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FractionsGrid – A Schema-based approach to the teaching and learning of the concept of fractions

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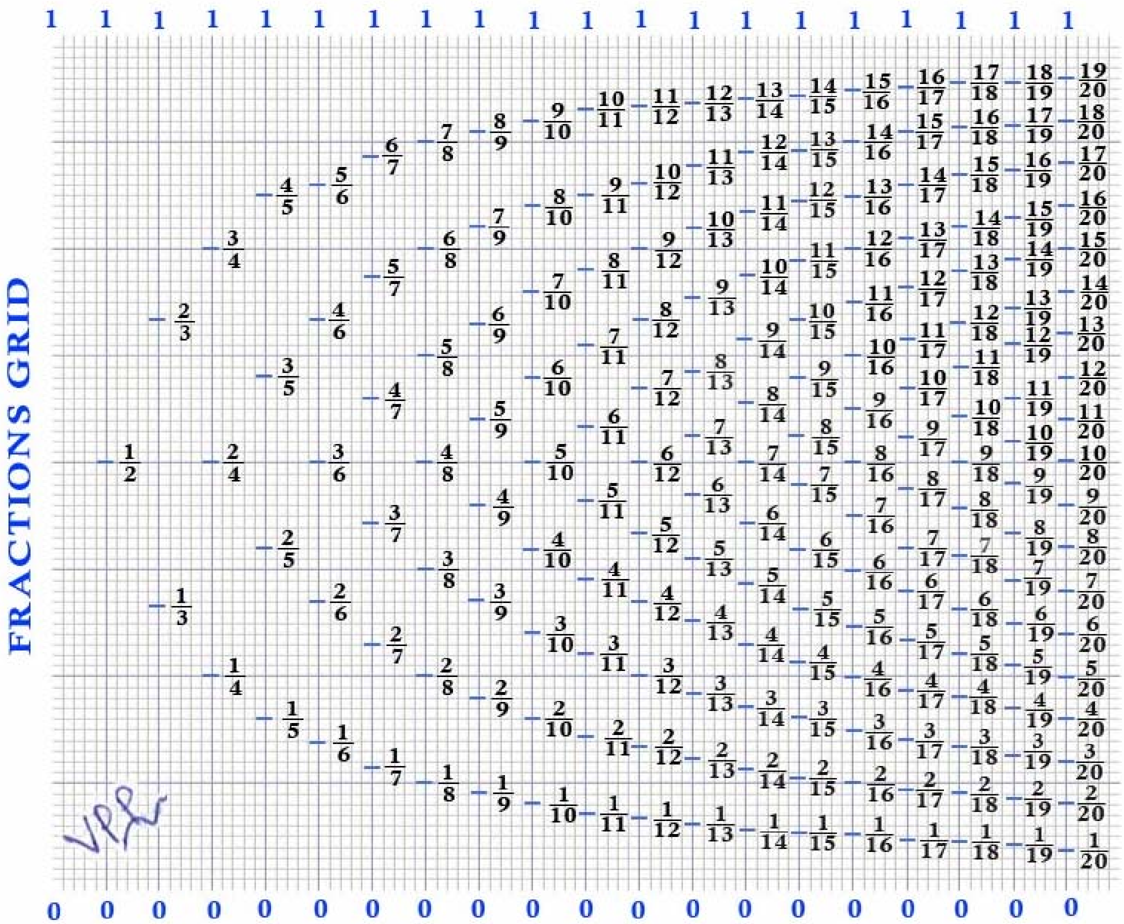

The FractionsGrid shown above is a concrete, didactic outcome of the TR-NYC methodology of teaching-research utilized in the teaching of the concept of fractions between Summer 2005[1] to the present. The FractionsGrid is simultaneously a tool being

- used to teach the concept of fractions, and

[1] One of the motivating elements in the development of the FractionsGrid, is the double line of the Realistic Mathematics Education movement.

- investigated for efficacy in the teaching of the concept of fractions

through the teaching-research cycles. The on-going teaching-experiment at BCC is the first semester-long teaching-experiment. All previous experiments at several sites were significantly shorter in duration.

The teaching-experiment investigation is being conducted in

- a fractions-based arithmetic course
- a transition course from arithmetic to algebra

The teaching-research questions posed from the point of view of bringing about a transformation in the way students view fractions is

- How can students make the needed connections between their own understanding of fractions with the "scientific"/mathematical concept of fraction?
- How can students see the connections that exist between the constituent parts of the real number schema?

i.e., how can students see the relations within and between concepts of the real-number-line schema?

Both courses are being taught at the community college. Students at community colleges are generally adults who have had prior academic exposure to the concept of fractions. They are also generally members of the labor force and hence utilize the part-to-whole concept as part of daily life, bringing those analogies to the class-discussions. Their strong intuitive understanding of the concept is generally related to a personal stake, such as

- the fraction of time spent for work versus that for education
- the fraction of credit hours completed

In the fractions-based class, the concept of fraction is a central concept, and in the transition class (arithmetic-algebra), the fraction concept is a Just-in-Time intervention, and both classes use the FractionsGrid and coordinate the use of this tool with the real number line. Knowledge of the real number line is a obvious pre-requisite, however, it has been diagnosed as sufficiently weak to necessitate regular reinforcement. In the development of the pre-requisite knowledge, the concept of fraction naturally emerges as a number on the number line, also called a rational number. The importance of the definitionof a fraction has been stressed by many authors, including (Wu, 2005).

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The connection with the real number line and the tool called the FractionsGrid is continually made so that the representation itself need not create any further obstacles to learning. Hence, during in-class assignments students are urged to use any representation they are at ease with, and in some questions their attention is deliberately brought to focus on considerations of the real number line representation.

Use of the FractionsGrid as a teaching tool begins by observing of patterns, which leads to decoding patterns, i.e.,
given the pair of fractions $\frac{101}{103}$ and $\frac{205}{207}$,
students must determine which is the larger fraction, with justification. Variations of such problems continue for homework.

Observation of increasing and decreasing patterns then leads to observing those patterns which are neither increasing nor decreasing, or are constant. The notion of equivalent fractions is developed, and is a good illustration of how the spontaneous concept (equal sizes) is taken to the scientific end ($\frac{a}{b} = \frac{ac}{bc}$, for non-zero and in this case positive c), through the observation of patterns. The most common equivalent pattern "sequence" provided by students through classroom prompts is

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \ldots$$

The sizes of each number in the sequence above is compared. It is checked whether there is complete consensus that the sizes of each of these fractions is the same. Next, students are asked which is the simplest of the fractions in the above sequence and the answer is unequivocally $\frac{1}{2}$. Now, students are prompted for a new way of writing the fractions that appear in the sequence so that each fraction in its representation also shows the fraction $\frac{1}{2}$. It may take some questioning to arrive at another way of writing $\frac{2}{4}$ as $\frac{1.2}{2.2}$ and a little less questioning to arrive at the fraction $\frac{3}{6} = \frac{1.3}{2.3}$.

The rest follow. Here, prior knowledge of fractions is helpful in enquiring what the common number in both numerator and denominator means and how each of the fractions is equivalent to the first simplest fraction $\frac{1}{2}$.

The accompanying Powerpoint presentation provides a visual tour of some of the operations on fractions.

As the teaching-experiment progresses, it is students' questions that direct the teaching and the introduction of the needed connections between subconcepts to be elucidated at length. One such concept that emerged from student-directed questions and work is the notion of the unit length. When asked to represent a list of fractions on the same grid/number line, students use intermediate steps, in which they represent each of the fractions in the list individually, however, the size of the unit they use for each individual representation varies and hence when they combine their work to one number line, the order of the fractions is not maintained. This required that the discussion in class focus on the notion of unit length, and this discussion directly leads to new ways of counting that are possible when we talk about fractions. In the operations on the set of natural numbers, the notion of counting by ones is now extended to counting by halves, thirds, etc, however, the unit that is used for these new ways of counting differs in size. The unit forms an integral part of the discussion and problem-solving and is helpful in reduction of the difficulties pertaining to common denominators, equivalent fractions, ordering and adding and subtracting. In a large-scale study (Promoting Rigorous Outcomes in Mathematics and Science Education) at Michigan State University, the authors point out the difficulties with fractions encountered by students between 3^{rd} and 12^{th} grades in the states of Michigan and Ohio. The scores are the lowest in grade 3 on the following concepts:

- common denominators
- multiplying
- equivalent fractions
- adding and subtracting.

References:

PROM/SE, 2006, *Research Report Vol. 1, 2006*, Michigan State University. http://www.promse.msu.edu/

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