**Mathematics Teaching-Research Journal On-Line**
A peer-reviewed scholarly journal
Editors: Bronislaw Czarnocha (Hostos Community College)
Vrunda Prabhu (Bronx Community College)
Anne Rothstein (Lehman College)
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Editorial, MT-RjoL, V2. N1.
March 5, 2007

Respect for the mind of the learner, or comradeship in learning are natural guiding principles in any teaching-research. This necessitates the continual improvement of the diagnosed existing situation, and hence action upon the existing situation with the goal of the improvement of learning.

The improvement of classroom learning is the focus of this issue of the teaching-research journal. It is evident in the articles by all authors; in the tools of the trade; in the research interests of our featured researchers.

The aim of this MT-RJoL issue is to provide an insight into the possibility of traversing the Vygotskian Zone of Proximal Development to make contact with the intuitive/spontaneous concepts of the learner and provide the right "instruction" to take it to the required level of scientific/mathematical concepts, while forming their schema in the minds of students.

Of particular interest in the current issue are reports from Teaching-Research activities in Poland where a current project, called Krygowska Professional Development of Teachers Researchers (www.pdtr.eu) introduces the methodology to that country. These teachers' reports are complemented by the report of Bronx teacher Jacqueline Wright who shares her knowledge and experience concerning strategies of prompt classroom interventions. At the same time we feature two articles from college faculty, Nkechi Agwu and Tom Carey informing about recent advances in college pedagogies. The editors of the journal are tackling promised discussion of the tools of Teaching-Research supplemented by the considerations about the ethics of Teaching-Research.

Editors

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# Teaching-Research investigation: Planning Solution the Mathematics Word Problem.

Ewa Szczerba, Krygowska PDTR

I have planned to conduct a Teaching-Research investigation titled Planning the Solution of the [mathematics] Word Problem. Word problems are a persistent difficulty for my students. I noticed that during the class devoted to the analysis of the homework and written quizzes. Consequently I decided to investigate this difficulty deeper and at the same time to try to address it.
I would like to inform you about the part of the teaching experiment conducted in the context of the Professional Development of Teacher-Researchers, which was devoted to this difficulty. I have formulated the following Research Questions:
1. Which solving techniques are preferred by the students?
2. How well students understand the written text?
3. To what degree students are able to write the relationships involved in the problem.

Together with the team of mathematics teachers participating in the Professional Development of Teacher-Researchers, I have designed the detailed sequence of diagnostic problems to find answers to these questions. Preliminary diagnosis of the main research issue that is of the Planning Solution of the Word Problem consisted of two stages.

**Stage 1**
Students get one typical word problem for independent solution during the class. I was hoping it will allow me to find out how many students will solve it, what will be the type of difficulties and when will they occur.
The first stage was conducted 9/25/06 with 16 students; it took 20 minutes.
**The problem**:
*Andrew is two times as old as Beata. In 10 years he will be 1.5 older than she will. How many years older is Andrew than Beata?*

**Results**:
There were 4 categories of results:
1. – absence of the attempt to solve?
2. – attempt to solve without algebra language
3. – attempt to solve using algebra

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4. – attempt to use trial and Error method

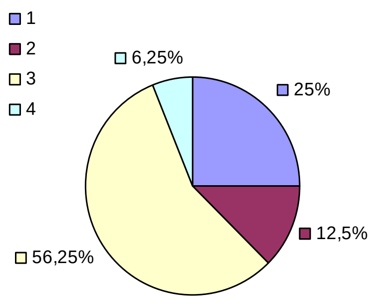


Fig. 1 The distribution of student answers into categories.

Category 1.
Students who didn't try at all had the following comments*:" I don't know how to solve this problem because it's too difficult for them*." „*I didn't know how to do it, had I had some example then maybe I would do it.*"

Category 2
Student, whose work was classified do the 2nd category writes two sentences which he considers essential:
Andrew is two times as old as Beata.
In ten years he will be 1.5 times as old..

Student analysis of the problem consists in re-writing these two conditions in that particular way, and that's why the student resigned from solving the problem. Farther, there is a coment crossed out by the student: *I think that one can not do that, because if Andrew is twice as old as she is.. In 10 years he will be 1.5 times as old , but Beata is also getting old so he can not be 1.5 times as old.*

I think the student does not understand here the changes in the relationship between the two variables, which take place with the passage of time, or, the student has difficulties with reading of multiplicative comparisons. The second possibility is also supported by the second's strudent quote. *Had Beata been 16 years old and Andrew 40 years old, then Beata would be 24 years younger*.

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We see that the student takes arbitrary pair, which fulfils the first [additive] condition, but he does not pay attention in his reasoning that 40 is not 2 times 24. The student writes only about the age difference. One can hypothesize that additive comparison is more understandable for the student than the multiplicative one.

Category 3
Has three subcategories: A. when algebraic notation takes time into account
B. when it does not
C. algebraic expression of the second condition is missing

Category 3A (12,5 % ). Algebra with time:
Example 1
*„Andrzej – 2x*
*Beata – x*

*+10 lat*
*Andrzej – 10 + 1,5x*
*Beata – 10 + x………………….”.*

Example 2
*„A=2B A=Andrzej*
*↓In 10 years B=Beata*
*A+10=B+10·1,5………….”*
Both have some errors and didn't receive a correct answer.

Category 3B. Algebra without time. (31,25% )
Example 1
*„age of Andrzeja – 2x*
*age of Beaty – x*
*In 10 years*
*Age of Beaty – x*
*Age of Andrzeja – 1,5x……”,*

Example 2
*„A=Andrzej's years Andrzej · 2 >Beata*
*B=Beat's years*

*10 years later* *x – as much is Andrzej older than Beata*
*Andrzej ·1,5 > Beaty*

*(2a – b) + (1,5a – b) = x …………….”*

*Category* 3C (12,5%):
*„x –age of Beaty*
*2x – age of Andrzeja”,*
i
*„Andrzej – x*
*Beata - 2x…..”*

We see that algebraic modelling of word problems is rather difficult for students [in my classroom]. They make many errors, don't continue solving at the moment of encounterin g difficulties. Stop solving at all.

Category 4 Trial and Error methods
The work of the students classified as Category 4 started algebraically, but the student resigns of algebraic language and starts quessing the ages. He takes numbers which fulfil first condition and changes the second one numerically. He doesn't get to the solution because of time constraint.

This first stage of the diagnostic investigation gave me a lot of information, despite the fact that not a single student solved the problem correctly. The first problem is understanding and incorporation of the time variable into solving the problem as well as the absence of understanding the relationships of difference and quotient. One needs to practice these relationson concrete numbers and then generalize from them. Only one student went the Trial and Error method.

**The Second Stage** took place 5/10/06. There were no practice exercises in between the trials. N=14 students. Time: 45 minutes

Student received the following sequence of problems:

**Solve the following problems:**
**1.**

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The sum of the ages of he father and his son is 60 years.. In 15 years father will be two times as old as his son. Find their ages.

**First Method of Solving:**

Is it possible for the father to have 40 years old and the son – 20 years? Explain your answer.

……………………………………………………………………………………………
………………………………………………………………………

**Second Method of Solving:**

Complete the table:

| | At present | in 15 years |
|---|---|---|
| Father's age | x | |
| Son's age | | |

State the equation
Solve the equation
Check
State the answer

**Third Method of Solving:**

Complete the table:

| | At present | In 15 years |
|---|---|---|
| Sum of the ages | | |
| Father's age | | |
| Son's age | | y |

State the equation
Solve the equation
Check

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State the answer

**2.**

The same problem can be written in a different way:
*In 15 years, the son will be………*

Dokończ to zadanie tak, by warunki zadania nie uległy zmianie.

**Discussion of the results**

The problem for the First Method of Solving was formulated so as to allow me to see whetehr students pay attention to the second condition. Moreover, the formulation of the question didn't require finding the solution to the problem but checking whether proposed solution is correct. The cognitive effort of the student could be smaller to answer the question. The method of solving was not specified.

The number of students, who didn't address the question at all significantly diminished in relation with the first stage.
There were 6 categories of answers:
1 – correct with full justification;
2 – correct with incomplete justification;
3 – correct with no justification;
4 – correct with wrong justification;
5 – incorrect;
6 – absence of the answer.

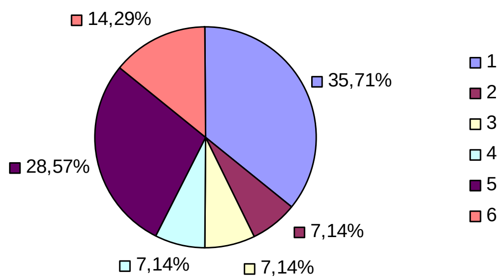


Fig. 2 Percent distribution into categories

Note that only 14.29% didn't give an answer comparing with 25% during the First Stage, indicating that this particular approach have awakened student problem solving initiative. The results of the Second Method of Solving confirm unusually useful charater of previous formulation

**The Second and Third Methods** were constructed so to enable the answer to the question, whether students can establish and write down noticed relationships in the mathematical language. Writing the same relationships in two ways should give me information whether the student really understands the problem.

**The work of students can be classified into 6 categories**:

1 – answer 20 i 40 years (as a consequence of the previous work);
2 – other wrong answers;
3 – correct answer by the Trial and Error method;
4 - correct answer by the Trial and Error method AFTER writing the algebraic equation;
5 – correct answer as a aresult of solving an equation;
6—absence of the solution.

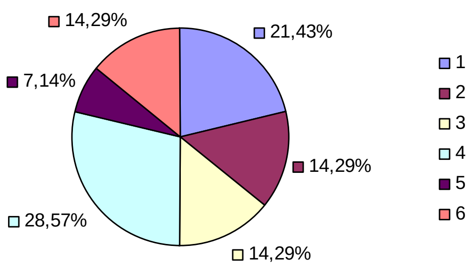


Fig. 3 Percent distribution. Second Method of solving

49% correct answers this time as compared to none in the First Stage;
42% of students used the Trial and Error method.
It seems that the First Method of solving, which suggested the process of checking the second condition with a given pair of numbers, was transferred to the solution of the Second Method's problem resulting in the dramatic incraese of correct solutions. In other words the First Method served not only as the diagnostic instrument but also as the succesful teaching strategy, demonstrating explicitly the duality of the investigation instruments of Teaching-Research methodology. At the same time the expectation of the Teacher-Researcher concerning the diagnosis were not fulfilled – not many students were able to write the algebraic equation. As we will see from the results of the Third Method, similar situation exists there as well.
Analyzing student errors, it seems that the major difficulty is in the algebraic description of the relationships. Especially difficult is the construction of the algebraic equation. Almost 43% used two unknowns although they don't know the method. Instead 49% solved it by the Trial and Error method, indicating that this method is more natural for my students at present than the algebraic method. From the point of view of teaching, the results sugegst that the route to the algebraic understandg is through the extension of T&E methods.

**Third Method of Solving.** Only one student solved the problem correctly, and we observe increase in the number of students who didn't touch the problem. Clearly the method was too challenging cognitively to the students. The question is why?

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Answering the last Research Question concerning the understanding of the text on the basis of the problem #2, we get 35% not attempting the problem and only 14% did it correctly. It seems that th interpretation of the text is a major problem for my students. They understand it in pieces don't see the whole and miss the essential elements.

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# Diagnostic procedure in algebra course of the first grade of the Middle School

Teacher Jerzy Migon, Krygowska PDTR.

The students had not have as yet the algebra course in the Middle School; they had a bit, but not too much, of it in the elementary grades. Nonetheless, all of them, when asked for the area of a rectangle, recite: P = a*b (P because in Polish area starts with the letter P, from Pole).

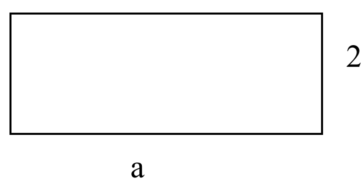
Equally easly they write formulas of the triangle, paralllelogram or trapezoid. That freedom of „letters" is, however, illusory.

I proposed 3,4 problems for them. The time of work = 30 minutes.

**Problem 1**. The area of the square is 9 $cm^2$. What is the length of the square?

**Problem 2.** Solve the equation $x^2 = 16$

**Problem 3**. Given rectangle has dimensions: length = a (cm), width = 2 (cm)




a) how much will the area increase, if the length increases by 3 (cm)?
b) how much will area increase, if the width increases by b (cm)?

The results confirm our misgivings.

Prob. 1 is solved by all students; they provide adequate drawings:

$P = a^2 = 9\ (cm^2)$ $\quad$ or $\quad$ $P = a \cdot a = 9$

$a = \sqrt{9} = 3\ (cm)$ $\quad\quad$ $3 \cdot 3 = 9$ , $\quad a = 3$

Even M triumphs: I did it.
The only problem was the absence of units.

Prob. 2. The difficulties start. Everyone find x = 4 because 4*4=16, or $x = \sqrt{4}$
Sometime there is a drawing of the square followed by the statement x = 4 cm.
We see that geometrical interpretation is omnipresent here favouring positive definite solutions. No one found the second solution x = -4 !

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Prob. 3 For the majority of the class the problem turns out too difficult. Angela is asking: how can I calculate how much the area of the rectangle had increased, if I don't know what it was in the beginning. The numrical value of the width is unknown…

Given width of the rectangle a associates with the unknown, hence I am providing a following intervention-hint: let's treat width **a** as a known quantity although arbitrarily chosen.

The intervention was quite succesfull: there appears one correct solution, and several interesting ones.

S1: : $a \cdot 2 = P$ , $(a + 3) \cdot 2 = P + 6$ ( this added length x 2 ) – explains

a)

Example: $5 \cdot 2 = 10$

) + 6

$(5 + 3) \cdot 2 = 16$

Answer:. The area will increase by 6 ($cm^2$)

b) $2 \cdot a = P$ , $(2 + b) \cdot a = P + b \cdot a$

Answer:. The area will increase by $b \cdot a$ ($cm^2$)

Distributivity of multiplication with respect of the addition gives some trouble. There is an additional drawing in S1 case which we ommit here.

S2 : a) area = $2 \cdot a = 2a$ , $2 \cdot (a + 3) = 6a$

Answer: area will incraese 3 times.

b) $(a \cdot 2) + b = 2a + b$

Odp. Area will incraese by **b** .

S3: : a) length = a (cm) , length = a + 3

$P = a \cdot b$ , $P = a \cdot 2$

$P = (a \cdot 2) + 3 =$ I don't know.

I am hurrying at the end of the class to discuss these problems. Why no one did the drawing for the problem #3? The letter a hiding the length of rectanle was a aproblem for 90% of the students. Absent this day S4 does it easily later but he is much above the level…

**Conclusions**:

1. Letters function for my students as part o formulas which indicate what operations to make to find the numerical value of the seeked quality such as area.
2. Problems are caused by a letter-parameter. Of course, here the technique of algebraic trasnformations magnifies this problem.
3. What's left to investigate is understanding of the letter as the unknown in the equation. One needs to quess it or solve the equation.

The class is writing a test in geometry a week later to which I am adding „my" two problems:

**Problem 4.** The region whose shape is a triangle has an area 2.5 ar. The basis of the triangle is 20 cm. What is the height of the triangle ?

**Problem 5.** The side of a triangle is a (m). The height dropped to this side is 10 (m). How much will the area of this triangle incraese of we incraese the side by 8 (m)?

**Results. 6 correct solutions**

Prob. 4. : S2 $P = a \cdot h / 2$ , $a = 20$ m , $h =$ ?
$20\text{m} \cdot h / 2 = 2{,}5\text{a}$ , $2{,}5\text{a} = 250\text{m}^2$ , $20\text{m} \cdot h / 2 = 250\text{m}^2$
$h/2 = 12{,}5$ , $h = 25$

4 incomplete solutions (wrong formulas for the area)
S5 : $x$ = height, $x \cdot 20 = 250$ , $x = 12{,}5$

Prob. 5 :

**S4 :** The area will increase by $8m \cdot 10m / 2 = 40m^2$ ( absence of a drawing )

**S1** : $P = 10 \cdot a / 2$ , $P = (a + 8) \cdot 10 / 2$ , $8 \cdot 10 / 2 = 40$ ( absence of a drawing )
Area will increase by 40.

**S3** : $P = a \cdot 10 / 2$ , $P = (a + 8) \cdot 10 / 2$
Answer.: The area will increase by $(a + 8) \cdot 10 / 2$

**Conclusions**:
Nothing changed after a week. To say the truth, prob. 5 is a bit more difficult than the Prob.3. No drawing. But Prob.4, where the letter appears as the unknown to find comes out much better.

I gave the problems 4 and 5 to another, parallel class, just out of curiosity. I asked for the drawing to be made in Prob.5. This time there 3 correct solutions and one one close to full solution. Same doubts about the „given length of value a…" – a contradiction, or…

Zadania 4 i 5 dałem też klasie Ib (równoległej do klasy badawczej) wiedziony ciekawością jak tu będzie ! ? Do zad. 5 dołączyłem polecenie : zrób rysunek do zadania. Tym razem są 3 rozwiązania prawidlowe oraz 1 rozw. bliskie prawidłowemu (błędny wzór) sztandarowego zadania 5 ! Te same watpliwości i rozterki odnośnie „danej długości wynoszącej a metrów" - sprzeczność, chyba ...

**S6 writes** : h = 10m, a = ? m ! She should have written a = a (m) ...
So easy it is to get into collision between letters. But the drawing saves S6 :

$$P = 10 \cdot 8 / 2 = 40(m^2)$$

Drawings are very important; we have to remind about them and require as an element of the solution. The drawing helped to avoid the issue of letters and algebraic tarnsformations. The problem is still in front of us, nonetheless, and subsequent exercises are supposed to eliminate it or diminish.

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# Mathematical Domino

Barbara Klawitter-Brzezinska
Beata Mielanczuk

**Mathematical Domino** is directed primarily towards the work with students of elementary grades, and especially to those students who believe and are convinced that mathematics is understandable only for the chosen. Working with students in classrooms with different intellectual level, we have attempted to encourage students to a different view on mathematics and to show that learning mathematics can give joy to everyone, not only those more able mathematically. We proposed mathematical domino. The effects didn't let us wait for too long. In fact it is those, mathematically weaker children forced us to create a domino for each following class. To our surprise they didn't ask for grades. They were happy that were able to play and to do the problems independently. They didn't ask for a break between classes forgetting about it completely, while engrossed in the game. Looking at their joy, happiness in their eyes one could not disappoint them. It wasn't easy. Consequently, the presented collection of domino games from different areas of mathematics had been developed on the basis of authentic needs of pupils. We would like to encourage teachers, colleagues to work wit this game. We are convinced that it will help students to get better results in their learning; we assure that well planned classroom with the domino game will not be wasted.

**How to work with the mathematical domino?**

The essential aspect of the game is exactly the same as in the standard domino game. Each piece has two parts, one of them is the answer to a problem stated on a different piece, the other is a new problem. For example, if one half of the piece has $8\frac{1}{5}$ and the other half of the piece has $\frac{34}{40}-\frac{20}{40}$, the first one is the answer to a problem stated in another domino piece, for example $8\frac{4}{5}-\frac{3}{5}$, while the answer to $\frac{34}{40}-\frac{20}{40}$ is still on a different domino piece. The collection of pieces in on set is designed so as to create the closed chain. This can be done by one, two or more students playing the game. It can also be done by one student as a homework. The number of players is arbitrary but it is limited by the number of domino pieces. Different sets have different number of pieces; not too many though in order not to discourages students. And not too few, to be able to work in a group. The rules of the game can be exactly the same as in a standard game but also one can decide on different rules. Students can use books or notebooks so that they can definitely finish each game. Playing the game they have the possibility for self-control; finding the correct piece they convince themselves that they did it correctly. The game is designed so that that it's easy step-after-step process, and repetition of rules and formulas allow for self-verification. It provides the possibility to trace one's own errors and correct them, hence students are stimulated for self-reflection.

Playing the game creates the conditions for independent action through:

- Choice and planning of rules of the game
- The plan of the game
- Checking one's own computations.

Playing the game shapes and develops student imagination. Often, creating the loop, students try to give it an original shape. Thanks to the game-like nature of the exercise, understanding and retention of different mathematical issues becomes easy and mobilizes the child to independent work. The game develops intellectual activity and motivates creativity (especially when students design their own domino sets).

**Mathematical domino** is helpful not only for students, for whom mathematics is more difficult but also for those children who like and enjoy mathematics, and would like to calculate faster. Many of the domino calculations can be performed faster in memory without writing in the notebook.

The game was designed in 3 versions, A,B,C. The version A being the easiest, B – more difficult although it contains some elements from A. The role of the version A is to encourage the student to reach for more difficult versions. The version B is designed for a student, who having done the version A, would like to try something more difficult. Similar relations is between the version B and the version C. Independently of these relationships, each set has its own independent structure. The sets are not arranged accidentally. Each example and its order is chosen so that the student, almost without noticing it, was solving more difficult problems. The game can be used as a review, as the beginning of the class or as a homework.

The authors wish you nice and fruitful play!

| | | | | | | |
|---|---|---|---|---|---|---|
| $\frac{5}{17} - \frac{3}{17} + \frac{9}{17}$ | $\frac{9}{31} - \frac{7}{31} + \frac{4}{31}$ | $5\frac{1}{12} - 2\frac{7}{12} + \frac{5}{12}$ | $13\frac{8}{15} - (6\frac{1}{15} + 4\frac{2}{15})$ | $5\frac{7}{18}$ | $7$ | $1$ |
| $\frac{1}{10}$ | $\frac{11}{17}$ | $12\frac{11}{15}$ | $\frac{6}{31}$ | $10\frac{3}{4} - x = 5\frac{1}{4}$ | $(2\frac{3}{4} + 4\frac{1}{4}) - 3\frac{3}{4}$ | $3\frac{1}{8} + (4\frac{5}{8} - 2\frac{3}{8})$ |
| $x - 4\frac{9}{15} = 8\frac{2}{15}$ | $5\frac{3}{8}$ | $8\frac{1}{4} - 3\frac{3}{4} + 2\frac{2}{4}$ | $7\frac{2}{13}$ | $(19\frac{16}{21} - 2$ [illegible]$) + 1\frac{4}{21}$ | $4\frac{4}{5}$ | $18\frac{1}{7}$ |
| $3\frac{1}{4}$ | $2\frac{2}{5} - \frac{4}{5} + 3\frac{1}{5}$ | $5\frac{1}{2}$ | $\frac{1}{7} + \frac{5}{7} - \frac{2}{7} + \frac{3}{7}$ | $2\frac{11}{12}$ | $4\frac{5}{18} + (6\frac{8}{18} - 4\frac{6}{18})$ | $3\frac{1}{16} + \frac{7}{16} - 3\frac{7}{16}$ |

## 32. Dodawanie i odejmowanie ułamków zwykłych o jednakowych mianownikach

| | | | | | | |
|---|---|---|---|---|---|---|
| $\frac{1}{2} + \frac{3}{5}$ | $\frac{7}{24}$ | $\frac{13}{18} + \frac{5}{6} + \frac{11}{12}$ | $9\frac{3}{10} - 5\frac{5}{6}$ | $5\frac{14}{15} - 2\frac{2}{9}$ | $1\frac{43}{45}$ | $x - 7\frac{3}{18} = 3\frac{11}{12}$ |
| $1\frac{19}{30}$ | $\frac{1}{2} + \frac{3}{4}$ | $2\frac{2}{5}$ | $12\frac{11}{18}$ | $\frac{4}{7}$ | $11\frac{7}{36}$ | $6\frac{2}{7} + 4\frac{3}{14} + 1\frac{1}{4}$ |
| $\frac{2}{3} - \frac{1}{4}$ | $4\frac{1}{8} - 2\frac{5}{6}$ | $x + \frac{3}{5} = 1\frac{6}{35}$ | $1\frac{1}{4}$ | $3\frac{32}{45}$ | $1\frac{7}{24}$ | $\frac{4}{5} + \frac{5}{6}$ |
| $1\frac{1}{10}$ | $2\frac{17}{36}$ | $\frac{5}{12}$ | $3\frac{7}{12} + x = 5\frac{59}{60}$ | $\frac{11}{12} - \frac{5}{8}$ | $\frac{7}{15} + \frac{3}{5} + \frac{8}{9}$ | $3\frac{7}{15}$ |

## 34. Dodawanie i odejmowanie ułamków zwykłych o różnych mianownikach

| | | | | | | |
|---|---|---|---|---|---|---|
| $\frac{\square}{150}$ | $\frac{\square}{12}$ | $\frac{3}{4}$ i $\frac{9}{14}$ | $\frac{7}{8}$ i $\frac{4}{5}$ | $\frac{\square}{20}$ | $\frac{3}{4}$ i $\frac{5}{9}$ | $\frac{\square}{42}$ |
| $\frac{2}{3}$ i $\frac{3}{4}$ | $\frac{1}{4}$ i $\frac{3}{10}$ | $\frac{\square}{45}$ | $\frac{\square}{15}$ | $\frac{5}{8}$ i $\frac{4}{6}$ | $\frac{\square}{40}$ | $\frac{\square}{28}$ |
| $\frac{7}{10}$ i $\frac{5}{6}$ | $\frac{\square}{30}$ | $\frac{3}{7}$ i $\frac{5}{6}$ | $\frac{5}{9}$ i $\frac{3}{5}$ | $\frac{4}{5}$ i $\frac{13}{15}$ | $\frac{\square}{18}$ | $\frac{\square}{60}$ |
| $\frac{9}{11}$ i $\frac{4}{5}$ | $\frac{\square}{36}$ | $\frac{4}{9}$ i $\frac{1}{2}$ | $\frac{\square}{55}$ | $\frac{\square}{24}$ | $\frac{5}{12}$ i $\frac{11}{20}$ | $\frac{17}{50}$ i $\frac{37}{75}$ |

## 33. Znajdowanie wspólnego mianownika

| | | | | | | |
|---|---|---|---|---|---|---|
| $\frac{1}{2} \cdot 2$ | $2\frac{2}{5}$ | $\frac{4}{5} \cdot 2$ | $11$ | $\frac{7}{8} \cdot 10$ | $4$ | $8\frac{1}{2} \cdot 7$ |
| $65$ | $\frac{4}{3} \cdot 3$ | $2\frac{7}{10}$ | $\frac{2}{5} \cdot 3$ | $2$ | $4 \cdot 2\frac{3}{4}$ | $8\frac{3}{4}$ |
| $1$ | $5\frac{1}{4}$ | $1\frac{1}{5}$ | $59$ | $3 \cdot \frac{4}{5}$ | $\frac{1}{2} \cdot 4$ | $7 \cdot 9\frac{2}{7}$ |
| $\frac{3}{7} \cdot 5$ | $6\frac{5}{9} \cdot 9$ | $\frac{7}{12} \cdot 9$ | $3 \cdot \frac{9}{10}$ | $59\frac{1}{2}$ | $2\frac{1}{7}$ | $1\frac{3}{5}$ |

## 35. Mnożenie ułamków zwykłych przez liczbę naturalną

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# Selected Middle Grade Intervention Strategies

**Jacqueline Wright**

According to the New York State Content Learning Standard for Mathematics, basic operations are introduced in grade 3 and further developed in grades, 4 and 5. Therefore, expectation is that middle grades students will be familiar and fluent with addition, subtraction multiplication and division of whole numbers. This level of proficiency is foundational for teaching more complex topics, i.e., representation of repeated multiplication in exponential form, representation of exponential form as repeated multiplication, identification of the multiplicative inverse (reciprocal) of a number.

Middle school teachers do not expect to introduce multiplication and division operations as if they are completely unfamiliar to students. However, at the beginning of the current school year, several teachers reported that student performance on informal and formal assessments highlighted significant gaps in student readiness for upper elementary mathematics instruction. Specifically, students demonstrated major gaps in understanding basic whole number operations (especially with multiplication and division). Discussions within the mathematics department (with teachers, lead teachers, mathematics coach and administration) resulted in two main suggestions for dealing with this issue: (1) re-teach basic operations before teaching grade specific topics and (2) identify and implement prompt intervention strategies. For the most part, several teachers re-taught basic operations before moving on to grade specific topics. As the school's mathematics coach, I did my own teacher research about intervention strategies for older students. When I heard about this online teacher research journal, I decided to share my findings. What follows are selected intervention strategies; I do not claim that this effort was comprehensive. There is so much more to read and reflect on. I do plan to share this paper with colleagues at our regularly scheduled common planning sessions. My hope is we will dialogue about their practicality.

## Prompt Intervention

Sigrid Wagner identifies prompt intervention as intervention that occurs before a student fails[1]. Wagner writes that this intervention is necessary when students' progress toward mathematical proficiency is in danger.

***The Cyclopedic Education Dictionary*** defines intervention instruction as educational assistance given to students. Assistance (which includes but is not limited to extra help, additional time, a modified teaching approach, remedial aid, etc.) preempts the need for referral for special needs evaluation.

In ***Elementary and Middle School Mathematics, Teaching Developmentally,*** Van de Walle states that children who have not mastered basic algorithms by sixth or seventh grade have seen and practiced basic operations countless times throughout their years in school. Thus, they will more probably ***not*** benefit from more drill instruction. Clearly, struggling students need something better. Van de Walle makes the following suggestion to help middle school students.

1. Recognize that more algorithmic may be ineffective. It is unwise to subject struggling students to fact drills unless they are comfortable with drill and have experienced success with them.

2. Find out what students do when they encounter math activities that they do not know how to approach. Do they count on their fingers? Add up the numbers in the margins?

3. Provide hope. Children who have experienced difficulty with fact drills often believe that they cannot learn facts without counting on their fingers. Let these children know that you will help them and that you will provide some new ideas that will help them as well.

4. Build in success. Begin with easy strategies, and introduce only a few new concepts or skills at a time.

Additionally, Janzen[2] outlines these additional intervention strategies.

5. Use quick warm-up activities in class. Warm-up activities are good for reviewing prerequisites and gauging student mastery of concepts. Begin lessons by having students complete several problems that cover prerequisites. While students are working on warm-up activities, teachers may conference with selected students.

6. Student writing in math provides insights about their misunderstandings and identifies gaps in understanding. Writing provides students with opportunities to think about key

prerequisites. Prompt students to use math journals to record steps they used to solve problems. Student explanations can also be used as a form of error analysis to help identify gaps in understanding.

7. Utilize multiple techniques to assess depth of student understanding. For example, application problems are a good mechanism through which students demonstrate mastery of specific skills or concepts and readiness to move on to new concepts. Application problems also help identify students who have not thoroughly mastered specific concepts and who may require intervention to avoid moving on to a new concept or skill too quickly.

8. Small group instruction is also a beneficial intervention strategy. Working in a small group or with a partner may be less intimidating and encourage struggling students to ask questions and admit confusion.

Invariably, students benefit from explanations from their peers. In fact, student to student explanations often make more sense than those offered by teachers. When students work independently or with others, teachers have opportunities to assess student learning informally.

9. Differentiating or varying instructional approaches is another way to help struggling students. When students do not understand a concept that is presented concretely, illustrate it by using symbols, pictures, graphs, models, manipulative or technology. Varied instructional strategies included in lessons enhance student chances of grasping concepts.

10. Use as many representations of the concept as possible: try manipulatives, models, real life examples, technology and symbolic representations. Middle school students often need more interventions because they have difficulty grasping abstract concepts of higher level mathematics. The use of multiple representations can help address their needs.

11. Help students see the value and application of the mathematics they are studying by presenting as many practical applications as possible. By relating a math topic to something relevant to student's life increases student interest and makes the math more

meaningful. This may be especially beneficial for struggling students who do not see how the math they are studying relates to them.

12. In addition to the intervention strategies mentioned above, provide students and parents with tutoring options that include the United Federation of Teacher's Dial- A-Teacher Program, Supplemental Education Services (SES), Saturday school programs, online help, alternative homework assignments and local after school program options.

Finally, it is important that all stakeholders in the education process (students, parents, teachers, administrators and the other members of the school community) know that students can and must be taught how to learn. This includes teachers planning lessons that encourage and motivate students to put forth effort. All students can learn, and teachers must be willing to provide different ways for them to do so. Professional development for teachers who are providing prompt intervention is central to the implementation of intervention strategy. In the final analysis, the success of intervention strategies requires the commitment of all members of the learning community.

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## Using the Concept of Upper and Lower Bounds to Find Square Roots

Nkechi Agwu

### Introduction

This paper provides a short lesson that can be used to reinforce students' understanding of square roots. Through this lesson, students will review the definition of square roots and the chronological history of the square root symbol. They will use two ancient Egyptian methods involving upper and lower bounds to approximate square roots and possibly facilitate their understanding of the concept of a limit. Students' will solve quadratic equations by applying these two methods. This lesson requires students to have some pre-requisite knowledge of real and imaginary numbers, perfect squares, and upper and lower bounds, or the instructor to introduce the afore-mentioned mathematical ideas. The concepts of upper and lower bounds and limit are fundamental ideas in mathematical analysis, so early reinforcement of these concepts are crucial for student success in calculus.

### Definition of Square Root

Let x and N be real numbers, with $N \geq 0$.

Then x is the square root of N, if and only if the square of x is N.

Symbolically $x = \pm \sqrt{N} \rightarrow X^2 = N$.

The principal square root of N is $\sqrt{N}$.

If $N < 0$, then $\sqrt{N}$ is not a real number. It is an imaginary number.

For example, $\sqrt{-16} = \sqrt{[(16)(-1)]} = (\sqrt{16})(\sqrt{-1}) = \pm 4i$ or $(\pm 4i)^2 = -16$

**Examples:**

(1) Find the square root of 9.

Answer: $\sqrt{9} = \pm 3$ since $(\pm 3)^2 = 9$

(2) Find the principal square root of 9.

Answer: The principal $\sqrt{9} = 3$

(3) Find the square root of –9.

$\sqrt{-9} = \pm 3i$ since $(\pm 3i)^2 = -9$

**Chronological history of the square root symbol**

A discussion of the chronological history of the square root symbol provides students' with the perspective of mathematics as an evolving discipline and with an appreciation of the contributions of different people and cultures to the development of mathematical concepts. The chronological history of the square root symbol provides a

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global perspective to the concept of square root. The history indicates that the square root symbol whose origin can be traced back to the ancient Egyptian civilization (3000B.C. – 400 A.D.) in Africa showed up later in the 15th century mathematical traditions of Europe in France and Germany, undergoing various evolutionary forms and finally transitioning to the symbol that we use in today's modern society.

(1) In ancient Egypt, A'homose in the *Rhind Papyrus* used the symbol "⌈" to denote square root.

(2) In France, Nicolas Chuquet (d.1487) used "R²6" for the square root of 6.

(3) In Germany, Michael Stifel (ca.1487-1567) developed the symbol "√·" and later refined it to "√ ".

(4) In France, François Viéte (1540-1603) used "l" to denote square root, but his follower Franz van Schooten adopted the radical sign "√ " in 1646.

(5) In the 16th century, Descarte used the symbol "√ " to indicate square root. This is the symbol we continue to use today to indicate square root.

**Finding Square Roots Using Ancient Egyptian Methods of Upper and Lower Bounds**

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In ancient times, our ancestors did not have calculators. So, they used approximate methods to calculate the value of the square root of a number. Given below are a few of these methods.

**Method 1 - Upper and Lower Bounds**

Step 1

Determine the number of the digits in the integer part of the square root.

This will involve upper and lower bounds for the integer part that are powers of 10.

For example, for $10^n$ where n is a whole number, there is a number x for which $1 \leq x < 100$.

Observe that $1 \leq \sqrt{x} < 10$.

The square root of x will only have one digit left to the decimal point.

Step 2

Select the number of decimal places of interest.

Then write the square root as A.BCD…, where A is the integer part and .BCD… is the decimal part.

Step 3 and subsequent steps

Proceeding step by step using upper and lower bounds as explained below.

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Determine the square root to the given number of decimal places.

**Example 1**: Find the solution of the quadratic equation $X^2 = 1000$.

**Solution:** To answer the question, we need to find the square root of 1000 to the second decimal place.

Step 1

Since $10^2 < 1000 < 100^2$ then, $10 < \sqrt{1000} < 100$.

The square root of 1000 has 2 digits number left to the decimal point, so it has the form as AB.CD.

Step 2

Find out the first digit number A.

$10^2 = 100$, $20^2 = 400$, $30^2 = 900$, $40^2 = 1600$,…

900 is too small for 1000, and 1600 is too big for 1000, so we can easily tell the square root of 1000 lies between 30 and 40, and the first digit in its integer is 3, written as 3B.CD…

Step 3

Find out the second digit B.

$30^2 = 900$, $31^2 = 961$, $32^2 = 1024$,…

961 is too small for 1000, and 1024 is too big for 1000, so we can tell the square root of 1000 is between 31 and 32, and the second digit must be 1, written as 31.CD…

Step 4

Find out the first decimal number C.

$31.1^2 = 967.21$, $31.2^2 = 973.44$, $31.3^2 = 979.69$, $31.4^2 = 985.96$, $31.5^2 = 992.25$, $31.6^2 = 998.56$, $31.7^2 = 1004.89$,…

You will see that $998.56<1000<1004.89$, so the square root of 1000 lies between 31.6 and 31.7. The first decimal number is 6, the value is written as 31.6D…

Step 5

Find out the second decimal number D.

$31.61^2 = 999.1921$, $31.62^2 = 999.8244$, $31.63^2 = 1000.4569$,…

The square root of 1000 lies between 31.62 and 31.63, the second decimal number D is 2, written as 31.62…

Now, we've found out the square root of 1000 to the second decimal places, so the solution is x = 31.62… to the 2nd decimal place. This is fairly accurate, but it is terribly slow and inconvenience, since it requires a huge amount of calculation. Also errors may occur due to the miscalculation.

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**Exercises:**

Find an approximate solution for the given quadratic equations to the given decimal places. Use the square root method involving upper and lower bounds.

(1) Solve the quadratic equation $x^2 - 1570 = 0$, remain accuracy to 2nd decimal place.

(2) Solve x for $x^2 - 442 = 0$, remain accuracy to the 1st decimal place.

(3) Solve x for $x^2 - 8 = 0$, remain accuracy to the 3rd decimal place.

(4) Solve x for $x^2 - 56.32 = 0$, remain accuracy to the 2nd decimal place.

(5) Solve x for $x^2 - 1.72 = 0$, remain accuracy to the 3rd decimal place.

**Answers**: (1) 39.62 (2) 21.0 (3) 2.828 (4) 7.5 (5) 1.311

**Method 2 - Trial and Error Method for a Perfect Square**

When you are asked to find the square root of a number that has a perfect square root, you can use the method below.

**Problem:**

Solve x for $x^2 - 1156 = 0$.

$X^2 = 1156$

$X = \pm\sqrt{1156}$.

STEP I:

Determine two perfect squares, a close lower bound of 1156 and a close upper bound for 1156. Since $30^2 < 1156 < 40^2$, then $30 < \sqrt{1156} < 40$.

STEP II:

Look at the last digit of the number that you are working with and try to find out several possible choices for the principal square root.

The number 1156 ends in six.

So, we determine the digits from 0 to 9 whose square ends in six.

$0^2 = 0$, $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$, $8^2 = 64$, $9^2 = 81$

This gives two possible choices for the last digit of the actual value of the principal square root of 1156.

We may guess the answer be 34 or 36, because they will end in 6. ($4^2 = 16$, $6^2 = 36$.)

Then we calculate the $34^2 = 1156$, $36^2 = 1296 > 1156$.

So, the square root of 1156 is $\pm 34$.

**Exercises:**

Find an approximate solution for the given quadratic equations by the trial and error method for a perfect square.

1) Solve x for $x^2 = 3844$.

2) Solve x for $x^2 = 7569$.

3) Solve x for $x^2 = 8281$.

4) Solve x for $x^2 = 2116$.

5) Solve x for $x^2 = 13225$.

**Answers:**

(1) 62 (2) 87 (3) 91 (4) 46 (5) 115

**Inquiry-based Activities:**

1. Ask students to reflect on the strengths and limitations of these methods.
2. Ask students to research other historical methods of finding square roots by approximation techniques.
3. Ask students to provide a comparative analysis of two historical methods of finding square roots using approximation techniques.

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# IPARK: Integrating Practice-based and Research-based Knowledge into Learning Resource Repositories

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Learning resource repositories are of growing importance in linking together the stages of knowledge production and improvement of practice in undergraduate STEM education. New educational materials are now frequently made available through repositories for reuse and adaptation by faculty, with networks of repositories of varying scopes appearing at the national, regional, state, and discipline levels. However, learning resources can only be effective when faculty have the motivation, time and expertise to incorporate them into effective learning designs that meet the needs of their students.

Recent studies of use and users of digital resource repositories have identified several clear needs including the following:

- *Reusing resources in new contexts*: "Faculty, including those active and enthusiastic in their use of digital resources, identified many obstacles to using these resources for teaching, including how to…reuse them in new contexts"[1]. For example, a recent NSF-funded project in Physics demonstrated the need to rethink instructional designs when resources from a research-intensive institution are reused in the differing context of a four year institution[2].
- *Time and expertise to adapt teaching practices*: for example, a study of Geoscience faculty noted that "While many faculty have a general knowledge of teaching methods, they are most interested in the application of these methods to the specific topics they teach, and they prefer to learn about teaching methods within such a context… This required a design…that would capitalize on faculty use of the web to find materials for class as a mechanism for bringing them into contact with materials that could be used later to support their redesign of a course[3]."

The concept of "learning object" repositories reflects the view that resources targeting a single learning activity will be easier to reuse in multiple contexts than resources bundled together into a course level package. However, successful reuse still requires that faculty design appropriate learning activities for student learning, including effective tasks, roles, feedback, etc.

Of course, a deeper understanding of student learning is a key element in adapting learning resources and teaching practices. While much of the pedagogical content knowledge incorporated in repositories reflects the practical know-how of exemplary teachers, there are few links to the growing body of research knowledge about learning and teaching. Our new approaches will integrate research-based knowledge with the exemplary practice of teachers.

Teaching strategies for the effective use of the resources are currently being shared and extended through evolving repository elements for pedagogical content knowledge. MERLOT has been a leader in integrating teaching expertise and exemplary resources in its repository[4], including the following formats for contributions of knowledge and experiences on teaching and learning:

- *Authors' Snapshots* to document teaching strategies from the resource originator[5];
- *Peer Reviews* for expert assessments of the effectiveness and roles of the resource[6];
- *Assignments* for contributions of teaching expertise from faculty[7];
- *Comments* for contributions of experiences from teachers and students.

Other repositories are also extending their formats for pedagogical knowledge, including representations for generic learning designs via *Activity Sheets* in the Teach the Earth portal[8], community expertise through *Expert Voices* in next generation prototypes of the National Science Digital Library[9], and student *Skills*, *Misconceptions* and *Assessments* in pilot studies for extensions of the Digital Library of Earth Science Education[10]. The impact of the pedagogical knowledge represented to date has been promising, e.g., the *Activity Sheets* facility is being reused across multiple repositories, including MERLOT, in a current NSF-funded project *Pedagogical Services for Digital Libraries*.

These existing initiatives do not directly address the need to tailor learning designs to suit specific learning contexts, nor do they provide a link to research results and community scholarship. As outlined in the previous section, these are key elements in developing faculty expertise for more effective learning designs. In the IPARK project, we plan to address these needs by developing, evaluating, refining and disseminating the following innovations for pedagogical content knowledge associated with exemplary learning resources in our repositories:

In our planned work in the IPARK project, we will develop several kinds of resources to accelerate the dissemination and implementation of important new faculty-developed teaching strategies and learning resources, including the following:

1. *Analytic Reviews*: an extension to existing Peer Reviews of learning resources, to provide a digest of research on learning issues associated with the resource and related teaching strategies [e.g., challenges students encounter with the topic, misconceptions that have been shown to impede their learning, etc.]
2. *Community Teaching Portfolios*: An evolving collection of faculty experience reports about how they adapted that resource for use in a variety of specific settings (different courses, different types of students, different faculty teaching strategies);
3. *Guides to Best Evidence*: A summary, written for faculty, of evidence relevant to the use of the learning resources and appropriate teaching strategies; the summary will be designed to help faculty decide how to adapt their teaching strategies to engage students with the subject matter content through the resource, and highlight particular experience reports from other teachers which demonstrate effective approaches for their context;

You can see prototypes of each innovation at the project wiki, http://castl.merlot.org. The IPARK project is an outgrowth of a collaboration amongst MERLOT institutions within the *Institutional Leadership Program* for the Carnegie Foundation's Academy for the Scholarship of Teaching and Learning[11]. The IPARK group is leading the program's *Online Teaching Commons* initiative, and the 96 campuses and systems participating in this program provide an additional audience for the results of the IPARK project.

The structure and processes for creating and refining these resources will support the engagement of *Communities of Inquiry* for ongoing development, management and dissemination of these resources. Our intent is to create a cycle of resource creation and upgrade that can be sustained within the normal reward systems of academic institutions, and the normal work processes and institutional support for MERLOT.

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**An Introduction to the Ethics of Mathematics Teaching-Research.**
B. Czarnocha

**Introduction**. (How People Learn?,1999) is the document produced by the collaboration of the National Research Council of U.S. with its National Academy of Sciences, which contains the full description of the research knowledge on the subject of learning at the end of the twentieth century. Its subdocument, How People Learn, Bridging Research and Practice considers the question: "what would be required for insights of research to be incorporated into classroom practice?"
Unusually long time of the integration of educational research into teaching practice is considered to be one of the essential obstacles in the progress of the reform in education. (How People Learn?, 1999) observes that, despite formidable obstacles, this slow process has been punctuated by moments of direct influence of research, which has taken place "…when teachers and researchers collaborate in design experiments, or when interested teachers incorporate ideas from research into their classroom practice. "
Whereas, we have certain number of reports of such collaborations between teachers and researchers in the literature (Kelly and Lesh,2000), we have very few acounts of single teachers doing clasroom research necessary for the effective process of introducing research results into classroom practice. Those few that remain offer some striking differences on the attitude to and principles of integrating research and teaching practice.

**The Improvement of Classroom Practice Principle**. One of the central differences between Teaching-Research practiced by the classroom teachers on one hand, and teacher educators leading collaborative teams of teacher and researchers is the attitude towards the improvement of teaching in one's own classroom. (Raymond and Lienenbach, 2000) state in the section Goals of Collaborative Action Research:"*A primary goal of the collaborative action research identified by many is to bridge the gap and strengthen the relationship between universities and schools. Collaborative research between university researcher and classroom teachers present opportunities for a more action-oriented approach to teacher enhancement. As teachers are encouraged to reflect upon and systematically examine aspects of their classrooms, they are likely to make changes based on observations that lead to the improvement in their classrooms*". In other words the classroom improvement of learning might possibly be a by-product of the researcher-teacher collaboration, whose main goal is to bridge the gap between university and schools. In order to bridge that gap so that both teachers and researcher receive appropriate satisfaction and fulfillment of their professional goals, we have to see very clearly what are the expectations and conditions of work of each professional community.

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For example, Jim Minstrell, the classroom teacher of mathematics states, on the other hand, that"*…the more immediate of the two is the improvement of the teaching practice. That is, when teachers engage in research on their teaching, they do so to get better at what they do. The second purpose is to seek an improved understanding of the educational situations in which they teach so that they could become the part of the knowledge base of teaching and learning* (Feldman and Minstrell, 2000). In other words the likelihood of classroom improvement is not enough for the classroom teacher; instead it is his or her primary goal with reflection and examination of the classrooms as the tools for that improvement.

**Differences in Interest**. These subtle differences in Teaching-Research approaches are, of course, natural; while educational reserchers' primary concern is the investigation of teaching and learning processes in their generality, the task of the teacher is to investigate the most effective methods of improving learning in his or her mathematics classroom, and beyond. Similar subtle differences between the classroom research approaches of a researcher and a teacher point to significant differences in the formulation of the research questions. (Cobb & Steffe, 1983), assert that the primary interest of a experimenter engaging in a teaching experiment lies in "*investigating what might go in children's heads*" and in "*hypothesizing what the child might learn*". (Czarnocha, 1999) responds that „ *In contrast to the interest of the experimenter, the teacher's interest here is to find means and ways to foster what students need to learn in order to reach a particular moment of discovery. Since however such moments can occur only within the students' autonomous cognitive mathematical structures, the teacher has to investigate these structures during a particular instructional sequence. In this capacity he or she acts as a researcher.*"

**Ethical Differences**. While abstractly these differences are simply changes of emphasis between the two components forming Teaching-Research, yet the same differences have strong ethical implications as to what kind of research should or should not be conducted in the classroom. (Pawlowski, 2003), one of the few teacher voices exploring the ethics of classroom research offers following suggestions on the matter: „*A professional investigator has as its responsibility to explore a research problem in its entirety, to grasp as many as possible of its aspects, continuously doubt and concientiously document its investigations. A teacher, responsible for children, pupils in his classroom, has as its primary responsibility to use the best available to him or her methods. All the doubts that the teacher has the right to, must be decided with the help of the criterion of its responsibility to children undertaken in the best of faith; his right to planned meandering*

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*or to conduct the control measurements is strongly restricted. The teacher has no right to conduct the „negative" experiments directed to show that some configuration of factors leads to to worse results. If the teacher had found by [experience] that some dydactic procedure is better than a standard one and knows about (because of course he or she has the right and responsibilityy to rely on one's own memory), but did not collect yet an adequate documentation to provide the evidence for its observations, nonetheless he doesn't have right to return to the method recognized as less succesfull – to return only in order to document its observations*."

He follows these remarks in a similar vein to Minstrell's, saying „*The main goal and the decisions criterion for the teacher must be, correctly diagnosed well-being of the student for whom the teacher is responsible. Only in the second instance, its goal can be the usefulness of the observation from a general, objective point of view. It is the objective investigative constraint, and not as one can think in a simplified manner, the limitation concerning only teacher-researchers, who are involved in other tasks. That's why didactic investigations are governed by a rigid system of values which must be respected during any such investigation independently of who is conducting them."*

We would like to pause and reflect upon the last statements of Pawlowski, because of the seriousness of their implications for the methodology of classroom research. For, if we accept Pawlowski's statement that the ethical considerations of classroom research constitute objective contraints upon it, which needs to be fulfilled not only by a teacher-researcher but by any investigator doing action research in the classroom, then we have a strong criterion with respect to which the design of any teaching-research collaboration needs to be judged. In particular, the length of the teaching-research cycle of the design, implementation, assessment and analysis, followed by the redesign has to be carefully measured relatively to the time a particular cohort of students is in a particular grade or a course, which in general have the length of a semester or a year.

**Ethics of the Teaching Experiment in the Classroom**.In a recent paper, (Czarnocha and Prabhu, 2006) offered the following principled considerations governing the structure of the classroom Teaching Experiment

*Ethical Principle*

*Those students who were the subjects of the Teaching Experiment leading to the increase of knowledge about their learning, should be the first beneficiares of the new understanding.*

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*The ethics of the classroom teaching experiment formulates strong conditions upon the length of the Teaching Experiment, which require classroom ingenuity of the Teacher-Researcher to satisfy. The Teaching Experiments need to be so situated within the regular cycles of work so that the students who were the subject of investigation are also its immediate beneficiaries. In simple cases, one can state as a guide a minimum of two cycles per unit of the classroom instruction, a semester or a year. Two cycles assure the refinement of instruction based on the qualitative or quantitative analysis, and hence its improvement through incorporation of results of investigation.However one can bypass this simple condition in special circumstances which still allow for the satisfaction of the ethical principle. For example, if a teacher teaches the same cohort of students the next unit cycle in the school then he/she has the opportunity to introduce the results of research after one unit and still fullfill the requirement. Or as a teacher, TR apprentice in one of the TR teams of the Socrates Project describes her way of dealing with the problem of the control group through the parallel class taught by the same teacher. The ethical problem teachers encounter here is that in reality it is impossible to have two parallel classes and to implement new instruction one believes in, in only one of them and not both. In other words she doesn't want the control class,whose students are used as object of comparative assessment, not to receive the benefit of the Teaching Experiment conducted in the parallel experimental class. The TR apprentice had divided the teaching experiment into parts within the year and having received the confirmation or rejection of her hypothesis in one class in a given part of the curriuculum, she immediately was introducing the improving technique to the other class, but only for that part. This way she was able to assess the effectiveness of the innovative instruction in its components, and at the same time to satisfy the ethical principle.*

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# Using Errors to Diagnose Math Misunderstandings

Anne Rothstein

This article takes as a given that students use an information processing model that can be discovered to solve mathematical problems. This model includes the following components:

*Input -- the raw material of the problem or example as presented*
*Sensation*
*Reception*
*Perception*
*Processing -- the internal actions that lead to solving the problem*
*Temporary Storage*
*Short Term Memory*
*Encoding*
*Retrieval*
*Solution*
*Output -- the observable work and answers provided by the student*
*Response Organization*
*Emit response*
*Feedback*

The trick of using student errors to diagnose misunderstandings is to require that students show all their work and/or talk through or write the steps they use to solve mathematical problems. It may be helpful to provide students with a math solutions template with which to organize their work. A sample template that students might use to organize their work is provided at the end. By going through the template with the questions you can identify where the student may have misunderstandings or problems and then target those areas.

## Math misunderstandings may be present at any stage of information processing.

The following questions are meant to help the teacher review the student's work and determine if the various elements needed to reach a correct answer are present in the shown work submitted by the student. It is not sufficient to look only at the answer but to go through the work and see that each step of the process is accurate. Correct answers, as you know, can be gotten using incorrect steps and intermediate answers.

☐Does the student read the problem correctly?
☐Can the student copy numbers and formulas accurately?
☐Can the student substitute numbers in the formula properly?
☐Does the student have spatial or visual difficulties?
☐Does the student demonstrate signs of dyslexia?
☐Is the student's work in proper sequence?
☐Does the student write legibly and clearly?
☐Can the student line up the number columns?
☐Does the student apply steps in proper order?

☐Does the student understand mathematical language?
☐Does the student's native language lead to incorrectly expressing order of operations?

☐Does the student know basic math facts, procedures and rules?
☐Does the student understand the order of operations?
☐Does the student recognize the type of problem?
☐Does the student choose the correct formula?
☐Does the student work precisely in working through the problem?
☐Can the student follow the steps or elements of the problem?
☐Can the student identify which aspects of a problem (as in a word problem) are important?

☐Can the student estimate the answer?
☐Does the student use the correct value label during intermediate steps?
☐Does the student use the correct value label in the answer?

□Can the student evaluate the produced answer with the reasonable answer?

□Can the student ignore irrelevant information?

□Can the student focus on the form of the final answer?

□Does the student understand the directions given for problem solution?

□Can the student express, orally or in writing, the directions?

□Can the student read and follow written directions?

□Can the student solve problems that require multiple steps?

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**Student Work Template**

Rewrite the problem you are going to solve:

Write the formula you will use to solve the problem:

Substitute the numbers into the formula:

Describe how you will solve the problem:

What is the approximate answer you expect to get once you work the problem?

Solve the problem showing the exact steps you to get the answer:

*How confident are you that the answer you got is correct? Why?*

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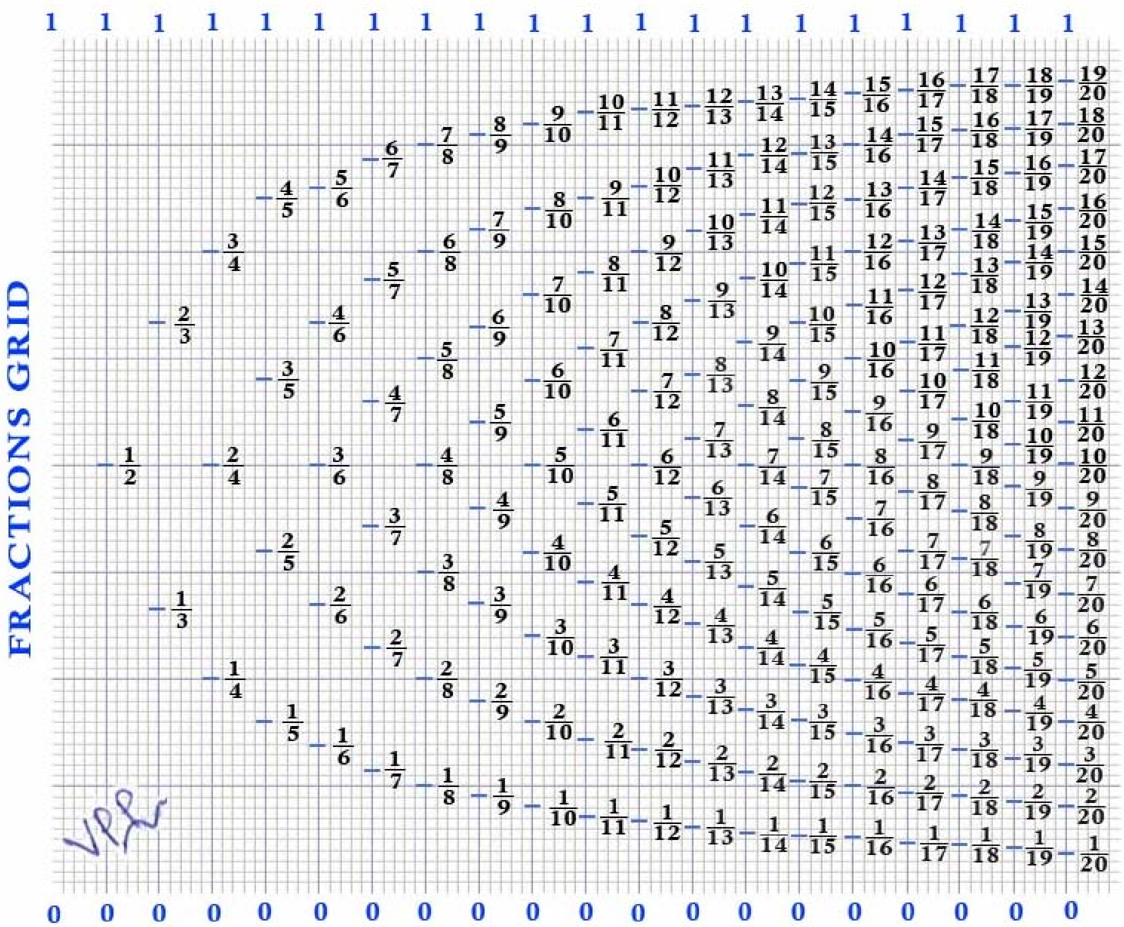
# FractionsGrid – A Schema-based approach to the teaching and learning of the concept of fractions

**Vrunda Prabhu, Bronx Community College, City University of New York**




The FractionsGrid shown above is a concrete, didactic outcome of the TR-NYC methodology of teaching-research utilized in the teaching of the concept of fractions between Summer 2005[1] to the present. The FractionsGrid is simultaneously a tool being

- used to teach the concept of fractions, and

[1] One of the motivating elements in the development of the FractionsGrid, is the double line of the Realistic Mathematics Education movement.

- investigated for efficacy in the teaching of the concept of fractions

through the teaching-research cycles. The on-going teaching-experiment at BCC is the first semester-long teaching-experiment. All previous experiments at several sites were significantly shorter in duration.

The teaching-experiment investigation is being conducted in

- a fractions-based arithmetic course
- a transition course from arithmetic to algebra

The teaching-research questions posed from the point of view of bringing about a transformation in the way students view fractions is

- How can students make the needed connections between their own understanding of fractions with the "scientific"/mathematical concept of fraction?
- How can students see the connections that exist between the constituent parts of the real number schema?

i.e., how can students see the relations within and between concepts of the real-number-line schema?

Both courses are being taught at the community college. Students at community colleges are generally adults who have had prior academic exposure to the concept of fractions. They are also generally members of the labor force and hence utilize the part-to-whole concept as part of daily life, bringing those analogies to the class-discussions. Their strong intuitive understanding of the concept is generally related to a personal stake, such as

- the fraction of time spent for work versus that for education
- the fraction of credit hours completed

In the fractions-based class, the concept of fraction is a central concept, and in the transition class (arithmetic-algebra), the fraction concept is a Just-in-Time intervention, and both classes use the FractionsGrid and coordinate the use of this tool with the real number line. Knowledge of the real number line is a obvious pre-requisite, however, it has been diagnosed as sufficiently weak to necessitate regular reinforcement. In the development of the pre-requisite knowledge, the concept of fraction naturally emerges as a number on the number line, also called a rational number. The importance of the definitionof a fraction has been stressed by many authors, including (Wu, 2005).

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The connection with the real number line and the tool called the FractionsGrid is continually made so that the representation itself need not create any further obstacles to learning. Hence, during in-class assignments students are urged to use any representation they are at ease with, and in some questions their attention is deliberately brought to focus on considerations of the real number line representation.

Use of the FractionsGrid as a teaching tool begins by observing of patterns, which leads to decoding patterns, i.e.,
given the pair of fractions $\frac{101}{103}$ and $\frac{205}{207}$,
students must determine which is the larger fraction, with justification. Variations of such problems continue for homework.

Observation of increasing and decreasing patterns then leads to observing those patterns which are neither increasing nor decreasing, or are constant. The notion of equivalent fractions is developed, and is a good illustration of how the spontaneous concept (equal sizes) is taken to the scientific end ($\frac{a}{b} = \frac{ac}{bc}$, for non-zero and in this case positive c), through the observation of patterns. The most common equivalent pattern "sequence" provided by students through classroom prompts is

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \ldots$$

The sizes of each number in the sequence above is compared. It is checked whether there is complete consensus that the sizes of each of these fractions is the same. Next, students are asked which is the simplest of the fractions in the above sequence and the answer is unequivocally $\frac{1}{2}$. Now, students are prompted for a new way of writing the fractions that appear in the sequence so that each fraction in its representation also shows the fraction $\frac{1}{2}$. It may take some questioning to arrive at another way of writing $\frac{2}{4}$ as $\frac{1.2}{2.2}$ and a little less questioning to arrive at the fraction $\frac{3}{6} = \frac{1.3}{2.3}$.

The rest follow. Here, prior knowledge of fractions is helpful in enquiring what the common number in both numerator and denominator means and how each of the fractions is equivalent to the first simplest fraction $\frac{1}{2}$.

The accompanying Powerpoint presentation provides a visual tour of some of the operations on fractions.

As the teaching-experiment progresses, it is students' questions that direct the teaching and the introduction of the needed connections between subconcepts to be elucidated at length. One such concept that emerged from student-directed questions and work is the notion of the unit length. When asked to represent a list of fractions on the same grid/number line, students use intermediate steps, in which they represent each of the fractions in the list individually, however, the size of the unit they use for each individual representation varies and hence when they combine their work to one number line, the order of the fractions is not maintained. This required that the discussion in class focus on the notion of unit length, and this discussion directly leads to new ways of counting that are possible when we talk about fractions. In the operations on the set of natural numbers, the notion of counting by ones is now extended to counting by halves, thirds, etc, however, the unit that is used for these new ways of counting differs in size. The unit forms an integral part of the discussion and problem-solving and is helpful in reduction of the difficulties pertaining to common denominators, equivalent fractions, ordering and adding and subtracting. In a large-scale study (Promoting Rigorous Outcomes in Mathematics and Science Education) at Michigan State University, the authors point out the difficulties with fractions encountered by students between 3rd and 12th grades in the states of Michigan and Ohio. The scores are the lowest in grade 3 on the following concepts:

- common denominators
- multiplying
- equivalent fractions
- adding and subtracting.

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**Agency, Identity and Achievement: Building Understandings of the Emergence of Scientific Literacy in Marginalized Student Populations**

Research Interests of Rowhea Elmesky

Science education, as it has been actualized for disadvantaged populations, contributes to the social reproduction of individuals' positions in society. Since success in the area of science serves as a central key for unlocking doors to higher education and to a vast range of career opportunities, there are severe consequences when culturally marginalized and economically disadvantaged students "fail" (according to standard measurements) to achieve highly in science. Marginalized youth living in poverty in large urban centers are among those most disadvantaged in schools and in society at large, and hence represent the focal population with whom my research interests lie.

I hold strong convictions that science education should not be about stratifying students or maintaining the current stratifications in place. Thus, I maintain a research focus that searches for understanding how science education can be a transformational force in the lives of all children. While my interests include studying the role that macro-level factors (e.g., societal racism, adequate science classroom facilities and resource access) play in shaping inequitable science classrooms, I predominantly conduct research that builds understandings of how teaching and learning practices (meso-level) and classroom interaction dynamics (micro-level) shape the emergence of science literacy among economically disadvantaged, culturally marginalized youth. I articulate a research agenda that places attention more centrally upon the students and on building understandings of how their embodied resources are accessed and appropriated, both consciously and unconsciously, in ways that may mediate their modes of participation in science class and lead to their empowerment or disempowerment as science learners as well as to their identification with science as a discipline. Specifically, I am committed to: 1) developing new ways of defining and recognizing student achievement in science classrooms; 2) studying student practices as resources that can promote and/or truncate their power to act (agency) as science learners; and 3) formulating understandings of how and why students' identities may shift to include or exclude perceptions of being successful science learners.

In conducting science education research, I utilize sociocultural theoretical lenses to a) illuminate classroom practices as forms of enacted culture, b) acknowledge the dialectical relationship between classroom structures and student agency, and c) view identity as a mediated outcome of classroom participation. My focus upon students' embodied practices has arisen from understanding that much of what occurs in the classroom is unconscious and originates from multiple realms both outside and inside of school. I study embodied resources as ways of being that develop in multiple places and may include specific knowledge, values, skills, morals, aspirations, rituals, beliefs, goals or interests as well as manners of interacting, communicating or moving. Identifying how students access and activate these resources toward their own goals in science learning contexts becomes central in considering how youth can re-position themselves with power, authority and respect in the science classroom. That is, by studying how they access their own resources in learning science, I can learn how students may experience shifts in their identification with science as a discipline.

Research concerned with expanding the existing conceptualizations of science and science achievement, recognizing student agency, and enhancing students' identification with science calls for particularly unique research designs. That is, since marginalized children living in economically disadvantaged circumstances represent a silenced group in schools (and in society), I am interested in considering how the research process (and not just the research findings) can be transformative. To this end, I focus upon utilizing critical ethnography since it is concerned with the empowerment of those involved and monitor the authenticity of the study, or the extent to which the research is educative, empowering, fair and catalytic, by introducing unique research group dynamics and nontraditional approaches to data and artifact collection and analyses. Thus, the roles of the research participants can expand as data are collected, produced and analyzed in a joint effort. In addition, I value holding research meetings with the teacher(s) and student researchers in the form of cogenerative dialogues or collective conversations in which all members accept responsibility for understanding classroom activity, with a commitment to blur power differentials amongst the researchers, to provide a space for building common theoretical language, and for developing analytical skills such as video microanalysis.

In conclusion, for decades upon decades, scientific literacy and science education have been situated within a culture of science that values the individual over the collective, the abstract over the contextualized, and the objective over the emotional. Unless students learn to communicate scientific literacy in ways that are recognized and acknowledged in the dominant society, science as a field remains inaccessible by the majority of marginalized populations. In my research endeavors and associated publications, I attempt to augment the limited literature that implores the nation to expand our definitions of scientific literacy, by recognizing and acknowledging students' ways of being as useful resources for mediating their participation in science classrooms and forming a positive association with science. Hence I remain committed to studying how scientific literacy and identity emerge through multiple forms of participation and are socially mediated by agentic access and appropriation of resources in hopes of making important strides toward improving the quality of science education experiences for marginalized children.

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# Research Interests of Serigne Gningue
## Lehman College

Dr. Serigne Gningue's scholarship and teaching can best be summarized by the search for equity in mathematics learning, through the teaching of mathematics content and methods, and through professional development beginning with preservice teacher preparation and extending to the development of experienced teachers. He began his career more than 18 years ago, teaching mathematics to middle school students. As his career in teaching developed, he participated in the design of programs meant to address the needs of gifted students and to create avenues for average students in urban settings to further develop their talents. The success of the implementation of the program at his school prompted his school district to have him design the curriculum for an Advanced Mathematics/Science Summer Institute for three years (1996-1998). His role as the curriculum planner and staff developer (training, working, and sharing with teachers of the Institute) set the foundations of his belief that the work he now does as a teacher educator has far reaching implications on the performance and success of school children, even more so than most people believe. Such experiences also made him realize the importance of promoting the use of methods that develop students' critical thinking ability through problem solving, in particular, the development of more effective methods of teaching algebraic concepts to students well before the onset of the high school years.

In the article he co-authored with Lisa Evered (Developing Mathematical Thinking Using Codes and Ciphers, 2001), they illustrated an example of how and why teachers must engage their students in non-routine problem solving activities that enrich their mathematical experiences to persuade them that mathematics can be both fun and serious at the same time. In his dissertation (The Use of Manipulatives in Middle School Algebra: An Application of Dienes' Variability Principles), and then through a series of articles accepted for publications in the New York State Mathematics Teacher Journal and the International Journal "For the Learning of Mathematics," he described how Dienes' and Bruner's theories of mathematics teaching and learning can be applied to introduce algebra to middle school students.

His search for equity in mathematics learning prompted an investigation of a model about ways of developing avenues for mathematical talent in the New York City public schools and in urban settings in general. The grant-supported project focused on one particular middle school whose principal believed in giving opportunities to all of his students, most of whom were minorities or coming from low income families, to learn advanced mathematics. The high school mathematics career of selected students from the middle school who graduated between 1995 and 1999 are analyzed and described. Results obtained strengthen his belief to promote the idea that algebra can and should be taught to all students as a course in $8^{th}$ grade, and his commitment to support children in urban settings who are historically underrepresented in mathematics and science.

In his current work as Lehman College's Coordinator of the graduate program in Mathematics Education, he has been coordinating the recruitment of mathematics teachers through the Teaching Opportunity Program (TOP), a CUNY scholarship program that recruits and trains change of career people to teach mathematics and science in New York City. His work also focuses on developing coursework, ensuring professional development of teachers through the New York City Mathematics Project (NYCMP), through the Professional Development School (PDS), and mentoring and supervising the large number of new teachers Lehman College has been getting each year through the TOPS and FELLOWS programs.

These experiences have enhanced his teaching and his ability to design effective learning experiences for Lehman's students, and have led to publications and presentations on the creation of PDSs, and on the effectiveness of training teachers to integrate technology in the teaching of mathematics concepts.

For Dr. Gningue, technology and the pedagogical changes resulting from its introduction to the curriculum have a decisive impact on our understanding of what should be included in the mathematics curriculum. For him, mathematics educators have the arduous task of keeping up with the advances and incorporating them into activities and lessons, and of influencing decision makers to invest in computing technology and in the training of teachers, especially in urban settings. That's why Dr. Gningue developed a course that looks at the issue in technology education and that uses technology as a means to develop concepts or to enhance the mathematics we teach.

Dr. Gningue's experience working with middle school students, and the large number of studies that have shown an inequitable distribution of course taking among high school students that favor racial/ethnic groups other than Blacks/Hispanics have been the driving force behind his current work. He wants to contribute to the elimination of the achievement gap in high school between majority and minority students. He is developing and testing a manipulatives-based algebra curriculum for use in urban sixth and seventh grades to enable the introduction of algebra as a course in $8^{th}$ grade. Blacks and Hispanics, mostly schooled in large urban public school districts which are more difficult to run and manage, often feel the consequences of their districts' lack of qualified teachers and inadequate resources and articulation for offering an algebra course to all its students before ninth grade. In contrast, the more affluent suburban districts usually offer an algebra course in eighth grade to all its students. According to Dorsey et al. (1994), the key to narrowing the differences of achievement between majority and minority students resides at the middle school level. The earlier students in middle school take their first-year algebra, the higher their overall mathematics proficiency by the time they reach Grade 12. Results from his grant-funded study, which analyzed the high school mathematical career of selected middle school students who were offered an algebra course in eighth grade, confirm such findings. The results have also led to a second funded study whose goal is to investigate the effectiveness of a program he designed which offers first-year algebra to average eighth grade students. The project (1) offers a manipulatives- and problem solving-based algebra after-school program for average and below-average Grade 7 students, (2) provides staff development in both content and pedagogy in the teaching of algebra in Grade 7 to middle school mathematics teachers, (3) places participating students into accelerated Regents-level eighth grade mathematics classes the following year, and (4) tracks participating students' mathematics career in high school. He expects results to perhaps enable him to determine whether providing average seventh grade students with a carefully designed environment for developmental trajectories of learning from the concrete, visual instances of algebraic concepts to their abstract and formal expressions (through an application of Dienes' and Bruner's learning theories) will develop their "intuitive" assumptions about the unfamiliar notation system into algebraic proficiency. The culmination of this project should result in a publication of algebra textbooks for use in the middle grades.

In his pursuit of equity in mathematics learning in the city of New York, Dr. Gningue has also participated in the New York City Department of Education Mathematics Advisory Panel chaired by Dr. Uri Treisman, Professor of Mathematics at the University of Texas and Director of the Charles A. Dana Center who was appointed by Chancellor Klein. The goal of the Panel was to provide advice and to support the City's ongoing mathematics initiative. He continues to be committed to preparing urban teachers who teach in ways that reflect a commitment to the well-being and learning of all students, by engaging teachers in activities that help them understand how school children differ in their approaches to learning, how to create instructional opportunities that are adapted to diverse learners, how to use a variety of instructional strategies that encourage students' development of critical thinking, problem solving, and performance skills.