Democratization of Mathematics - Excellence in the Discovery of Number

Bronislaw Czarnocha and Vrunda Prabhu
Hostos Community College, Bronx Community College
CUNY

Introduction

Mathematics is a beautiful science. It shows us the possibilities of our minds while at the same time, it informs us about the depth and profoundity of the Universe around us. To grasp a mathematical concept and to use it succesfully in the process of understanding the world – "that's a treat of the Gods".

There are two phrases that characterize contemporary debates in mathematics education literature: "democratization" and "mathematics for all". How is "mathematics for all", to be understood?
Mathematics for All, or equity means that every child goes through the process of grasping mathematical concepts and using them in the process of understanding, of naming the World for herself/himself.
The Curriculum and Evaluation Standards for School Mathematics (NCTM ,'89) addresses issues of equity directly by naming its absence, a „social injustice". The Principles and Standards for School Mathematics (NCTM ,2000) make equity the First Principle of the reform in the U.S. What is not stated, however, as (Allexsaht-Snider and Hart,2001) point out, nor has „become clear how teachers and educators in individual [classrooms], schools and districts might accomplish equity in mathematics with their students." Equity in the individual classroom of mathematics depends on teachers' understanding of the term in its minutest detail. Hence, again the question forces itself to the forefront of the discussion: What does Mathematics for All mean in a mathematics classroom?

Is it, the
- Minimal mathematics knowledge which all can understand? or, the
- Maximal mathematics knowledge that could be understood with the proper instructional guidance?

Is it the lower or upper level of the classroom's Zone of Proximal Development?
Where is the democracy? What is the difference between the two meanings in the questions above?
The first meaning above is democratic because everyone gets the same amount of knowledge, however, more questions arise:

- Are everyone's needs met?
- Is each student's potential attained?

In the second question above, the phrase "maximal mathematics knowledge that could be understood with the proper instructional guidance" the emphasis is on finding such means of instruction that allow every student the possibility to fulfill her/his needs and potential. Democratization of mathematics education must mean creating a level playing field in which there is complete and free access to the highest mathematics and this is provided and available for all. Finally, (Skovsmose, 1990) asks more directly: "are undemocratic tendencies served by introducing students to unrelated bits of knowledge putting the teacher (and the book) in a special role of authority?"
What does it mean, the democratization of mathematics in a mathematics classroom?

Voices in the discussion

The previous issue V2N1 of MTRJoL posted an interesting article by a colleague from Kingsborough CC, who proposes a new approach to the college course of mathematics for liberal arts students. A pragmatic mathematics course which is focused on practical mathematical areas; where examples do not illustrate a theory, but are the main focus of active interest of students. It consists of 40 practical problems that serve as a springboard to introduce mathematical skills and methods, and it sounds like a well thought-through approach worthwhile of closer familiarity.
The author, together with (Amit and Klein, 2002), observes that the major hindrance to mathematics is NOT their weak intellectual faculties, but their "frightful attitude towards mathematics". Such an observation is very hopeful: once there are no essential mental difficulties, than there should be no problem in organizing the education to eliminate the fear of mathematics while preserving the mathematical sophistication. The author, however, asks: Will these students comprehend and appreciate the full picture of mathematics?
Is the absence of the grasp of real mathematics in the traditional education system really evidence that a „true" mathematics course is not appropriate for the students? Perhaps

students' difficulties could be alleviated by teaching via sophisticated dydactic processes? Maybe students are not motivated to learn? In such a case helping them to find or rediscover the appropriate motivation and utilize it within the didactic contract of the class could be a possibility. Does the honest analysis indeed suggest that students will not understand the mathematical theory and therefore we should not spend the effort to facilitate abstract, generalized thinking? Or maybe it says that the act of teaching as well as its learning environment needs to be changed to enable the abstract grasp of some fundamental concepts.

According to (Vygotsky,1987,Ch.4) the first moment of generalization for a child comes at the tender age of 2 years, when suddenly there is integration between the child's linguistic and cognitive development so that the child discovers the first generalization, that everything has a name. The child is so eager to learn the words-names and points to the things they refer to, that she/he cannot stop, and what results is the famous „terrible twos" period. Every child passes through that period, hence every child can generalize and grasp the abstraction. So how come they don't learn - school pupils, college students? Is it that they do not care, that education has become "irrelevant"? Perhaps, because they don't care, they are not interested, frightened of mathematics. But why not? In their own words, this is what students from remedial classes state:

" My experience with math have been pass horrible ever since I hit freshman year in high school. I can remember I was an A+ student until 8th grade and received 90, 100 on all my math tests. I am not sure what I missed but ever since it's been basically down the hill and I guess I have partial blame, because I gave up after I felt there is no help for me."

"I can remember one accident in the elementary school, I believe. It was such an embarrassing moment. We were learning about the fraction. I, of course, didn't get it, but the rest of the class did. Mrs X told me to get up in front of everyone to the chalkboard and humiliated me in front of everyone, several times….I thought then and there, I will never be good at anything."

Feder goes on to state:
"Being that the studies underscore the failings of the traditional methods for the lower twenty to thirty percent of students" (Feder, 2007), our approach is effective with the lower 20-30% and all others. The approach described in this article for remedial mathematics courses (Arithmetic and Transition from Arithmetic to Algebra), combines the practical utility and the beauty of mathematics, both of which Feder points out to, as essential for both groups

of students. However, there is no distinction of who receives what, but instead all students have the chance to experience beauty of mathematics with its practical utility. The theme of democratization is a necessary fact that community college instructors of mathematics have to encounter daily, given that the major task of the community college is to provide basic mathematics from the pre-college school system, that which the school system failed to develop.

Role of Community College Professors

Community college professors (especially at CUNY) hold an unique position. They are PhDs in their disciplines. They can transform their classrooms into Teaching-Research laboratories, where they can investigate students' learning and can design appropriate instructional sequences to improve the learning. In this process, every student can get the benefit of the instruction from her/his intuitive level. Classroom research leads to the design of instruction which every student has access to. When community college professors apply their research competencies to classroom investigation of students' learning, and design instruction on the basis of that, that takes students' from their intuitive understanding to the sophistication and beauty of mathematical concepts, at that moment, democratization takes place.

Community college students are a special audience. The students are generally in the labor force and they may have children of their own. The responsibilities of daily life are heavy on many of them. Their investment in education as the gateway to a better life is strong. With this background, most students and teacher-researcher form a partnership, a camaraderie, in which a joint goal of deep learning and elimination of negativity toward mathematics becomes the driving force.

A Didactic Contract between students and teacher-researcher

Most documents that are prescriptive in framing national agendas and are broad in their scope in thinking about the needs of the nation at large, recommend that "students attain their potential". We describe here brief elements of courses at the community college, where attainment of students' potential is termed attaining one's Excellence and is couched in a game. The partnership with students in the game whose objective is to attain one's own Excellence, translate the national agenda's to concrete classroom work ethic, motivation level, etc. Every course begins with a syllabus and the syllabus is the transactional document or contract between students and teacher-researchers. It sets the tone and allows to return to it many times in the semester to re-invigorate the low enthusiasm that may be present at any time. The process of democratization starts with the contract where responsibility is properly distributed between student and instructor. The teacher-researchers have noted two difficulties in day-to-day classroom work of students: attendance and attentiveness. The latter is directly contributed by the technology of cell-phones and text-messaging. A deeper difficulty is the process of synthesis, viz., seeing the pattern apparent in several cases to create a method of attacking the problem from a new vantage point.

The main features of the syllabus used in our classes (with the larger goal of democratization) are:

- the class is named a learning community, and appears as the heading on the syllabus, Learning Community : What's Up? – A Handshake.
- A grading scheme with no negative-classifications (i.e., every student has a full "bank-balance" of points on all the syllabus items and only loses those that he/she does not earn, i.e., the final exam has a full score till the actual occurrence of the final exam, at which point those points lost are deducted from the final exam score.)
- 1 reading for the semester, with the objective of addressing the fear of mathematics (the Research Report, (Making the Grade: Fractions in your school, 2006) of a large-scale Mathematics and Science Partnership study conducted in the states of Ohio and Michigan in which a test on fractions was administered to third through twelfth graders and the passing rates with low passing rates.)
- Focus on the development of thinking technology and schema formation (quote of Einstein appearing within the syllabus: "What precisely is thinking? When, at the reception of sense-impressions, a memory-picture emerges, this is not yet thinking, and when such pictures form series, each member of which calls for another, this too, is not yet thinking. When however, a certain picture turns up in many of such series then - precisely through such a return - it becomes an ordering element for such series, in that it connects series which in themselves are unconnected, such an element becomes an instrument, a concept.")
- Establishing a connection between the classroom and the larger world through the notion of global competitiveness (and in the middle of Spring 2008, global competitiveness was connected with attainment of one's own Excellence)

The search for one's own Excellence is a repeating theme which is addressed each time a need for it arises in the classroom and this need arises often:
"I don't know how to do this. I am no good in math. I have never been good". Such a common response from students to being asked to think how a certain problem on the board should be solved, would prompt the teacher-researcher to introduce the Excellence remind-

er: "Please do not be impolite to yourself, to your Excellence. There is nothing you cannot do, let's think together". While the first time such an exchange between a student and teacher-researcher takes place, it might be a source of surprise to many students in class, the inclusiveness and openness of the conversation and the openness of the thinking of mathematics creates some sort of "breaking the ice" of negativity toward mathematics from past experiences. The second time such an exchange occurs, the number of students willing to think together has already risen. The terms, "one's own excellence", "thinking rather than remembering", "memorizing vs. thinking", "handshake", are soon colloquial classroom terms and one student's negative comments are very often countered by another's friendly reminder, "do not put yourself down, remember your excellence". The need to bring student's attention to their own thinking, i.e., developing one's own meta-cognitive skills is an essential element of successful learning of mathematics, and both the cognitive as well as the affective aspects must be brought to bear upon this development.

The learning environment, which is consistently being cultivated the entire semester, draws from:

- The teacher-researcher's past experiences teaching mathematics and in particular knowing what has worked well, and what has not. (this falls in the domain of the craft knowledge that every teacher possesses by virtue of the daily teaching experience)
- Learning from the existent literature, those elements that other teachers and researchers have found to be useful
- Learning from the mathematics in the design of the instructional sequences, to ferret out which parts of the mathematics in question would not lend itself to discovery given the conventions adopted by the mathematical community and making accommodations for this phenomenon within the instructional sequence
- the learning theory as a guide in the development of concepts, which is taken into account in the design of the instructional sequence and, importantly, in the daily classroom discourse. Hence, there is a need to have not a one-voice classroom, but an open discussion, in which everyone learns and progresses, even though the progress may be at different rates. That concept or aspect of the problem at hand that is difficult, or non-transparent for the particular student, when voiced, has the chance of being addressed directly on the spot, or via a short discussion followed by a brief interventional mini-instructional sequence for the difficult concept. (In the calculus classrooms of the Bronx, under the NSF-ROLE#0126141, this was successfully addressed by the creation of a lab book, in which students kept their mini-instructional sequences on topics such as the summation symbol, or logic, or algebra, etc, which were proving to be obstacles in learning the current calculus concept of the class).

Mathematics of the Handshake

In Spring 2008, in a class of only 7 students, particularly resistant to learning (11 on original roster with 2 dropouts very early in the semester, 1 dropout from a student who wished to engage the entire class in discussion about non-mathematics and even read the newspaper headlines; 1 student with about 5 class attendances). Of the 7 students who remained in the class, the level of brightness was openly evident to the instructor. All 7 students were very pleasant and in their openness revealed the sources of their own difficulties. The students' from the public schools in the Bronx revealed the weakest "number sense", so much so that recognizing a prime number, knowledge of multiplication tables, etc. was a formidable task. A student who had failed the Regents' test in high school several times expressed great distaste at mathematics. However, pleasingly, the student thrived in the course and went on to demonstrate her real talent for mathematics in the provided learning environment. A student who could only be described as an "overachiever" excelled beyond all expectations of the instructor and at least 3 of the 7 students' have promised to be "ambassadors" of the Learning Community program in the following semesters.
The experience of teaching this difficult class, whose attention was very difficult to capture consistently, that led to the booklet Excellence in the Discovery of Number. Each of the eight topics (the missing student was included in the booklet because it was uncertain whether she would be returning, and the few times she had been in class, in spite of the general hostility toward mathematics, but directed at the instructor, she had demonstrated a quickness of mind that was excellent). Each of the 7 elements of the Excellence in Discovery of Number, arose from the interests students in the class had demonstrated to the instructor and these are as follows:

1. Natasha's Dots
2. Stephanie L's Primes
3. Nancy's Triangular Numbers
4. Sylvester's Continued Fractions
5. Gissell's Triangle
6. Amy's Plots
7. Stephanie A's Money Measures

A few examples of these problems are included in the Appendix.

In the first of the above, patterns of dots are created that lead students' while continuing the pattern to recognize the multiplication table. The second pattern develops the recognition of which of the first few natural numbers correspond to a prime number. Recognition of

prime and composite numbers naturally follows, and the following fact is introduced : Every composite number can be written as a product of its prime factors, which results in quite a stir among students' as this now opens the possibility of factoring large numbers quite easily. The pattern of triangular numbers serves as an introduction to basic geometry of shapes. Given students' aversion to fractions also developed in prior schooling, the continued fractions are a bit of an intrigue and the fascination of the continued fraction is used to develop the needed facts about operations on fractions. Pascal's Triangle as is known in traditional mathematics books is Gissell's Triangle and the multiplication of binomials with high powers and various coefficients follows from this pattern. A discovery approach to graphing developed as a separate module begins with the developed knowledge of the number line thus far and leads to graphing "Amy's Plots".
Democratization occurred via the Excellence in Discovery of Number, booklet in several ways:

- It brought back ownership of mathematics to the students. A student who had failed the Regents several times in high school and whose hatred of mathematics was a contrast to her very pleasant nature, after the assignment which assigned her a mathematical concept, arrived in class after Spring Break and announced to the instructor: I have three questions, and proceeded to ask them. The questions she asked were about generally difficult concepts in mathematics that come from the convention of usage. One such question asked "How does one graph y = 1, how can it be a line". This provided an opportunity for a whole class discussion on what are the possible meanings of y = 1 and how can we look at the context to determine which of these meanings are relevant to the problem under consideration.
- A student whose interest in her work was phenomenal, and who maintained her diligence and retained interest in an otherwise turbulent classroom with many distractions, excelled in the class to an extent that would not be able to be captured by merely looking at the test scores or quizzes and attendance, etc.

The booklet, Excellence in the Discovery of Number, provided the needed scaffolds for the individual students to work on their own difficulties and jump over them easily, for example, a common difficulty for students is the multiplication table severely unaddressed in prior schooling, which with the pattern of dots coupled with the decomposition of any composite number as a product of primes, becomes a way to once again begin to have fun with number. Apart from the much needed scaffolding, the booklet connected with the concept map of the course and provided students with a schema for the course, and ways to navigate that schema.

The question is often asked, how can all this fit into the semester and does it cut into the course syllabus. The answer to both parts of the question is that it is not difficult to fit into the semester since most parts act toward the accomplishment of the common goal – high level of learning by the students and marked improvement from their own work and attitudes toward mathematics at the beginning of the semester. Second, it is not possible in the tough environment that students and instructors operate to just cover the syllabus items – the failing rates are the evidence that this does not work. Third, the "named" mathematics described above, not only creates an inclusive environment, it also brings back the joy of learning which is openly evident in each of the 4 classes taught by one teacher-researcher, and finally the levels of accomplishments of the students are the most important reason why such a system is considered worthwhile for continuation.

Why does an Excellence in the Discovery of Number fall in the category of democratization of mathematics education, or why should it be characterized as having the democratizing aspects?

1. The booklet has several instances where students can "test" their own discovery, i.e., they can verify for themselves what they know and what they do not, and when the latter, can find a way to do so. In this process, they discover their own thinking through observation. their problem-solving skills are improving.
2. It creates the playing field "more" level, i.e., absent pre-requisite knowledge is not a deterrent, but is being learnt in a Just-in-Time based intervention. The precision of the approach from the natural patterns to the formal multiplication table , combined with the judicious fact that every composite number can be factorized in terms of its prime factors, serves to alleviate the difficulty that absent skill which was not developed by the school system.
3. It develops a conceptual/computational bridge starting with patterns and connecting to the course syllabus items that appear on the concept map of the course, creating opportunities for the connections between the various concepts to have several avenues toward schema formation.

Recommendations from CC professors

The integration of research and practice that happens in the classroom of a teacher-researcher creates the democratization element embedded in the formation of the learning environment. By investigating students' mathematics thinking and designing mathematics activities on the basis of these investigations, CC professors of mathematics are creating new knowledge, whose application allows every student to reach their invisible yet present intellectual potential. Consequently, Community College professors of mathematics play a fundamental role in spreading the democratization process across CUNY and beyond. To make this process effective there is a need to create an avenue that allows them, access to the university infrastructure to conduct large scale studies with the assistance of graduate and CC students to create a new public knowledge arising directly from the source, to address the problems head-on.

References

Allexsaht-Snider, M. and Hart, L. (2001) "Mathematics for All":. How Do We Get There? *Theory into Practice*. Volume 40, Number 2, p.93-105, Routledge, Taylor and Francis Group

Amit, M., & Fried, M.N. (2002). High-Stakes Assessment as a Tool for Promoting Mathematical Literacy and the Democratization of Mathematics Education. J*ournal of Mathematical Behavior*, 21, 499-514.

Making the Grade:Fractions in your School. (2006). http://www.promse.msu.edu/research_results/PROMSE_research_report.asp. May 2006

Scovsmose, O. (1990) Mathematical Education and Democracy. *Educational Studies in Mathematics*, 21, p. 109-128

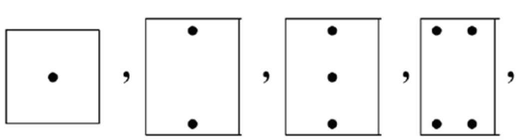
Feder, E. (2007) A Pragmatic Mathematic: A new skills-for-life mathematics course addressing the NSACS's revelation of the dismal quantitative literacy of America's college graduates. *MTRJoL*, V.2. N.1

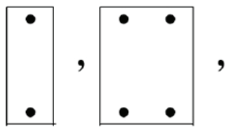
Vygotsky, L. (1986) *Thought and Language*. The MIT Press, Cambridge, Mass.

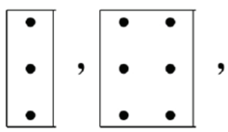
Appendix

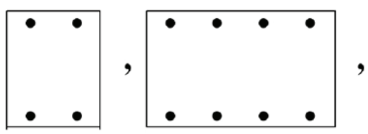
Natasha's Dots : Meaning of Number

Question 1. *What are the next few terms in the patterns below?*









Question 2. *Does the number 20 fit in the pattern of the third row above? Does it fit in the pattern of the fourth row?*

Sylvester's Continued fractions

Consider the pattern below:

$1, 1+\frac{1}{1}, 1+\frac{1}{1+\frac{1}{1}}, 1+\frac{1}{1+\frac{1}{1+\frac{1}{1}}}, \cdots$

The first term of the sequence is 1.

The second term of the sequence is $1+\frac{1}{1}$.