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Editors: Bronislaw Czarnocha (Hostos Community College)
Vrunda Prabhu (Bronx Community College)
Anne Rothstein (Lehman College)
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Diagnostic procedure in algebra course of the first grade of the Middle School

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The students had not have as yet the algebra course in the Middle School; they had a bit, but not too much, of it in the elementary grades. Nonetheless, all of them, when asked for the area of a rectangle, recite: P = a*b (P because in Polish area starts with the letter P, from Pole).
Equally easly they write formulas of the triangle, paralllelogram or trapezoid. That freedom of „letters" is, however, illusory.
I proposed 3,4 problems for them. The time of work = 30 minutes.
Problem 1. The area of the square is 9 cm^2. What is the length of the square?
Problem 2. Solve the equation $x^2 = 16$
Problem 3. Given rectangle has dimensions: length = a (cm), width = 2 (cm)

a) how much will the area increase, if the length increases by 3 (cm)?
b) how much will area increase, if the width increases by b (cm)?

The results confirm our misgivings.
Prob. 1 is solved by all students; they provide adequate drawings:

$P = a^2 = 9\ (cm^2)$ $P = a \cdot a = 9$

or

$a = \sqrt{9} = 3\ (cm)$ $3 \cdot 3 = 9$, $a = 3$

Even M triumphs: I did it.
The only problem was the absence of units.

Prob. 2. The difficulties start. Everyone find x = 4 because 4*4=16, or x = $\sqrt{4}$
Sometime there is a drawing of the square followed by the statement x = 4 cm.
We see that geometrical interpretation is omnipresent here favouring positive definite solutions. No one found the second solution x = -4 !

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Prob. 3 For the majority of the class the problem turns out too difficult. Angela is asking: how can I calculate how much the area of the rectangle had increased, if I don't know what it was in the beginning. The numrical value of the width is unknown…

Given width of the rectangle a associates with the unknown, hence I am providing a following intervention-hint: let's treat width **a** as a known quantity although arbitrarily chosen.
The intervention was quite succesfull: there appears one correct solution, and several interesting ones.
S1: : $a \cdot 2 = P$, $(a + 3) \cdot 2 = P + 6$ (this added length x 2) – explains
a)
Example: $5 \cdot 2 = 10$
$) + 6$
$(5 + 3) \cdot 2 = 16$

Answer:. The area will increase by 6 (cm^2)

b) $2 \cdot a = P$, $(2 + b) \cdot a = P + b \cdot a$

Answer:. The area will increase by $b \cdot a$ (cm^2)

Distributivity of multiplication with respect of the addition gives some trouble. There is an additional drawing in S1 case which we ommit here.
S2 : a) area = $2 \cdot a = 2a$, $2 \cdot (a + 3) = 6a$
Answer: area will incraese 3 times.

b) $(a \cdot 2) + b = 2a + b$
Odp. Area will incraese by **b** .

S3: : a) length = a (cm) , length = a + 3
$P = a \cdot b$, $P = a \cdot 2$

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$P = (a \cdot 2) + 3 =$ I don't know.

I am hurrying at the end of the class to discuss these problems. Why no one did the drawing for the problem #3? The letter a hiding the length of rectanle was a aproblem for 90% of the students. Absent this day S4 does it easily later but he is much above the level…

Conclusions:

1. Letters function for my students as part o formulas which indicate what operations to make to find the numerical value of the seeked quality such as area.
2. Problems are caused by a letter-parameter. Of course, here the technique of algebraic trasnformations magnifies this problem.
3. What's left to investigate is understanding of the letter as the unknown in the equation. One needs to quess it or solve the equation.

The class is writing a test in geometry a week later to which I am adding „my" two problems:

Problem 4. The region whose shape is a triangle has an area 2.5 ar. The basis of the triangle is 20 cm. What is the height of the triangle ?

Problem 5. The side of a triangle is a (m). The height dropped to this side is 10 (m). How much will the area of this triangle incraese of we incraese the side by 8 (m)?

Results. 6 correct solutions

Prob. 4. : S2 $P = a \cdot h / 2$, $a = 20$ m , $h = ?$
$20m \cdot h / 2 = 2{,}5a$, $2{,}5a = 250m^2$, $20m \cdot h / 2 = 250m^2$
$h/2 = 12{,}5$, $h = 25$

4 incomplete solutions (wrong formulas for the area)
S5 : x = height, $x \cdot 20 = 250$, $x = 12{,}5$

Prob. 5 :

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S4 : The area will increase by $8m \cdot 10m / 2 = 40m^2$ (absence of a drawing)

S1 : $P = 10 \cdot a / 2$, $P = (a + 8) \cdot 10 / 2$, $8 \cdot 10 / 2 = 40$ (absence of a drawing)
Area will increase by 40.

S3 : $P = a \cdot 10 / 2$, $P = (a + 8) \cdot 10 / 2$
Answer.: The area will increase by $(a + 8) \cdot 10 / 2$

Conclusions:
Nothing changed after a week. To say the truth, prob. 5 is a bit more difficult than the Prob.3. No drawing. But Prob.4, where the letter appears as the unknown to find comes out much better.

I gave the problems 4 and 5 to another, parallel class, just out of curiosity. I asked for the drawing to be made in Prob.5. This time there 3 correct solutions and one one close to full solution. Same doubts about the „given length of value a…" – a contradiction, or…

Zadania 4 i 5 dałem też klasie Ib (równoległej do klasy badawczej) wiedziony ciekawością jak tu będzie ! ? Do zad. 5 dołączyłem polecenie : zrób rysunek do zadania. Tym razem są 3 rozwiązania prawidlowe oraz 1 rozw. bliskie prawidłowemu (błędny wzór) sztandarowego zadania 5 ! Te same watpliwości i rozterki odnośnie „danej długości wynoszącej a metrów" - sprzeczność, chyba ...

S6 writes : $h = 10m$, $a = ?\ m$! She should have written $a = a\ (m)$ ...
So easy it is to get into collision between letters. But the drawing saves S6 :
$P = 10 \cdot 8 / 2 = 40(m^2)$

Drawings are very important; we have to remind about them and require as an element of the solution. The drawing helped to avoid the issue of letters and algebraic tarnsformations. The problem is still in front of us, nonetheless, and subsequent exercises are supposed to eliminate it or diminish.