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Teaching non-traditional Applications in Engineering Mathematics: A Case Study

Norbert Gruenwald, Gabriele Sauerbier (Wismar University of Technology, Germany), Tatyana Zverkova (Odessa National University, Ukraine), Sergiy Klymchuk, (Auckland University of Technology, New Zealand)

This paper presents and analyses engineering students' attitudes towards a new experimental course Mathematical Modelling of Survival and Sustainability. The course has a multidisciplinary character and is very practical but its context is not typical for students majoring in engineering. The aims of the course are: to introduce students to some of the techniques, methodologies and principles of mathematical modelling for ecological, environmental, socio-economic and military systems; to involve the students in solving real-life problems adjusted to their region emphasising the aspects of both survival (short term) and sustainability (long term); to encourage students to pay attention to environmental issues. On one hand, the context of the course is not directly related to engineering. On the other hand, chances are that most of the graduates in engineering will be dealing with mathematical modelling of environmental systems in one way or another in their future work because nearly every engineering activity has an impact on the environment. Students' comments on the above issue are analysed in the paper.

FRAMEWORK AND INTRODUCTION

There are many papers devoted to investigating undergraduate students' competency in different steps of the mathematical modelling process. We mention some recent research. A measure of attainment for stages of modelling has been developed in (Haines & Crouch, 2001). The authors expanded their study in (Crouch & Haines, 2004) where they compared undergraduates (novices) and engineering research students (experts). They suggested a three level classification of the developmental processes which the learner passes in moving from novice behaviour to that of an expert. One of the conclusions of that research was that "students are weak in linking mathematical world and the real world, thus supporting a view that students need much stronger experiences in building real world mathematical world connections" (Crouch & Haines, 2004). It echoes with the findings from a study of 500 students from 14 universities worldwide (Klymchuk & Zverkova, 2001). The study indicated that the students felt difficult to move from the real world to the mathematical world because of the lack of practice in application tasks. An investigation of undergraduate students' working styles in a mathematical modelling activity has been done in (Maull & Berry, 2001) and a study on the development of transferable skills in undergraduate mathematics students through mathematical modelling in (Nyman & Berry, 2002). Some relationships between students' mathematical competencies and their skills in modelling were considered in (Galbraith & Haines, 1998) and in (Gruenwald & Schott, 2000).

In this paper we will consider engineering students' attitudes towards modelling of problems from areas that are non-traditional for engineering students, namely ecological, environmental, socio-economic and military systems. In this study, practice was selected as the basis for the research framework and, it was decided 'to follow conventional wisdom as understood by the people who are stakeholders in the practice' (Zevenbergen & Begg, 1999). The students' mathematical and modelling activities in the class as well as their attitudes were the research objects.

The idea of the experimental course arose from an ongoing joint project of three universities - from Germany, Ukraine and New Zealand (Gruenwald, Sauerbier, Zverkova, Klymchuk, 2005). The project is based on innovative pedagogical strategies in teaching applications in engineering mathematics. The aim of the project is to develop a set of ***real*** environmental models suitable for engineering students. The models in the above project have the following features:

- Each model is environmental. Students can see possible catastrophic consequences of human activities by changing the values of parameters and interpreting numerical solutions of mathematical models. It can help them broaden their vision, pay attention to the environment and prepare them to take ethical responsibility in future.
- Each model is adjusted to the region where the students study. We assume that this psychological strategy will help students to relate to the models in a *personal* and an *emotional* way and increase their motivation and enthusiasm.
- Each model is developed by professional mathematicians working in industry and is based on a real practical problem.
- Each model is adapted and presented in a way understood by engineering students.
- Each model is a little bit beyond the scope of the mathematics course the students study. So they need to learn on their own some new concepts. We assume that this discovery learning strategy can help the students enhance their investigation and research skills.

THE COURSE

The above project was extended to the development of a new experimental course Mathematical Modelling of Survival and Sustainability that was taught as an optional course to engineering students at Wismar University, Germany in 2006. There were many concerns among some engineering staff regarding suitability of the course for engineering students. 11 students enrolled in the course - from year 2 to year 4. The course had a multidisciplinary character and was very practical. Matlab was used throughout the course. Learning and teaching strategies of the course were: lectures, discussions, exercises, group work and practical computer sessions. Students had to do two individual projects and one group project. They had to give an oral presentation of their group project. Expected students' benefits (learning outcomes) after completing the course were:

- Improving students' generic mathematical modelling skills
- Developing students' skills of analysing aspects of survival and sustainability of ecological, environmental, socio-economic and military systems
- Improving students' multidisciplinary and interdisciplinary competence
- Increasing students' confidence in using computer software
- Improving students' conceptual understanding of related mathematics topics
- Enhancing students' ability to deal with real practical problems
- Developing students' team work skills
- Creating students' ethical responsibility to environmental issues
- Making connections to the local industry
- Enhancing students' problem solving skills

EXAMPLES OF THE MODELS

Below are two out of many models that were used in the course.

<u>1. Model of Bird Population on Rügen Island.</u> The population of rare birds on Rügen Island is modelled by the following differential equation $\frac{dP}{dt} = r(M - P)(P - m)$, where $P = P(t)$ is the number of birds, t is time in years, M is the maximum sustainable population, and m is the minimum population below which the species becomes extinct, r is a positive constant. The values of the maximum and minimum population are $M = 1200$, $m = 100$ and the value of the constant r is 0.001.

Questions:

a) Show that the differential equation can be rewritten in the form $\left[\frac{1}{1200 - P} + \frac{1}{P - 100}\right] dP = 1.1dt$ and solve this equation with pen and paper.

b) How many birds will be in 5 years if there are 300 birds now?

c) Use Matlab to find the solution of the differential equation with the initial condition $P(0) = 300$ analytically (dsolve) and numerically (ode45). Plot the graph of the solution.

d) Estimate from the graph when the rate at which the bird population changes is highest.

2. Populations of Birds and Insects on Poel Island. Populations of birds and insects on Poel Island are modelled by the following system of differential equations:

$$\begin{aligned}\frac{dx}{dt} &= 0.4x - 0.002xy \\ \frac{dy}{dt} &= -0.2y + 0.000008xy\end{aligned} \tag{1}$$

Questions:

a) Which of the variables, x or y, represents the bird population and which represents the insect population? Explain.

b) Find the equilibrium solutions and explain their significance.

c) Find an expression for dy/dx and solve this separable differential equation with pen and paper.

d) Use Matlab (pplane) to draw the phase trajectory corresponding to initial population of 100 birds and 40,000 insects. Use the phase trajectory to describe how both populations change with respect to time.

e) Use part d) to make rough sketches of the bird and insect populations as functions of time. How are these graphs related to each other?

Suppose equations (1) are replaced by the equations:

$$\begin{aligned}\frac{dx}{dt} &= 0.4x(1 - 0.000001x) - 0.002xy \\ \frac{dy}{dt} &= -0.2y + 0.000008xy\end{aligned} \tag{2}$$

f) According to equations (2), what happens to the insect population in the absence of birds?

g) Use Matlab (pplane) to draw the phase trajectory for equations (2) corresponding to initial population of 100 birds and 40,000 insects. Use the phase trajectory to describe how both populations change with respect to time.

THE STUDENTS' ATTITUDES AND PERFORMANCE

After completion of the course the students were asked to answer the question: Do you think this course is suitable for engineering students and if so, why? All 11 students answered 'Yes'. They gave several reasons. The main two main reasons were:

- Improving knowledge (in mathematics, Matlab, mathematical modeling) that is useful for engineering - 10. Typical students' comments: "You consolidate your mathematical knowledge", "Raise knowledge about differential equations and especially how to build them", "Increasing skills in Matlab", "In my opinion many problems or predictions in the 'engineering world' could be handled/solved with the techniques that you can learn here", "Because you learned how to put some problems into a mathematical system", "To see new ways (models)", "In the course you can better make a statement for normal problems about the life", "The mathematical models all around us and the true way for an engineer is to understand how a model from the nature react if you change one parameter".

- Practical and interesting – 7. Typical students' comments: "To get practical problems", "It is very important to use practical part in the course as it is done here to help students to understand what are they going to do in their future jobs", "Of course it deals not with typically engineering problems but after all it was an interesting subject", "I think that every subject which have a lot of practical things is very useful. This mathematical course was very useful for me and I think, that in our university everyone must study mathematics in this way".

All 11 students received excellent or very good final grades. Their learning was measured through two individual projects, one group project and the final exam. Their oral group presentations were independently assessed by two lecturers.

CONCLUSION

The above students' comments indicate that the students were very enthusiastic about the (non-traditional) course. From informal talks to the students we received a strong indication that their enthusiasm and positive attitudes towards the course significantly contributed to their high performance in the course. It was a risk to offer such non-traditional course to engineering students. In spite of concerns of some engineering staff and to their surprise the students were very positive about the course. They were mature enough to value the new knowledge in mathematics and modelling they received from the course that can be applied in engineering. They also enjoyed the practicality of the course that enhanced their problem solving skills. From discussions with the students we understood that all our expected learning outcomes were achieved to a significant extend. It was a good lesson for us as teachers that students' feedback should be taken into account when designing curricula for their study.

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