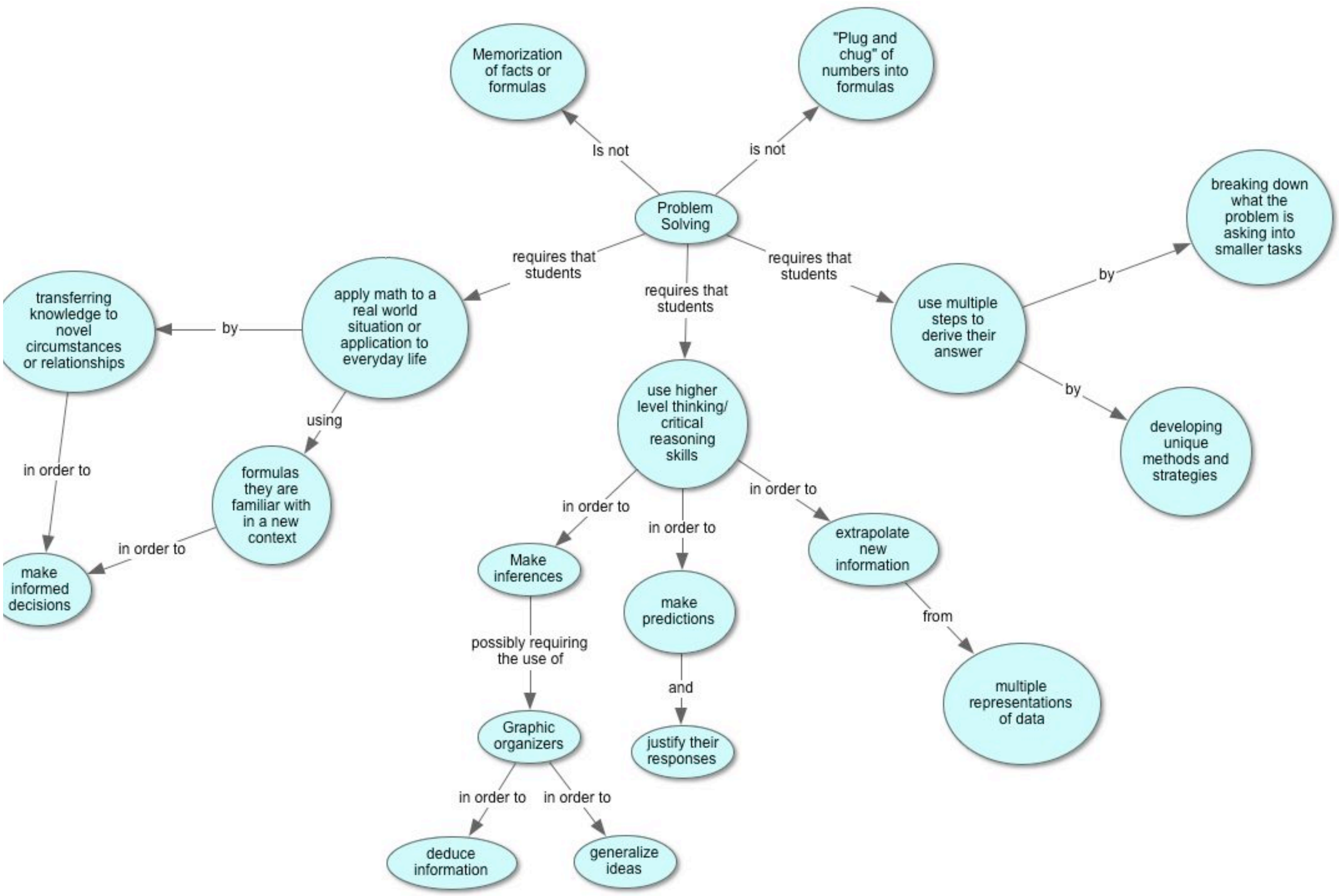

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NCTM Definition of Problem Solving

After comparing and contrasting participant responses, we will now turn our attention to the nationally recognized definition of problem solving as presented by the National Council of Teachers of Mathematics (NCTM). According to the NCTM website, problem solving should not be an isolated, additional component of the K-12 mathematics curriculum but an integrated and integral part of the way effective mathematics teachers prepare and deliver content. It is described as not only a systemic goal of mathematics instruction but also a "means of doing so" (National Council of Teachers of Mathematics, 2010). To NCTM, problem solving is not just giving students more challenging problems but giving them problems that enhance their "mathematical dispositions" and ability to analyze and interpret the mathematics of everyday life. Demonstrating problem solving abilities in accordance with the national standards for school mathematics means that students are able to:

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

(National Council of Teachers of Mathematics, 2010)

By comparing and contrasting the prevalence of these problem solving components

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with the ones suggested by the educators in our study, it is obvious that there are both additions and elaborations to the ideas offered by our survey of educators. Below we will provide a more detailed analysis of our findings as compared to the NCTM definition.

Comparison to the NCTM Definition

The first part of the NCTM definition of problem solving states that students are able to demonstrate their problem solving abilities when they "build new mathematical knowledge through problem solving." The interesting thing about this definition as compared to the results of our survey is that only one of our 10 participants wrote about the development of "new mathematical knowledge," though four out of 10 teachers surveyed described the skills required to build such knowledge using terms such as "logical reasoning," "critical thinking," "deduction" and "decision making." In congruence with the NCTM standards, 4 out of 10 participants wrote that problem-solving abilities afford students opportunities to stimulate new understandings, one going so far as to state that problem solving provides students a "scaffolded approach" to synthesize new understandings.

The second part of the NCTM problem solving definition states that instructional programs should enable students to "solve problems that arise in mathematics and in other contexts" (National Council of Teachers of Mathematics, 2010). This was a sentiment echoed by 4 out of 10 participants, though none of our participants used the exact words "other contexts." As described earlier, those teachers who viewed problem solving as transfer of knowledge also classified the Regents questions as exemplars or

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non-exemplars of problem solving abilities based on the context of the problem. One of the major difference between our teachers' understanding of problem solving and the NCTM definition is that none of our participants wrote that solving problems abilities could be demonstrated through application of thinking skills to other academic areas or fields of mathematics. None of our participants wrote about "cross over tactics" as a viable way to express problem solving abilities, though it is included in the NCTM standards.

Interestingly, the third part of the NCTM definition was another area of divergence between the teachers in our sample and the nationally recognized standard for problem solving. NCTM states that instruction in problem solving should enable students to "apply and adapt a variety of appropriate strategies to solve problems" though "multiple strategies" though this was only mentioned by one of our 10 participants. In fact while NCTM writes specifically about students' use of "diagrams, looking for patterns, or trying special values or cases" as problem solving skills, four out of 10 teachers in our study wrote that Regents question #1 (which required a use of a diagram) was not problem solving because the diagram was provided. Instead of the strategies required, the teachers in our survey wrote about the complexity of each problem and the number of steps required to solve as a criterion for problem solving. The length of the problem (not the strategies required to solve each problem) was mentioned in five of 10 of the surveys.

The final component of the NCTM definition of problem solving states that teaching problem solving skills means that students are required to "monitor and reflect on the process of mathematical problem solving" (National Council of Teachers of

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Mathematics, 2010). None of participants in our survey described students' meta-cognitive processes or reflection on decisions made. 7 out of 10 of our participants stated that higher-level critical thinking skills were required, however none made mention of the process of monitoring or reflecting on these thinking skills.

Implications for Teaching and Discussion

With significant implications for curricular design, instructional planning and lesson delivery, a great deal of what has been discussed through our research will challenge teachers to align classroom practice with the intentional teaching of problem solving called for in the Principles for School Mathematics. Given that our research has found noteworthy variation between the conceptions of problem solving put forth by NCTM and the notions described by our sample of secondary teachers, it is unlikely that students are graduating from our schools with the ideal problem solving abilities called for by this document. If these standards are to serve as a starting place for national mathematics standards, than a great deal of professional development will be required for math educators to flesh out what problem solving looks like in classrooms. Our research suggests that it is not simply enough to mandate that teachers focus on problem solving, but one must be very explicit to ensure that all teachers have a common conception of this idea. It may take a great deal of time before problem solving will become a central focus in the mathematics curriculum, but it will never occur without first establishing what it looks like in the classroom.

Given the apparent disagreement over what problem solving looks like in the

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mathematics classroom, more conversation is needed to ensure that curricular design and instructional planning has not only a problem solving focus, but the same problem solving focus. To ensure vertical curricular alignment, this instrument could be used to begin conversation and concept clarification regarding these ideas. This common understanding will enhance student learning as it will ensure that all instruction is focused on the same end. It is only once all teachers are on the same page that one can begin to see the benefits described in the NCTM standards.

These research findings can be generalized to many of the concepts found in the concept map on the previous page. Administrators and instructional leaders should proceed with great caution if pushing for initiatives regarding even concepts as obvious as “real world examples,” “higher order thinking skills,” or “critical reasoning” since our study shows that math teachers in fact disagreed upon what these concepts looked like in the classroom. It may be beneficial for teachers to not only have conversations to clarify their ideas, but then to observe other mathematics teachers who are taking the initiative of implementing these teaching practices as a starting point for further concept clarification.

Areas for further research

Since the 1980’s there have been much research demonstrating the connection between a teacher’s view of problem solving and his/her classroom practice (Thompson, 1984, 1985; Raymond, 1997). While this paper has been able to speculate as to how these teachers’ beliefs about problem solving manifest themselves in classrooms, more research is needed to identify each teacher’s fidelity in designing activities that provide

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his/her students opportunities to problem solve. Identifying problem solving techniques frequently employed by the teacher in correlation to his/her responses to the survey could help to support or debunk the previous research cited above. Teacher and student interviews, anecdotal records and classroom observations would be needed to establish a connection between one's beliefs about problem solving and how (or if) this process comes to life in classrooms.

One interesting aspect of this data, which is beyond the scope of this paper but could be explored to reveal interesting data patterns is the correlation between the grade level of the teacher and his/her answers to the survey. It may be interesting to note if the grade level taught has any effect on the perceived sophistication of the questions in the teacher's survey. For example, if a seventh grade math teacher stated that a question requires problem solving but a ninth grade teacher stated that the same question does not, is it possible that this discrepancy occurred because the seventh grade math teacher does not realize what is developmentally appropriate for a ninth grade Integrated Algebra student? Similarly, if we gave high school teachers the same survey but with all seventh grade mathematics questions, would they state that none of them required problem solving because they seem so simple relative to the mathematics they teach? This area of bias could be eliminated if we had chosen all ninth grade math teachers to view these ninth grade Integrated Algebra questions, but even still it may be impossible to eliminate all biases given the subjective nature of this concept.

In addition to this data, the researchers have also collected a dozen of these problem solving surveys completed by graduate students (some who are practicing teachers)

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enrolled in a 13 week Problem solving in Mathematics course at the University at Buffalo. Using the work of Zeitz (1999) as their curricular guide, these students investigated what it means to teach general problem solving strategies such as symmetry and extreme tactics, pigeonhole and invariant tactics, and cross over tactics in secondary classrooms. One could compare the problem solving perceptions of teachers who have taken the course to the problem solving to perceptions of teachers who do not have this prior knowledge to look for any differences in their classification. This would help us to speculate as to whether this course had any effect on how these graduate students viewed problem solving, and would allow us to see if their answers were more sophisticated than those who have not had extensive problem solving training. One might expect to see more similarities among participants who have had a similar experience, but it would allow one to compare the perceptions of those who have had previous experiences with problem solving to those who have not, and speculate as to the differences.

Improvements that could be made to this study

This study could be improved with a larger sample size and more in depth analysis of the rationale behind the teachers' problem solving classification. An interview protocol could have been used to get a better grasp on why these teachers classified these examples as they did, and if any of the biases identified above occurred during their classifications. In order to determine if grade level taught influenced one's classification of the questions, we could have surveyed elementary school teachers or even pre service teachers to account for variables such as grade level or teaching experience. In addition,

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people with no mathematics teaching experience would be valuable in gaining an additional perspective as to what 'problem solving' means outside of the mathematics community.

Additionally, we could have enhanced the questions that our participants classified by choosing additional, similar types of regents questions to make a more expansive survey. Multiple questions of a similar type would help us see if simple wording had an impact on teachers' perceptions of its problem solving nature, or if their classification was based on a conceptual understanding. By adding answer choices to open ended problems or eliminating answer choices from multiple-choice questions, we also could have seen if these variables made a difference in participants' classifications.

Conclusion

Having the ability to problem solve is essential to success in the 21st century (Wagner, 2008; Hanushek, Jamison, Jamison, & Woessmann, 2008). It is an integral part of everyday life, job success and future opportunities. Being able to problem solve and think critically gives students an advantage in terms of conceptual growth and depth of understanding - it opens doors into more advanced, abstract and creative thinking (Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier & Wearne, 1996; Bottge, 2001). In our study, we considered teacher perceptions about the concept of problem solving and how this perception is manifested in the secondary mathematics classroom. By asking practicing teachers to classify examples of Regents questions of exemplars or non-exemplars of problem solving, we were able to generalize how our sample of

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teachers viewed the teaching and assessment of problem solving skills.

Though insightful, we found that the teachers in our survey has conceptions of problem solving that were accurate but more narrowly focused than the definition set forth by the National Standards for School Mathematics. The majority of teachers in our sample lacked the recognition of specific problem solving strategies as well as meta-cognitive components to problem solving, such as reflection and monitoring of thinking skills. We concluded that further professional development and concept attainment is needed among secondary mathematics teachers, departments and instructional leaders. It is only with a clear, common understanding of what problem solving is and how it can be assessed that vertical curricular alignment and systemic curricular modifications can be made. We challenge the education community to integrate problem solving into their existing mathematics program with a common, intentional and systemic focus.

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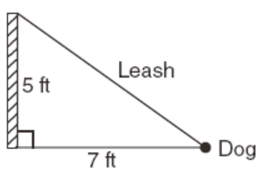
Appendix A

In your own words, what does it mean to "teach" problem solving?

Would these exercises aid in your assessment of a students' problem solving abilities? If yes, why? If no, why not?

1 The end of a dog's leash is attached to the top of a 5-foot-tall fence post, as shown in the diagram below. The dog is 7 feet away from the base of the fence post.



How long is the leash, to the *nearest tenth of a foot*?

(1) 4.9
(2) 8.6
(3) 9.0
(4) 12.0

2 The width of a rectangle is 3 less than twice the length, x. If the area of the rectangle is 43 square feet, which equation can be used to find the length, in feet?

(1) $2x(x - 3) = 43$
(2) $x(3 - 2x) = 43$
(3) $2x + 2(2x - 3) = 43$
(4) $x(2x - 3) = 43$

3 How many different sandwiches consisting of one type of cheese, one condiment, and one bread choice can be prepared from five types of cheese, two condiments, and three bread choices?

(1) 10
(2) 13
(3) 15
(4) 30

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4 Perform the indicated operation: $-6(a-7)$

State the name of the property used.

5 Megan and Bryce opened a new store called the Donut Pit. Their goal is to reach a profit of \$20,000 in their 18th month of business. The table and scatter plot below represent the profit, P, in thousands of dollars, that they made during the first 12 months.

t (months)	P (profit, in thousands of dollars)
1	3.0
2	2.5
3	4.0
4	5.0
5	6.5
6	5.5
7	7.0
8	6.0
9	7.5
10	7.0
11	9.0
12	9.5

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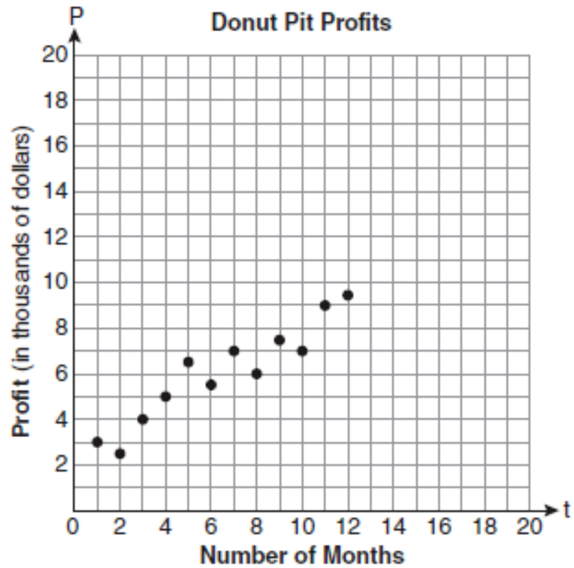

Appendix B

	Summary of Reponses
What does it mean to “teach” problem solving?	- “more than ‘plug and chug’ ” (2) - “real world situations”/ “everyday life” (4) - “ come up with (different) ideas and methods” - “manipulate mathematical ideas to come up with a solution” - “critical thinking skills” (2) - questions with “multiple steps” - “teach students how to read carefully and convert lengthy problems.” - “to help people think about situations in multiple ways and to guide people to make informed decisions.”
Question #1	Yes – (6)

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	Rationale: - "They need to know what to do with the picture" (2) -"The students need to know what equation to use." - "Yes because it is making them think of ways to break down the problems." - "It is a real world application." (2) No- (4) Rationale: -"It merely requires students to regurgitate a memorized formula" -"Maybe if they had to draw the picture, but as it is I would say no" (2) - "The diagram shows everything, in the question is already provided."
Question #2	Yes – (8) Rationale: -"Requires an understanding of how algebra can be used to describe geometric phenomena." - "They have to think about how to set up the equation." - "Requires critical and abstract thinking." - "Students need to break down what they need to do." - "Requires thinking and analyzing." - "Students must use formulas they are familiar with in a new context." - "Requires that you translate between words and mathematical notation." - "Requires students take out all the main points from the question so they can answer the question precisely." No- (2) Rationale: - "Not a real life example." (2)
Question #3	Yes – (7) Rationale: -"It does not tell students how to go about finding a solution." - Multiple steps- "I would have students draw a diagram." -" Requires analyzing and thinking." - "Requires students to utilize graphic organizers or a formula derived from these organizers to solve the problem." - "Real life problem." - "(Students) would have to convert the questions to a list of known facts." - "Definite connection to the real world." No- (3) Rationale: -"This is a pretty rote/ straight forward problem." - "Provides students with the answer options." - "Only requires that you recognize the counting principle."
Question #4	Yes – 0 No- (10) Rationale: -"Only tests to see if you have your properties of real numbers memorized."

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	- "They are not thinking about a problem and how to solve it." - "This is just the distributive property." - "This does not even require plugging numbers into a formula." - This problem is mere memorization." - "This is not real world." - This is just a test of memory." - "It does not require critical thinking or have any applications." - "It requires only algebra skills." - "More of a task-oriented problem."
Question #5	Yes – (5) <u>Rationale:</u> -"Gives students options to get to the answer in different ways." - "Makes them think about how to break down the problem." -"Requires thinking and analyzing." - "The problem requires students justify an answer as well as think about data and graphical analysis." - "It asks them to make a prediction." No- (3) <u>Rationale</u>: -"You do not have to think about how to go about solving the problem." - "It is more an assessment of a student's ability to read and derive information from a mathematical situation (math literacy)." - "In order for students to answer this question they are required to know a lot more than just problem solving skills." Yes and No – (2) <u>Rationale:</u> - "The idea of drawing a line of best fit is not, but the idea of extrapolating information from a graph is." - "Drawing a best fit line is not. Predicting and making inferences is along with justifying an answer."