Mathematics Teaching-Research Journal On-Line
A peer-reviewed scholarly journal
Editors: Bronislaw Czarnocha (Hostos Community College)
Vrunda Prabhu (Bronx Community College)
Anne Rothstein (Lehman College)
City University of New York
Volume 2 Date March 5 2007

Teaching-Research investigation: Planning Solution the Mathematics Word Problem.

Ewa Szczerba, Krygowska PDTR

I have planned to conduct a Teaching-Research investigation titled Planning the Solution of the [mathematics] Word Problem. Word problems are a persistent difficulty for my students. I noticed that during the class devoted to the analysis of the homework and written quizzes. Consequently I decided to investigate this difficulty deeper and at the same time to try to address it.
I would like to inform you about the part of the teaching experiment conducted in the context of the Professional Development of Teacher-Researchers, which was devoted to this difficulty. I have formulated the following Research Questions:
1. Which solving techniques are preferred by the students?
2. How well students understand the written text?
3. To what degree students are able to write the relationships involved in the problem.

Together with the team of mathematics teachers participating in the Professional Development of Teacher-Researchers, I have designed the detailed sequence of diagnostic problems to find answers to these questions. Preliminary diagnosis of the main research issue that is of the Planning Solution of the Word Problem consisted of two stages.

Stage 1
Students get one typical word problem for independent solution during the class. I was hoping it will allow me to find out how many students will solve it, what will be the type of difficulties and when will they occur.
The first stage was conducted 9/25/06 with 16 students; it took 20 minutes.
The problem:
Andrew is two times as old as Beata. In 10 years he will be 1.5 older than she will. How many years older is Andrew than Beata?

Results:
There were 4 categories of results:
1. – absence of the attempt to solve?
2. – attempt to solve without algebra language
3. – attempt to solve using algebra

Mathematics Teaching-Research Journal On-Line
A peer-reviewed scholarly journal
Editors: Bronislaw Czarnocha (Hostos Community College)
Vrunda Prabhu (Bronx Community College)
Anne Rothstein (Lehman College)
City University of New York
Volume 2 Date March 5 2007

4. – attempt to use trial and Error method

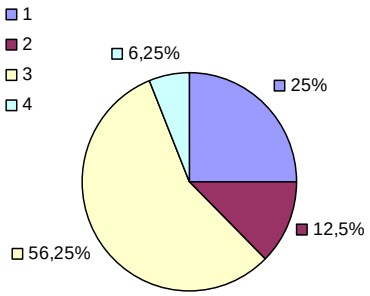

Fig. 1 The distribution of student answers into categories.

Category 1.
Students who didn't try at all had the following comments*:" I don't know how to solve this problem because it's too difficult for them*." „*I didn't know how to do it, had I had some example then maybe I would do it.*"

Category 2
Student, whose work was classified do the 2nd category writes two sentences which he considers essential:
Andrew is two times as old as Beata.
In ten years he will be 1.5 times as old..

Student analysis of the problem consists in re-writing these two conditions in that particular way, and that's why the student resigned from solving the problem. Farther, there is a coment crossed out by the student: *I think that one can not do that, because if Andrew is twice as old as she is.. In 10 years he will be 1.5 times as old , but Beata is also getting old so he can not be 1.5 times as old.*

I think the student does not understand here the changes in the relationship between the two variables, which take place with the passage of time, or, the student has difficulties with reading of multiplicative comparisons. The second possibility is also supported by the second's strudent quote. *Had Beata been 16 years old and Andrew 40 years old, then Beata would be 24 years younger*.

Mathematics Teaching-Research Journal On-Line
A peer-reviewed scholarly journal
Editors: Bronislaw Czarnocha (Hostos Community College)
Vrunda Prabhu (Bronx Community College)
Anne Rothstein (Lehman College)
City University of New York
Volume 2 Date March 5 2007

We see that the student takes arbitrary pair, which fulfils the first [additive] condition, but he does not pay attention in his reasoning that 40 is not 2 times 24. The student writes only about the age difference. One can hypothesize that additive comparison is more understandable for the student than the multiplicative one.

Category 3
Has three subcategories: A. when algebraic notation takes time into account
B. when it does not
C. algebraic expression of the second condition is missing

Category 3A (12,5 %). Algebra with time:
Example 1
„Andrzej – 2x
Beata – x

+10 lat
Andrzej – 10 + 1,5x
Beata – 10 + x……..".

Example 2
„A=2B A=Andrzej
↓In 10 years B=Beata
A+10=B+10·1,5……"
Both have some errors and didn't receive a correct answer.

Category 3B. Algebra without time. (31,25%)
Example 1
„age of Andrzeja – 2x
age of Beaty – x
In 10 years
Age of Beaty – x
Age of Andrzeja – 1,5x……",

Example 2
„A=Andrzej's years Andrzej · 2 >Beata
B=Beat's years

Mathematics Teaching-Research Journal On-Line
A peer-reviewed scholarly journal
Editors: Bronislaw Czarnocha (Hostos Community College)
Vrunda Prabhu (Bronx Community College)
Anne Rothstein (Lehman College)
City University of New York
Volume 2 Date March 5 2007

10 years later *x – as much is Andrzej older than Beata*
Andrzej ·1,5 > Beaty

(2a – b) + (1,5a – b) = x ……..”

Category 3C (12,5%):
„x –age of Beaty
2x – age of Andrzeja”,
i
„Andrzej – x
Beata - 2x…..”

We see that algebraic modelling of word problems is rather difficult for students [in my classroom]. They make many errors, don't continue solving at the moment of encounterin g difficulties. Stop solving at all.

Category 4 Trial and Error methods
The work of the students classified as Category 4 started algebraically, but the student resigns of algebraic language and starts quessing the ages. He takes numbers which fulfil first condition and changes the second one numerically. He doesn't get to the solution because of time constraint.

This first stage of the diagnostic investigation gave me a lot of information, despite the fact that not a single student solved the problem correctly. The first problem is understanding and incorporation of the time variable into solving the problem as well as the absence of understanding the relationships of difference and quotient. One needs to practice these relationson concrete numbers and then generalize from them. Only one student went the Trial and Error method.

The Second Stage took place 5/10/06. There were no practice exercises in between the trials. N=14 students. Time: 45 minutes

Student received the following sequence of problems:

Solve the following problems:
1.

Mathematics Teaching-Research Journal On-Line
A peer-reviewed scholarly journal
Editors: Bronislaw Czarnocha (Hostos Community College)
Vrunda Prabhu (Bronx Community College)
Anne Rothstein (Lehman College)
City University of New York
Volume 2 Date March 5 2007

The sum of the ages of he father and his son is 60 years.. In 15 years father will be two times as old as his son. Find their ages.

First Method of Solving:

Is it possible for the father to have 40 years old and the son – 20 years? Explain your answer.

……
……

Second Method of Solving:

Complete the table:

	At present	in 15 years
Father's age	x	
Son's age		

State the equation
Solve the equation
Check
State the answer

Third Method of Solving:

Complete the table:

	At present	In 15 years
Sum of the ages		
Father's age		
Son's age		y

State the equation
Solve the equation
Check

Mathematics Teaching-Research Journal On-Line
A peer-reviewed scholarly journal
Editors: Bronislaw Czarnocha (Hostos Community College)
Vrunda Prabhu (Bronx Community College)
Anne Rothstein (Lehman College)
City University of New York
Volume 2 Date March 5 2007

State the answer

2.

The same problem can be written in a different way:
In 15 years, the son will be……..

Dokończ to zadanie tak, by warunki zadania nie uległy zmianie.

Discussion of the results

The problem for the First Method of Solving was formulated so as to allow me to see whetehr students pay attention to the second condition. Moreover, the formulation of the question didn't require finding the solution to the problem but checking whether proposed solution is correct. The cognitive effort of the student could be smaller to answer the question. The method of solving was not specified.

The number of students, who didn't address the question at all significantly diminished in relation with the first stage.
There were 6 categories of answers:
1 – correct with full justification;
2 – correct with incomplete justification;
3 – correct with no justification;
4 – correct with wrong justification;
5 – incorrect;
6 – absence of the answer.

Mathematics Teaching-Research Journal On-Line
A peer-reviewed scholarly journal
Editors: Bronislaw Czarnocha (Hostos Community College)
Vrunda Prabhu (Bronx Community College)
Anne Rothstein (Lehman College)
City University of New York
Volume 2 Date March 5 2007

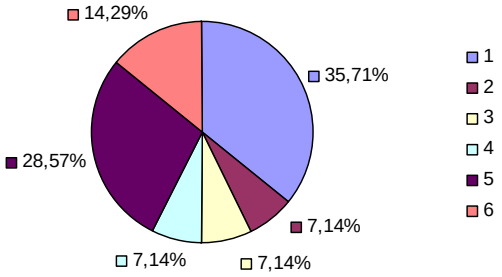

Fig. 2 Percent distribution into categories

Note that only 14.29% didn't give an answer comparing with 25% during the First Stage, indicating that this particular approach have awakened student problem solving initiative. The results of the Second Method of Solving confirm unusually useful charater of previous formulation

The Second and Third Methods were constructed so to enable the answer to the question, whether students can establish and write down noticed relationships in the mathematical language. Writing the same relationships in two ways should give me information whether the student really understands the problem.

The work of students can be classified into 6 categories:

1 – answer 20 i 40 years (as a consequence of the previous work);
2 – other wrong answers;
3 – correct answer by the Trial and Error method;
4 - correct answer by the Trial and Error method AFTER writing the algebraic equation;
5 – correct answer as a aresult of solving an equation;
6—absence of the solution.

Mathematics Teaching-Research Journal On-Line
A peer-reviewed scholarly journal
Editors: Bronislaw Czarnocha (Hostos Community College)
Vrunda Prabhu (Bronx Community College)
Anne Rothstein (Lehman College)
City University of New York
Volume 2 Date March 5 2007

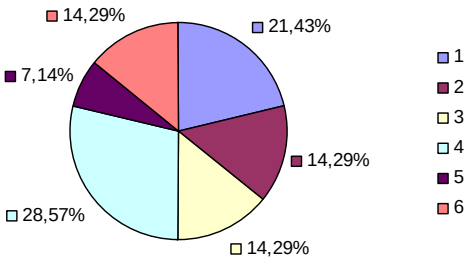

Fig. 3 Percent distribution. Second Method of solving

49% correct answers this time as compared to none in the First Stage;
42% of students used the Trial and Error method.
It seems that the First Method of solving, which suggested the process of checking the second condition with a given pair of numbers, was transferred to the solution of the Second Method's problem resulting in the dramatic incraese of correct solutions. In other words the First Method served not only as the diagnostic instrument but also as the succesful teaching strategy, demonstrating explicitly the duality of the investigation instruments of Teaching-Research methodology. At the same time the expectation of the Teacher-Researcher concerning the diagnosis were not fulfilled – not many students were able to write the algebraic equation. As we will see from the results of the Third Method, similar situation exists there as well.
Analyzing student errors, it seems that the major difficulty is in the algebraic description of the relationships. Especially difficult is the construction of the algebraic equation. Almost 43% used two unknowns although they don't know the method. Instead 49% solved it by the Trial and Error method, indicating that this method is more natural for my students at present than the algebraic method. From the point of view of teaching, the results sugegst that the route to the algebraic understandg is through the extension of T&E methods.

Third Method of Solving. Only one student solved the problem correctly, and we observe increase in the number of students who didn't touch the problem. Clearly the method was too challenging cognitively to the students. The question is why?

Mathematics Teaching-Research Journal On-Line
A peer-reviewed scholarly journal
Editors: Bronislaw Czarnocha (Hostos Community College)
Vrunda Prabhu (Bronx Community College)
Anne Rothstein (Lehman College)
City University of New York
Volume 2 Date March 5 2007

Answering the last Research Question concerning the understanding of the text on the basis of the problem #2, we get 35% not attempting the problem and only 14% did it correctly. It seems that th interpretation of the text is a major problem for my students. They understand it in pieces don't see the whole and miss the essential elements.