INTRODUCTION OF A DYNAMIC STUDENT RESEARCH PROGRAM IN APPLIED MATHEMATICS

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1 INTRODUCTION

This paper presents an innovative program whose objective is to educate undergraduates in methods of mathematical research and problem solving. The PRIME Research Program accomplishes this objective by creating a research environment and educational program based on the educational philosophy of George Polya. The program focuses on the primary themes of: Problem Solving, Informatics, Modeling, and Education. In order to assist students in developing their talents in mathematical research as undergraduates, the PRIME Research Program focuses on mathematical areas that are more readily intuitive to students, and do not require expansive background knowledge. The program creates an environment conducive to active student research by: 1) developing a strong interdisciplinary mentoring program to provide students with the necessary support to embark on a course of mathematical research; 2) revising course curricula in order to focus on problem solving techniques to be used in mathematical research; and 3) forming research seminars and discussion groups that are geared towards introducing undergraduates to various methods of mathematics research. The totality of the research environment is designed to introduce students into the world of mathematical research at an early stage of their mathematics education.

The paper elaborates on the details of the program in the following sections. Section 2 discusses the motivation and various objectives of the program. Section 3 describes the structure of the program. Section 4 discussed some of the unique features of the program. Section 5 concludes with sample research topics which are suitable for the program.

2 MOTIVATION AND GOALS

This section discusses the motivation of goals of the PRIME Research Program, with a specific focus on George Polya as a role model for the philosophy of the program.

2.1 Motivation

Providing research experience in the mathematical sciences is a vital component of preparing advanced undergraduate mathematics students for the rigors of graduate school and the real world. Even highly performing undergraduate students often have no exposure to mathematical research. Rather, success as an undergraduate is primarily based on excellence in coursework. A successful graduate may have received little training in how to think independently, invent new problems, and devise methods for solving problems in relatively unconstrained situations. Undergraduate students who participate in research apprenticeships are more likely to complete their studies and pursue graduate degrees [3]. Based upon this realization, the PRIME Research Program is designed to train students to participate in mathematical and interdisciplinary research, and thereby prepare them to succeed in graduate schools and research institutions. This increases their success rate in excelling in and graduating from such institutions.

2.2 George Polya as a Model

George Polya, the mathematician, the thinker, the educator, and the persona, is the fundamental motif of this program. His mathematical contributions to the areas of combinatorics, analysis, number theory, geometry, and mathematical applications are exceptional. His interests and contributions in field of mathematical education are inspiring and insightful. His depth, speed, brilliance, versatility, power, and universality are impressive. His crystal clear methodology provides an excellent tool for every mathematics researcher [25], [26], [27]. His warm and cheerful personality, which enabled him to successfully collaborate with his colleagues, provides insight into the nature of an environment which is ideal for collaborative mathematics research. He was a unique individual, a world class research mathematician, a master research scientist, and a superb educator, all at once [4]. These characteristics make George Polya the perfect model and inspiration for the many goals and objectives of the PRIME Research Program. The goals of the program can be divided into the following classes:

2.3 Goals

The goals of the PRIME Research Program can be divided into four distinct classes: research training; skills development, synthesis and integration, enrollment and retention.

Goal 1-Research Training

1. Foster a research-oriented academic culture among undergraduates. Give students an experience that explicitly illustrates the value and intrinsic rewards of research.
2. Encourage interdisciplinary interactions and discourse by introducing, training, and mentoring undergraduates to participate in professional and intense interdisciplinary mathematical research.
3. Go beyond the classroom setting to provide various opportunities for students to realize the full extent of their abilities. Foster imagination, discovery, and creativity among the students.
4. Encourage students to embark on independent research and participate in summer research programs, such as the REU.
5. Further student professional progress by providing exposure to research publications, presentations, and professional networking.
6. Introduce a new form of mathematical education consisting of formally educating undergraduates in methods of mathematical research. Apply and expand upon Polya's methodology to problem solving to inculcate in students a sense of direction in their mathematical research.

Goal 2-Skills Development

1. Train students to gather, organize, and analyze data using mathematical modeling.
2. Increase students' skills in solving challenging problems, by providing assistance and support from the faculty mentoring team, graduate students, and other fellow students.
3. Train students in improving their skills in effectively communicating research advances and discoveries, in both written and oral forms.
4. Give students the ability to engage in analytical discussion of research progress.
5. Educate students in the usage of appropriate analytical methods and computer tools in solving mathematical problems.

Goal 3-Synthesis and Integration

1. Create interdisciplinary faculty collaboration between mathematicians and scientists,

on the research level and on the mentoring level. Integrate different disciplines as they appear in real world problems.

2. Provide an occasion for faculty-student mentoring relationships that are oriented around proposing, planning, and executing research.

3. Create a research-based community which incorporates all the research teams into a unified group, under the leadership of a program director.

4. Create analytical discussion groups in which the participating students are engaged in peer reports and evaluations of each other's research.

5. Foster collaboration with researchers in the industrial world.

6. Incorporate into students' research an analysis of the relationship of scientific and mathematical research to mathematical education.

Goal 4-Enrollment and Retention

1. Increase the enrollment and retention of undergraduate and graduate students in mathematical sciences and statistics.

2. Provide motivation for students to choose research-oriented careers in academia and in industry by exposing them to the panorama of applications of mathematical sciences and statistics.

3. Create contacts for students with industry and government agencies, and thereby facilitate their attainment of high-quality research employment.

3 ANALYSIS OF THE PROGRAM

In order to achieve the above goals and objectives, the PRIME Research Program integrates the idea of modeling real world problems, and solving them by applying techniques from the areas of algorithms, heuristics, and problem solving. In this approach, the program follows Hans Freudenthal, who argued that mathematical education should be based in reality, around phenomena that "beg to be organized" through techniques of mathematical modeling. He opposed deductive approaches and, instead, favored development from the concrete to the general [7]. This approach motivates students to expand their mathematical horizons, and to pursue research careers in the academic world and in industry.

As an incentive to motivate students to do research, one of the key components of the PRIME Research Program is the theme of applied mathematics and its correlation to the physical sciences. Applied mathematics can be broadly divided into three mutually reinforcing components; (i) modeling - the derivation of governing equations from physical principles; (ii) algorithms – the design of methods for solving the model equations; (iii) unification - constructing a framework that incorporates diverse analytical methods into a few broad themes.

George Polya argued for the fundamental similarity between the practices of mathematics and science.
There are several valuable reasons to incorporate the study of applied mathematics into the study of the physical sciences. Firstly, the results and information of these sciences are formulated in mathematical terms. Consequently, one must use mathematics to apply this scientific knowledge. There is also a significant conceptual reason to incorporate applied mathematics in the study of the physical sciences. The ideas and procedures of the sciences have a mathematical origin. Therefore, the understanding of the associated mathematical concepts is essential for the understanding of these sciences themselves. Additionally, applied mathematics is an exciting and creative study in its own right, and thereby has very great appeal to students [20].

The interaction of mathematical modeling with the sciences involves three components which reinforce each other and together lead to an understanding of scientific phenomena: (i) experimentation and observation; (ii) theory; and (iii) modeling.

(i) Experimentation and Observation - Based upon careful experimentation and observation, there are currently enormous data sets produced that can only be analyzed by the use of deep statistical and computational tools. Indeed, there is a need to fashion new tools to properly study the data involved [31].

(ii) Theory – Once the scientific data is properly collected, an appropriate theory must be developed to explain the data. This theory is often expressed in mathematical language and grounded in mathematical theory. Thus, certain scientific phenomena can only be properly understood within the context of the mathematical theory upon which they are based.

(iii) Modeling- Once a scientific theory is developed, mathematical modeling is used to predict behavior and thereby validate the theory. However, if the mathematical predictions do not match up with the experimental results, the reasonableness of the theory is questioned, and sharper experiments and more focused observations are usually in order. The process than goes back to the first step and is repeated until a suitable theory is verified through modeling.

The nature of this multifaceted process suggests that science can only succeed if there is a close collaboration between mathematical scientists and the other disciplinarians [30]. An example of this collaboration is the many interdisciplinary programs in America which study Genetics, Bioinformatics, and Computational Biology. These programs are designed to provide a combination of discipline-specific and cross-disciplinary course work, in order to truly advance in these areas. The principles and methods involved in these programs provide a good example of the type of research which the PRIME Research Program strives to implement.

The structure of the program consists of three essential components: mentoring teams, curriculum development and seminars.

3.1 Mentoring Teams

One successful feature of the "Spend a Summer with a Scientist (SaS) Program [3]", and any program that attempts to educate students to do research, is a strong mentoring structure. Thus, the PRIME Research Program is built around multi-level research mentoring teams consisting of a research faculty, graduate students, former students, and current students. The faculty member selects suitable research problems for the students, elucidate the appropriate background literature, suggest directions for the research and supervise each project. The graduate students assist the undergraduates in carrying out the research itself. This hands-on interaction helps the undergraduates maintain a focus on the objectives of the research, and assists them in its implementation. After the first year, participating students who have completed their research projects are encouraged to join the mentoring teams in the ensuing year, offering their past experience and guidance. This multi-level structure is designed to ensure that the necessary support system is available to facilitate the advanced students' entry into the world of mathematical research. Additionally, the program calls for collaboration with industry to suggest and assist with real world problems for the mentoring teams. This is designed to provide the students with a valuable connection to the world of industry and mathematical research.

3.2 Curriculum Development

In order to provide students with invaluable skills for carrying out their research project, the

PRIME Research Program involves the development of a curriculum designed specifically for its students. Courses are offered to provide students with appropriate mathematical knowledge as background, mathematical methods for applying this knowledge, and a methodology of how to think mathematically. To achieve this goal stress is placed on mathematical reasoning and problem solving, with the following six important themes: combinatorial analysis, graph theory, discrete structures, algorithmic thinking, applications, and modeling. Students must understand mathematical reasoning in order to read, comprehend, and construct mathematical arguments. The intuition and motivation that lies beneath the theories and problems of modern mathematics are elucidated. Students will understand that even very complex and deep results can often be guided by very simple and common-sensical principles [29]. A method for uncovering the simplicity in various mathematical problems can prove to be a valuable tool for approaching mathematical problems, especially for beginners. As such, the students in the program are trained in the methodology developed by George Polya for approaching and, ultimately, solving mathematical problems. The courses offered to the students in the program are an effective complement to their research problems.

3.3 Seminars and Discussion Groups

Another facet of the structure of the PRIME Research Program is seminars and discussion groups. The seminars have two alternating components: talks by faculty members, and talks by students. The talks by the faculty members should serve as a stimulus, model, and resource for the student talks which follow. The topics of the faculty talks should complement the students' research projects. The student seminars initially consist of the students' presentation of known results which serve as background for their research, and eventuate in the students presenting the results of their own original research. The seminar component of the program is an invaluable tool in initiating the students into a research community. It provides a subtle pressure for the students to clarify their research in a way that will be understood by the attendees of the seminar, and prepares them for further courses of mathematical research.

In addition to the seminars, the program consists of informal analytic discussion groups involving the students and faculty. Each meeting consists of a given group reporting on the progress of their research. The report is followed by analytical discussion on the ideas presented by the participating group. These discussions consist of an evaluation of the content of the research, and the methodology used in pursuing the research. By presenting the research to peers and faculty for critical feedback, students gain experience in questioning, challenging, defending, supporting and proposing solutions in various areas of research. Being that the students are engaged in active inquiry, as opposed to passive listening, they learn to take more responsibility and initiative in their learning [22]. Together, the seminars and discussion groups are a vital complement to the student research and a significant tool in helping students mature into successful researchers.

4 UNIQUE FEATURES OF THE PROGRAM

Many features of the PRIME Research Program are modeled from research mentoring programs that were proven to be effective (for instance, [3]). This section describes some of the outstanding and unique key features of this program.

4.1 Choice of Research Topics

Research topics are carefully selected in manner appropriate for students who are early on in their mathematics education. This makes the program suitable for advanced undergraduate students, despite their minimal educational backgrounds. The nature of this program can even be suitable for advanced, motivated community college students. Specifically, the research problems are generally chosen from the areas of optimization, graph theory, and combinatorics. The motivation for this choice is the fact that students do not necessarily need a great deal of background knowledge to solve some research problems in these areas. By choosing problems that are removed from the technical obstacles of requiring a great deal of background knowledge, and modifying the problems to be suitable for undergraduates, we provide students with a challenging, but realistic and enjoyable research experience [25], [26], [27]. With these problems, students can experience the pride of working on a personal research project, and the unique satisfaction that comes from creating an original argument to resolve an open problem.

4.2 Mentoring Teams

The mentoring involves groups of students led by a mentoring team of research faculty and graduate students. This provides students with the research expertise of the faculty, together with a greater sense of comradery with graduate students.

4.3 Integration into the College Experience

Many research programs are limited to the summer. Students come from different colleges and participate in the research group for a short period of time. However, the PRIME Research Program is a part of the college itself, and the students are all in the same college or university. As such, students from the same cohort interact with each other for at least a full year. Even after the end of the year of research, students are encouraged to return, and participate as part of the mentoring team. This reinforces the unity and cohesiveness of the program. The seniors or graduate students advise the younger students in overcoming potential obstacles involved in research. They also provide encouragement and support to the participating students. Thus, the theme of unity and collaboration helps create a friendly, united community that is dynamically self-evolving and nurturing from its own fruits.

4.4 Education in Research Methodology

A new dimension of mathematical education is introduced to synthesize mathematical research and education. This is accomplished by designing a classroom setting around training in methods of mathematical research. Until the present time, research training has been introduced by practice problem solving. We are suggesting the creation of a new era of formal education in carrying out mathematical research. We are expanding on the pedagogy and philosophy of George Polya, which promotes the formulation of heuristics in mathematical education, and the study of methods to teach the art of discovery of solutions to mathematical problems.

4.5 Multifaceted Research Environment

By improving the course curricula and creating student seminars and discussion groups, students are immersed in a strong research environment which allows them to develop as researchers.

4.6 Interdisciplinary Focus

The strong focus on interdisciplinary applications of mathematics demonstrates to the students that mathematics is a powerful subject with broad connections to the sciences and other disciplines.

4.7 Exposure to the Real World

The collaboration with the industry and research institutions gives students important exposure to the real world and illustrates how mathematics is actually used in the world. It also provides them with important connections which might prove to be useful in their future pursuits.

4.8 Interactions throughout the Research Community

The PRIME Research Program unifies its various components into one community. The size and cohesiveness of the student community allows support to come from multiple individuals, rather than a single mentoring relationship. The presence of, and interactions with, other students in the program greatly increases the desire to attend or remain in graduate school. This is consistent with research on graduate school success and retention, which has found that interactions with peers (as well as with faculty) were a more important determinant of success in graduate school than were undergraduate background and personal characteristics [11], [15], [20].

5 SAMPLES

5.1 Sample Research Problems

A. Mathematical Games

The chess endgame King and Rook and Bishop vs. King and Rook has been called the "Headache Ending". It was discovered (by computer methods) that the ending requires 59 moves to win, in the hardest positions with optimal play. The standard for such "pawnless endings" is 50 moves to checkmate with no pawn moves or capture, otherwise the game is a draw. Of the space of over 120 million positions, many positions in this ending are drawn with best play. The challenge is to write a program (not a database) that employs search and heuristics to find the winning positions in these endings, and to do so optimally would be an even greater challenge.

B. Bioinformatics

Expanding the field of DNA computers to RNA

Adleman [1] introduced DNA based computing to solve mathematical problems. DNA is used as a data carrier, and techniques of molecular biology are used to operate on DNA. Though the theoretical foundations are strong [23], there have been relatively few experimental demonstrations of DNA based on computations [13], [19]. Additionally, a new method using RNA as a molecule for the construction of binary nucleic acid libraries was developed to compute solutions to mathematical problems [10]. The challenge is to develop the new method to solve other mathematical problems, and to construct other similar methods.

C. Heuristics (based on discrete space/lattices for quality of solutions) for multi-criteria combinatorial optimization problems: [16], [17]

(i) the knapsack problem, the multiple choice problem
(ii) the "shortest" path problem
(iii) the graph approximation problem
(iv) the traveling salesman problem (TSP)
(v) assignment/allocation problems
(vi) scheduling problems.

D. Graph Theory and Combinatorics

The Maximal Rectilinear Crossing Number of k-Regular Graphs of Order n, for n and k even
Students attempt to further the results of [5], [9] and compute the maximal rectilinear crossing number of k-regular graphs of order n, where both n and k are even. Students will design a program to use the database of Aichholzer and Krasser [2] to compute the configuration which maximizes this crossing number for small values of n and k. They use this data to conjecture a crossing number in the general case, and attempt to prove this value. A similar method may be used to compute the maximal rectilinear crossing number of the wheel graph, and the graph consisting of two cycles with a common vertex, building upon results in [14].

5.2 *Sample Seminar Topic (associated with sample research problem D)*

The Maximal Rectilinear Crossing Number of k-Regular Graphs of Order n, for k and n of opposite parity
Background regarding the k-regular graph and the maximal rectilinear crossing number are presented. Previous results regarding the maximal rectilinear crossing number of the cycle graph Cn [9] and the complete graph Kn [28] are discussed. The generalized star drawing of the k-regular graph is illustrated for k and n of opposite parity. It is demonstrated that this drawing maximizes the rectilinear crossing number for all drawings of the k-regular graph. We thus compute the maximal rectilinear crossing number of the k-regular graph of order n.

The unsolved case where both n and k are even (sample research problem D) is discussed.

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