MY STUDENTS HAVE TO BECOME 'INDEPENDENT LEARNERS', AND NOT BECAUSE I'M LAZY

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A group of secondary mathematics teachers are working together with a teacher educator who tries to live according to the words of Daniel D. Pratt (1988):
"(Adult) learner's dependency on a teacher should be temporally and situational, capable of being changed through an appropriate mix of direction and support.
Direction when they lack the necessary knowledge or skills to make informed choices.
Need for support comes from a lack of confidence in one's ability to accomplish the goals.
Direction and support are the keys to a teacher's role and to the relationship between learner and teacher."

And also:
"Whether learners are dependent or autonomous, the teacher must not do for learners what they can do for themselves and, conversely, must do for learners what they cannot do for themselves."

As a result of this background of the in-service provider, the topics to be discussed during the meetings were mainly chosen by the participants and could be said to be almost always a mixture of mathematical content, mathematical backgrounds, pedagogical aspects and psychological aspects of learning and teaching. Two topics are central in most of the meetings:

Topic 1: A collection of activities to engage students and to foster mathematical thinking. Included are activities for solving absolute value equations and activities to understand trigonometric identities.

Topic 2: A change in the way of working in a school for secondary education. From 'delivery' by the teacher too independent learning' by the students.

BEFORE THE MEETING

Half an hour before an in-school in-service meeting a math teacher asked me how she could explain the difference between a proof [1] and a justification in a situation in which graphing and absolute value were involved. This question was the starting point for an informal, but very abstract and general talk between the questioner, some of her math colleagues and me their in-service educator about what the essence of school mathematics (also called 'mathematics for teaching') consists of. Like in earlier meetings some of the participants challenged their colleagues to give examples of 'assuming', 'defining', and 'generalizing' which are some of the important processes of secondary mathematics (together with classifying, deducing, inducing, symbolizing, modelling, interpolating, extrapolating, and not to forget concretisizing and applying(?)).

And of course we talked – again - about mathematics as a body of certified knowledge (not economic, politic, religious or ideological, but strict scientific) according to the so called Olympus model of science, and/or mathematics as a human activity in which not differentiation or segregation but integration of human capacities is the kernel.

But, let me tell you about the meeting and the question that became central after the decision to become 'more practical' and do what we intended to do; discuss some ideas around the teaching of the mathematical topics for that meeting absolute value and trigonometry.

THE SITUATION

A staff meeting during a small scale project in a school for secondary education. In this project teachers, together with some University people, tried to change the way of working from 'delivery' to more 'independent learning'. The topics absolute value and trigonometry were chosen because all participating teachers had the idea that it was not possible to give students 'space for independent learning' while working on these topics.

The starting point for the meeting was chosen a week before: an observation made by one of the teachers.

"My 16 year old students almost all come into the classroom with the idea that 'absolute value' means 'taking away the negative sign'. As a result they change |x-2| in x+2, because they couldn't see it as 'distance to 2'.

During the discussion it became clear that none of the participating teachers was really interested in my questions about working with different definitions, such as: |x| as the 'larger' of the numbers x, -x, or |x|=√x2, nor with |x|= distance on the number line. They all worked with:

If x≥0, then |x|=x; if x<0, then |x|=-x.

Also: all of them wanted their students to 'know' at least that the graph of |f(x)| is for non-negative values of f(x) the same as the graph of f(x) and for negative values of f(x) the reflection in the X-axes (What is underneath the X-axes has to be turned over to the opposite)

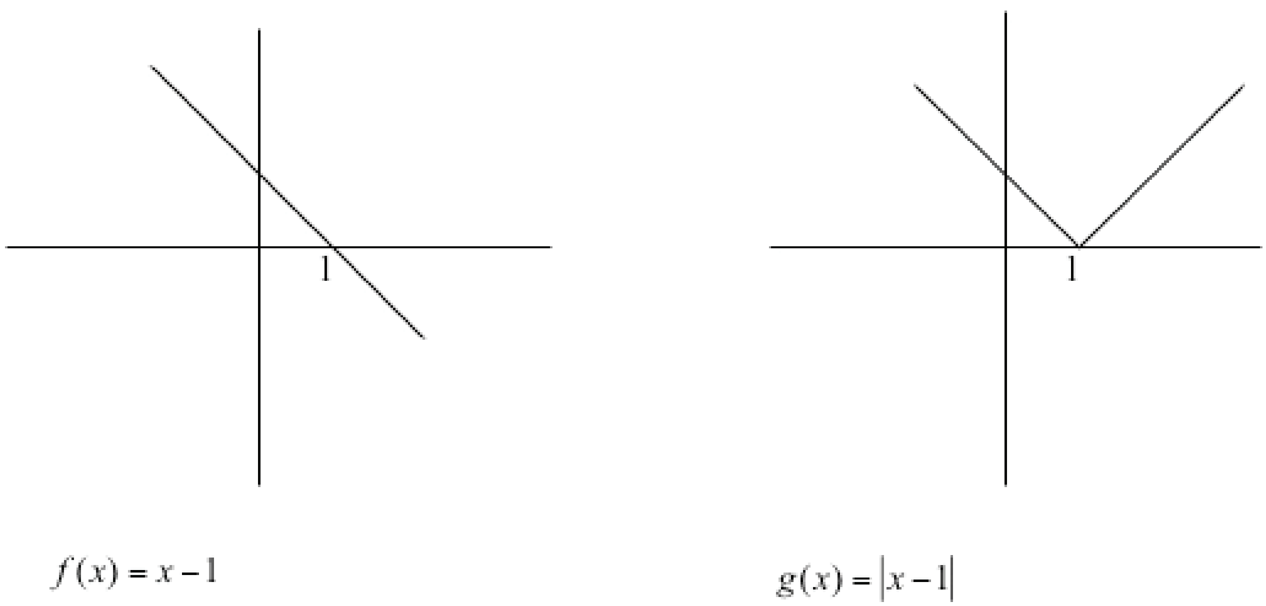


Figure 1

Most participating teachers started immediately with suggestions of what functions and graphs would challenge the students to look for the characteristic aspects of the graphs. They wanted to select tasks [2] from available materials and invent tasks themselves. All kind of suggestions were made around the idea of giving the students an A4 piece of paper with different graphs on it of discrete/concrete functions of different types like |f(x)|, f(|x|), |f|x||, f(|x-2|), etc., as well as a list of the functions (in different order) and the question to 'match' functions and graphs.

Others wanted to discuss more the pedagogical aspects. They expected the mathematical question to be easy to solve and wanted to concentrate on the more pedagogical question formulated by one of them (Alice): "How can I as a teacher avoid stepping in the trap-fall of telling instead of asking (challenging) questions?"

Discussing Alice's question

Although the majority of the participants preferred working on 'content' and develop tasks for the students Alice convinced them to go back to the pedagogical point of view [3].

She explained the background of her question by giving an example: "When I go to a (small) group of pupils working on this kind of exercises I try to ask the proper question to help them 'going', just like in a private lesson. But with pupils talking together at the same time (higgledy-piggledy) I often forget this principle. I direct myself automatically to one or some of the pupils. Sometimes I think 'this is/seems too difficult for this student and it would be better to do this together with the whole group and then it becomes delivery.(Me working hard and the students 'maybe' listening, but not doing) I have to unlearn this, but at the same time I want to be sure that something has been "seen" at least once in a proper way. So, I show them what is correct."

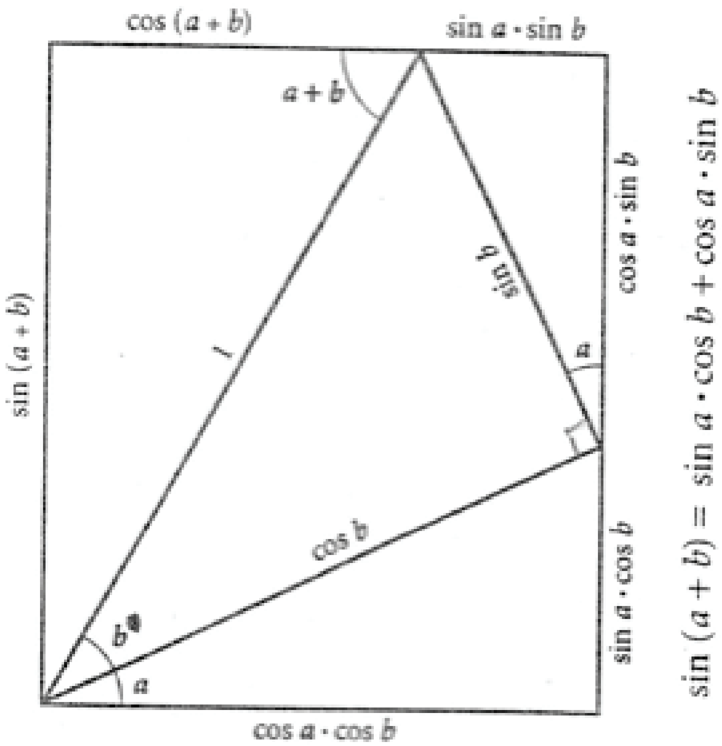
Alice is talking here about what she does (behaviour), about what she is not capable of to do but tries to do (competence) and – more hidden – her idea that it is good/better to teach by asking questions (conviction). Then she jumped to a specific content, not the one that was recognized by the group as being problematic (absolute value and graphs), but one from lower secondary she recognized as being – at least to her – problematic in the same way: "Easy to explain by a talking teacher, but difficult to develop/learn by a student independently".

Like with the question: 4a2x-a-1:a3, and why this can not be 'done' by working by following horizontal lines; 4x-1:=4, 2x-1:3=2/3, and axa:a=a.

A teacher is needed here, isn't it?

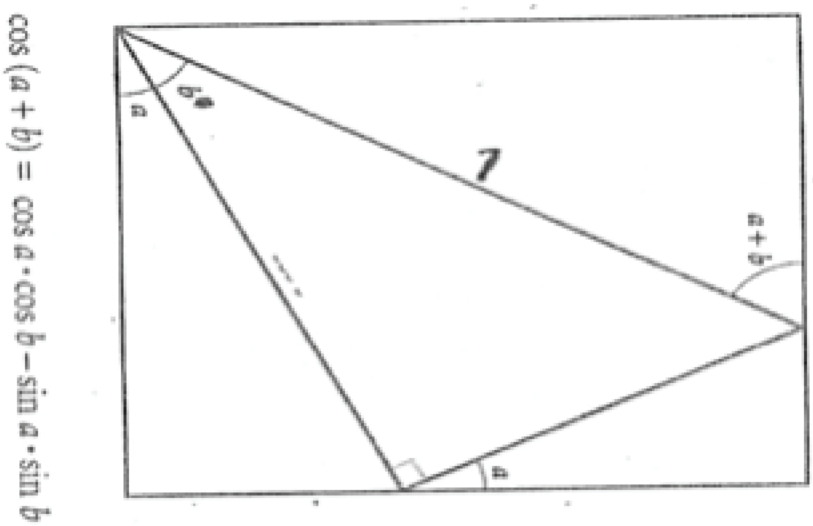
Other participants recognized the problem of not willing to force cognition or to force skills-development. They wanted to search for possibilities to avoid 'force' by giving the students well chosen tasks; maybe content dependent tasks.

Hessel, a teacher who likes his students (and himself) to use all kind of visualizations, reacted – indirectly – by jumping to another content [4]; without protest of anybody: "I recognize this also in other situations [5]. I have the same kind of problem when we are working on the reduction of sin (a + b) and of cos (a + b) using a graphical representation or just a drawing. (Figure 2.)



Firgure 2

A colleague from another school works with this 'picture' with only the length (1) of the 'hypotenuse' given and asks her students to give an expression for the other lengths using sin and cos. (Figure 3.) She often says I'm pushing my students too hard and that her way of giving more open tasks gives her students more opportunities to develop mathematically. Maybe, I'm just too protective, or too afraid not to help them enough".



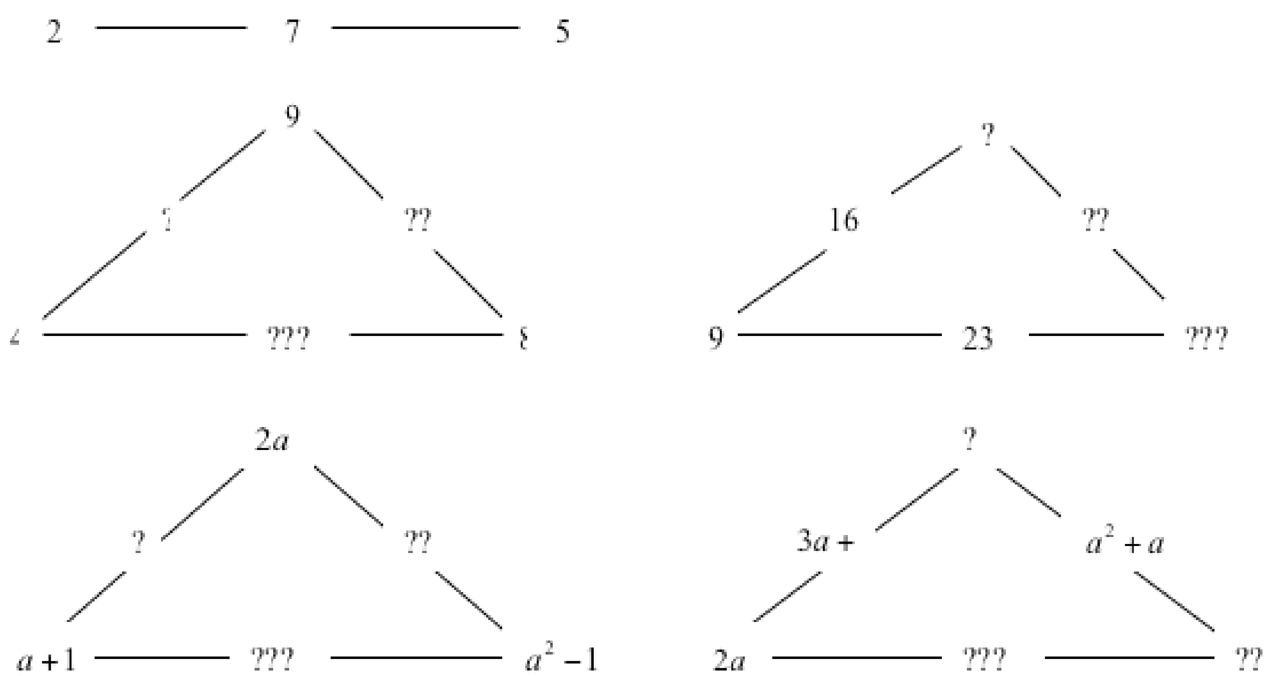
$$\sin(a+b) = \sin a \cdot \cos b + \cos a \cdot \sin b$$

Firgure 3

Looking back, looking forward

It is – like so often – only afterwards that I realized we didn't talk about the underlying conviction that 'teaching mathematics by asking questions is often better for the learners then teaching by 'telling'. This is an important conviction to reflect on together with another conviction that often seems to go hand-in-hand with the former one: 'pupils often/usually need a teacher to tell them what to do and how to do it'.

Bringing these two seemingly conflicting convictions in balance has been my entire professional live as difficult as using exploratory work with the aim to establish basic concepts and working methods. Nevertheless, it is still exciting and challenging to look for and develop questions that give students the opportunity to explore ideas and develop them in different directions. A nice example can be developed from the idea of 'Arithmogone' for young children with numbers and for older children with algebraic expressions.



Other ideas to consider are for instance:
The number-cracking machine that 'divides by 2' and then 'adds 5'
Is there a so called stay-the-same number?
What about other machines?

The participating teachers decided to have a try-out in upper-secondary with two of the 'possible criteria for choosing tasks that promote independent learning': connect graph and description and open questions (By a participant formulated as: questions with more than one possible answer or more than one way of 'attacking'.) [6] (See also appendix 2 for another example from lower secondary.)

As a challenge they used the material of appendix 1, although this material was different from most of the textbook material there students use (more closed, step by step questions and many more so-called context problems). The result of many try-outs was encouraging. It was an eye-opener for many teachers to see how their students were able to find the connections, although they almost never saw these kinds of questions. For those students that didn't see this kind of (polar) coordinates before, a hint about distance to the origin and angle with positive X-axes was in many cases enough. To be honest, it was more. It was the enquiring atmosphere as a part of a philosophy in which doing (taking care for enough 'brain' activities), reflecting (care for conscious learning), deepening (for 'different' and 'more'), and anchoring (for 'lasting' learning) are main aspects of the teaching-learning situation the teacher takes responsibility for.

Giving our students the opportunity to develop as independent thinkers forces us, their teachers, to look around and develop problem situations and change existing classroom exercises into challenges, like the attached ' star'-problem.(Appendix 2) This problem stimulates most students 'to inquire, search for solutions, in short to work independent from their teacher with question A. Only if they need extra help question B is added). The 'connect graph and description'-problems (Appendix 4) also worked well as an eye-opener for students to start a discussion with their peers about 'essential features' of the different graphs and 'how you can see that in the description'. Especially working with these last exercises made me realize that students (and many of the teachers) can work out the graph-functional-description-connection for those functions they didn't work on before. They started to explore, first by working systematically one by one, than skipped the ones they didn't recognize and came back to them later. Of course some students needed a hint or only the suggestion to 'try-and-improve' without being scared to do something wrong, because:

by doing only those things you already know or can do, you never learn something new. And working in an uncertain situation gives more possibilities to learn to survive than only being in safe situations. [7]

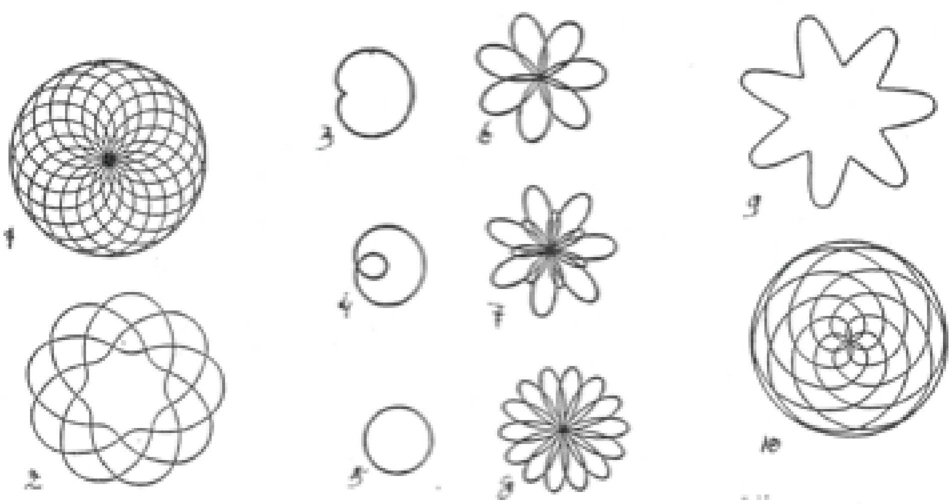
Looking for challenging exercises, to provoke discussions amongst students that provoke reflection is another task for teachers that want 'to go beyond the textbook'. Flexible creative teachers for flexible creative students!

Appendix 1

Can you identify the sketch that goes with each equation?

(a) $r = 1 + \cos\theta$.	(f) $r = 1 + \cos(7\theta/2)$.
(b) $r = \cos\theta$.	(g) $r = \cos(9\theta/10)$.
(c) $r = \cos(7\theta/2)$.	(h) $r = \cos(9\theta/10) + \frac{1}{3}$.

(d) $3r = 1 + 3\cos\theta$. (i) $r = 3 + \cos 7\theta$.
(e) $r = \frac{1}{2} + \cos(7\theta/2)$. (j) $r = \cos(3\theta/10)$.

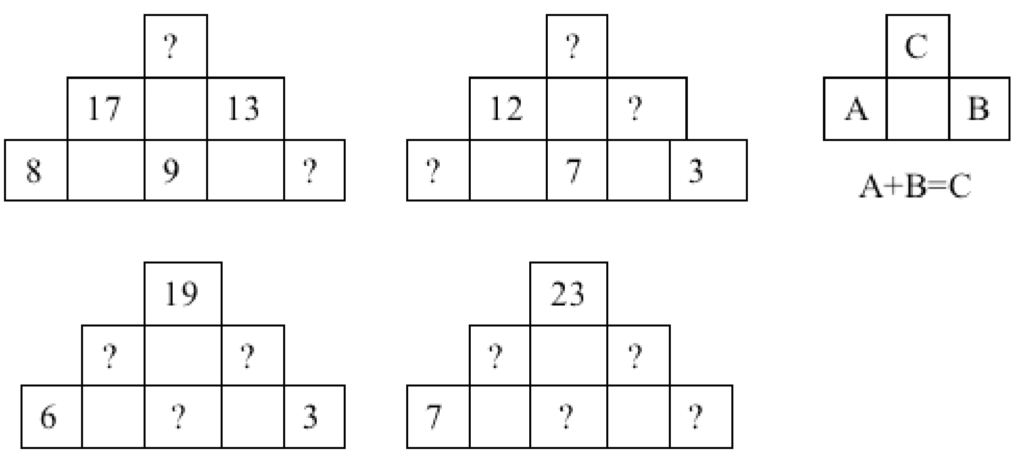


/ou think of a situation where any of these graphs/formulas has a meaning?
:re they can be seen as a model of?)

Appendix 2: Number Pyramid

A guiding principle for the choice of activities is that they 'don't end in themselves' but do allow further development. In the number pyramids which follows the numbers in each new level of the pyramid are derived from the level below by the addition rule shown on the right.

Find the missing numbers (the ? mark) in each case.



Some children working on the number pyramids use the basic idea: a + b = c explicitly, others 'just do'/'try some numbers'.

Using a + b = c explicitly means: If a and c are known numbers you can find b by a 'counting', 'addition' or 'building up' strategy. But also by 'subtraction', or the more general use of algebra (Solving the equation by adding -b to both sides of the equal sign).

The 'just try', 'counting', 'addition' and 'building up' strategies may work for many problems but it often does not work for more advanced or generalized problems. So we try to provoke the learner by offering a next problem that is almost impossible to solve without a more advanced / more general method.

And this is why the third problem is given. The third problem can be used as an obstacle that forces the learners to look back at the first two and look at the way the structure is symbolized by a + b = c. This is an important act of 'teaching/supporting learning', because: It will be a barrier to a learner's advancement if he/she is not initiated into the system which places emphasis upon the 'disembedded' use of language.

Important question:
Can we force a learner to look for an operation to model a problem; i.e. to work within the mathematical system of which the problem is an example?

The participants answer (after a long discussion): of course! But then we cannot stop with the above questions. Next questions have to be stimulated (some students don't need a teacher for this, but most do): there are in all these four pyramids 3 numbers given; is that necessary? Can they be placed everywhere? Is a solution possible with every chosen triple of numbers? Etc.

And in the direction of generalisation: how about a four level pyramid? A six level pyramid?

"the successful use of the student's own strategy with the easier items mitigates against the student's obtaining, or seeing the need to obtain, access to the formal methods of mathematics. In some cases, the concept or method being taught may even conflict with the intuitive notions that the student is already using.

If this is the case, then neither merely demonstrating the 'correct' method, nor working with the student's own strategies, is likely to be successful: the student sees neither need nor reason for the first, and the second is counter-productive."

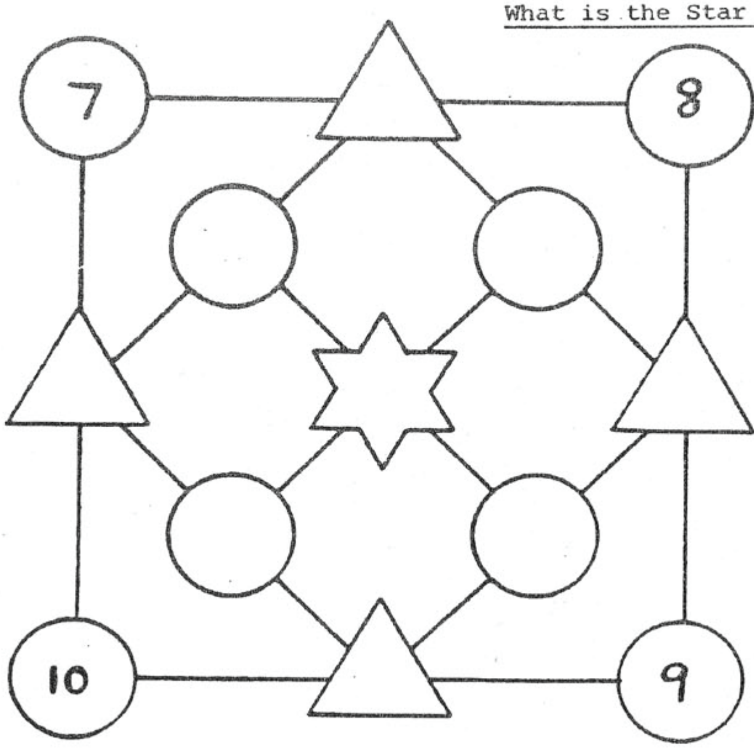
"Ways must therefore be found of working from the student's own strategies, but in such a manner as to ensure their replacement by the more 'mathematical' approach; perhaps more indirect ways are required."

Appendix 3: Star Numbers

Gillian Hatch
(1984)

Puzzle Card 12

What is the Star Number



To find the star number:

1. Add the pairs of numbers on the same side of the square.

2. Write each answer in the triangle between them.

3. Then add the numbers in the triangles. Write the answer in the circle between them.

4. Now you can add the circles to get the star numbers.

Now put a piece of tracing paper over the picture and find the star number.

Change the corner numbers to (6) (9) (11) (8) on another piece of tracing paper

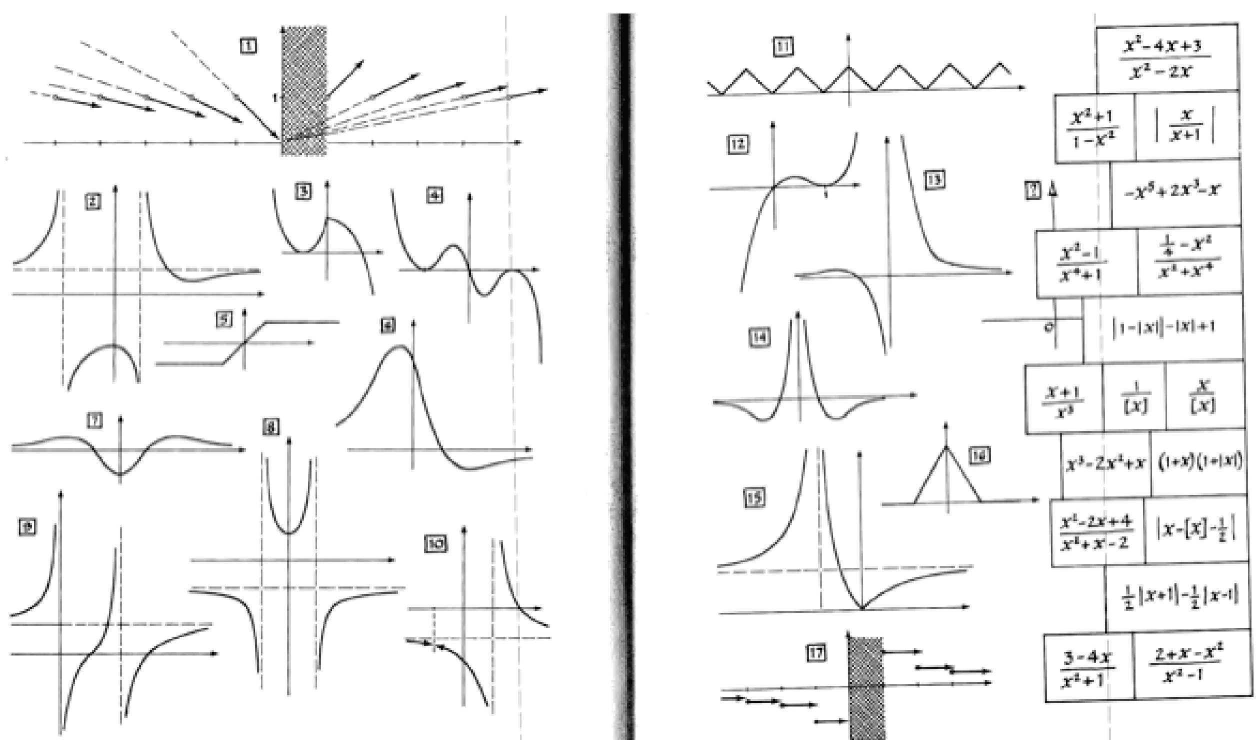
You should get the same star number.

A. Your problem is to find a quick way of working out the star number from the corner numbers.

B. Here are some corner numbers which may help.

1.	3	4	2.	3	7	3.	2	7
	7	5		4	5		4	5
4.	3	7	5.	4	7	6.	5	6
	5	5		5	5		5	5
7.	1	1	8.	1	2	9.	1	3
	1	1		2	1		3	1

Appendix 4: Connect graph and description



Extra question/exercise

20. Figure 3 represents the graph of the function $y = f(x)$.

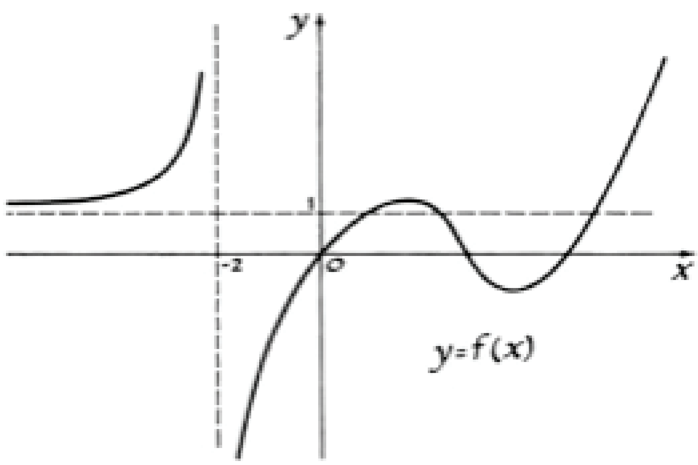


Fig. 3

Sketch the graphs of the following functions:

(a) $y = f(x) - 2$; (b) $y = f(x + 2)$;
(c) $y = |f(x)|$; (d) $y = f(|x|)$;
(e) $y = -3f(x)$; (f) $y = \frac{1}{f(x)}$;

NOTES

1. Proof is not a thing separable from mathematics, as it appears to be in our curricula; it is an essential component of doing, communicating, and recording mathematics. And I believe it can be embedded in our curricula, at all levels.
2. "In view of educational needs, it seems to be fruitful to conceive of the field of tasks as a spectrum extending between two poles: tasks, for which a complete procedure leading to the solution is known (often called 'exercises' or 'routine tasks'); and tasks (with Aporie: doubt, indecision, un-decidedness) for which such a procedure is unknown (often called 'problems' or 'non-routine tasks')." [Taken from Christiansen and Walther, Task and Activity. In: Perspectives on Mathematics Education.]

3. The teachers in this group very often jumped from ‘mathematical content choice’ to ‘ways of working in the classroom’ (classroom activities) and backwards. Often they forgot to mention their own role – to their own surprise - and maybe that was a reason they decided to go on with the question formulated by Alice.
4. He brought this example to the meeting because we agreed last time on the topics absolute value and trigonometry.
5. Abstraction is the ability to see likeness in things apparently dissimilar, and not as an act of disassociation with context.
6. They already formulated in the fore-last meeting: “To support the autonomy (independent learning) of the students, the learning and teaching strategy should be transparent for the students; this implies that at any point it should be clear to the students what learning activities they have to do, when and why (including the freedom they might have to make own decisions)” Furthermore they agreed upon having as many whole class or small group discussions as needed to help the students to become aware of their own learning processes to make them less directed at ‘production’ and more on ‘control’.
7. It is important for the learning situation that all suggestions are taken seriously by all students/teachers. This asks for a classroom atmosphere in which discussing suggestions and reflecting on activity ~ result connections are part of work, together with the space for spontaneous reactions on questions.

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Advances in knowledge come often, not by addition of new facts, but by a novel arrangement of known material. (Foster Kennedy 1947, Interrelationship of mind and body, Journal Mount Sinai Hospital, 9, 607-612.)