The Flow of Thought Across the Zone of Proximal Development between Elementary Algebra and Intermediate English as a Second Language.[i]

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Abstract

A teaching experiment in correlating the instruction of courses in Elementary Algebra and Intermediate ESL is described whose results suggest a measurable transfer of thought organization from algebraic thinking into written natural English. It is shown that a proper context to situate this new effect is the Zone of Proximal Development of L. Vygotsky.

Introduction

The relationship between Mathematics and Language teaching has been the topic of many papers and presentations [1], [2], [4],[9]. Yet the literature and research on the subject suffer from several shortcomings. First, the majority of the research deals with the role of language in learning mathematics, leaving the reciprocal relationship, that of the influence of learning mathematics on the development of language, almost totally unexplored. Second, although several benefits of writing as an instructional tool in teaching mathematics have been proposed - such as a better understanding of conceptual relationships [2] or the facilitation of "personal ownership" of knowledge [4], [10], there has been, till recently, little evidence to explicitly demonstrate these benefits [11]. Finally, there is a relative absence of theoretical considerations that could provide a context in which to properly situate the reciprocal relationship between the development of mathematical understanding and the mastery of language.

ESL literature presents us with more or less the same situation and focuses on the role of the mathematics teacher as " a teacher of the language needed to learn mathematical concepts and skills." [5]. The methodology of classroom practice based on this principle was formulated in [6]. An important theoretical distinction in the area of second language acquisition has been introduced by Cummins ([5]), who asserted that the process of language acquisition has at least 2 distinct levels: the Basic Interpersonal Language Competency (BILC) level of everyday use, and the Cognitive Academic Language Proficiency (CALP) level.

This presentation addresses the shortcomings listed above. A brief discussion of certain ideas of Vygotsky in [13] outlines a context in which the relationship between mathematics and language can be situated. This is followed by a new and interesting

result obtained during a teaching experiment at CUNY's Hostos Community College, in which an Elementary Algebra course was pedagogically linked with a course in English as a Second Language (ESL), most probably demonstrating an influence of mathematical thinking on the development of descriptive writing.

Theoretical Background

The literature contains sporadic hints about the relationship between mathematical understanding and the acquisition of language. To the degree that writing is like problem solving, as Kenyon [7] claims, one might think about this relationship as determined by a common set of problem-solving strategies. This point of view, which doesn't take into account the peculiarities of each of the disciplines, is strongly supported by Anderson's Adaptive Control of Thought theory [1].

A point of view that gives justice to the richness of relationships between thought and language might perhaps be found in (early) Vygotsky ([13]). There, thought and language are seen as being in a "reciprocal relationship of development." [8]. "Communication presupposes generalization…, and generalization… becomes possible in the course of communication" (p.7, [13]) – in other words, in order to communicate we need to think; and in order to think, we need to communicate. Such a view opens, in a very natural way, the possibility that thought - in our case, mathematical thought - might be able to shape natural language. One of the ways through which this process can take place is across the Zone of Proximal Development (ZPD). (Ch.6, [13]).

The ZPD arises in Vygotsky's thought through his distinction between spontaneous and scientific concepts. It represents the depth to which an individual student can develop, with expert help, his spontaneous concepts concerning a particular problem - as opposed to his ability to do it alone.

Valsiner had noted that the development of the ZDP can also be furthered if the environment is structured in a way that leads the student to use elements which are new to him, but reachable from his ZDP [14]. One of the essential characteristics of the upper level of ZDP, as compared to the level of the corresponding spontaneous concepts, is its higher degree of structural systematicity. In our experiment, the abstract character of elementary algebra has created exactly that type of ZPD with respect to the "spontaneous" level of natural English.

Experimental Realization

To confirm Vygotsky's highly dialectical view one would clearly need to detect the presence of two directions of developmental progression: the acquisition of English under the influence of mathematical thinking and the acquisition of mathematical understanding under the influence of the use of English. While the first direction is the main topic of this presentation, let us mention that the existence of the second has been confirmed, for the first time, in a recent experiment by Wahlberg [13]. Measuring the increase in the students' understanding induced by essay-writing; she observed a substantial (80%) increase in the experimental group as compared to the control group.

Elementary Algebra/Intermediate ESL teaching experiment[1]

The general goal of the ESL sequence at Hostos is to develop what Cummins ([5]) calls the Cognitive Academic Language Proficiency, and what Vygostky calls the language of "scientific concepts". Our experiment had two goals: to see how far Algebra can help in that process, and to investigate the cognitive relations in the acquisition of both.

Methodology

A group of seventeen students was enrolled in a class of Intermediate ESL and in a remedial class of Elementary Algebra taught in English. In the previous semester these students passed 2nd low level ESL as well as Basic Arithmetic, the first remedial mathematics course. The Algebra class was the only class they were taking in English, and thus constituted their only exposure to academic English and possibly to English in general. Although the classes were separate, the communication between the instructors was very tight, involving weekly meetings, exchanging materials, and visiting each other's classes. The methodology of the experiment was based on two assumptions. First, since we were interested in the influence of the algebraic language upon the natural one, we needed to verbalize the algebraic language to a maximum level possible. That meant we needed to make the symbolic notation of algebra explicit in speech and/or writing - to verbalize the procedural steps and the content of algebraic thinking. Second, these elements, having been made explicit in their algebraic context, needed to be transferred into the context of the ESL class, both on the semantic and the grammatical level.

As a result, student discussions in the Algebra class often involved a level of academic discourse somewhat above the students' capacity at the given time.

This effort the students had to make to communicate the comparatively abstract mathematical ideas in English is, we think, is at the root of their very particular linguistic improvement. At the same time, the ESL class deliberately involved discussions of the linguistic peculiarities of algebraic language, such as the role of word order and sentence structure. Below are examples of specific instructional strategies in both classes.

New algebra instructional strategies.

1. Verbalization of algebraic procedures.

Example: Solving linear equations: $2X + 5 = 9$

Solution Steps (to be written by students)

[1] -The description of the experiment and of its results is based on The Final Report of the ESL/Elementary Algebra Teaching Experiment supported by the New Visions Program Grant of CUNY, July 1998 – M. Pujol, B.Czarnocha

2X + 5 - 5 = 9 - 5 *First, I add -5 to both sides of the equations in order to eliminate +5 on the left side.*

2X = 4 *Second, I cancel the opposite numbers and add the like terms*

2X/2 = 4/2 *Third, I divide both sides by 2 in order to have X alone*

X = 2 *The answer is X = 2.*

2. Explication of algebraic symbolism through writing paragraphs

*a) Write a paragraph explaining the difference between 3*5 and 5*3.*

*What does it mean to you that 5*3 = 3*5?*

b) What is the difference in the meaning of the equality sign in

*3*16 = 48 and in X + 5 =12?*

3. Analysis of algebraic rules and principles.

a) Compare the rule for the addition of signed numbers with different signs with the rule for the multiplication of signed numbers with different signs.

c) Write an explanation to Jose who missed a couple of classes, about how to do the problem below. Explain to him the order of steps in the procedure, warn him against the possible errors he might make, remind him of the rules which justify your the steps of the solution. -[2(3X – 5Y) –3(Y – X)] – 4(2X + 3Y)=

New ESL instructional strategies.

The goal of the-math related ESL exercises was to extend the meaning and application of algebraic words and concepts into natural English.

a) Expressing the same Math concept in different ways in English

Example: **Solving Linear Equations**

Instruction: Fill in the blanks with the appropriate word.

x + 16 = 20

• *Sixteen .....**added**.....to an **...unknown ...** number .....**is**.. twenty.*

• *If an **...unknown ...**number is ...**increased**.. by sixteen, the ...**result**.... is twenty.*

• *The ...**sum**.. of an **.unknown** . number ..**and**.. sixteen ...**equals**.. twenty.*

• *Sixteen ...**more**... than an **...unknown ...** number is ...**equal**.. to twenty.*

These exercises allowed students to understand how a specific Math idea or concept could be phrased in many different ways in English. This provoked opportunities for thinking and internalizing the basic English sentence structure.

b) Word order exercises.

Instruction*: **Put the following words and expressions into the right order:***

1- temperature - by +10 - the - by - the - decreases - evening - degrees

2- multiply - he - the number - needs to- in the parentheses- by- the numbers -

3- much - how - you- your sister - to - money- owe- do?

4- makes - of - John - same- as - do - money - every month- the - I -amount

5- If - perform- the - you - you'll- the number 48 - multiplication- get-

Serious thinking and discussion about what was said and how it was phrased accompanied these exercises. Students learned that the way concepts and numbers are put together in the algebraic language is essential for understanding algebraic operations. By paying attention to the word order in algebra, students became sensitive to word order in English.

c) Editing exercises.

Instruction: Please correct the following paragraphs, written by different students, not only for correct ideas but for mistakes involving any of the grammar rules studied so far.

$(x^a)^b$ *-- I think that raise a power to another, I need to put the variable and then multiply the exponent.*

$(xy)^a$ *-- When you have two variable in the parenthesis and you raise to any power. You have to multiplied each variable with same power.*

--I need to get each variable to the power separately. I need to multiply the variable with the raise power.

--It is when you have two variable raising to a power. I solve this raising separately each variable to the power. For example, I do this because I multiply the variable by the exponent.

d) A long-term (6-week) essay, written on a word processor, with 3 drafts discussed with the instructor. The topic was In Between Two Cultures; the students were supposed to compare and contrast their life experience in the Dominican Republic and in New York City.

Data collection and analysis.

As has been stated above, the teaching experiment had two goals: to use algebra to help in the development of natural English, and to investigate the possibility of a cognitive relationship between the acquisition of both. Vygotsky suggests such a possibility when he asserts (p.160,[13]): "one might say that the knowledge of the foreign language stands to that of the native one in the same way as knowledge of algebra stands to knowledge of arithmetic… There are serious grounds for believing that similar relations do exist between spontaneous and academic concepts".

For the purpose of the present discussion, the main tool of analysis were the long-term essays on the topic In Between Two Cultures which the students wrote in the course of the semester. The process of writing was important because "Written speech assumes much slower, repeated mediating analysis and synthesis, which makes it not only possible

to develop the required thought, but even to revert to its earlier stages, thus transforming the sequential chain of connections in a simultaneous, self-reviewing structure. Written speech thus represents a new and powerful instrument of thought"[8].

To assess the changes in the students' written mastery of English, we first used the holistic assessment of the ESL instructor - a standard way of judging student essays in English courses. Next, we translated this judgment into syntactic components. Finally, we compared these with the corresponding components in the essays of a control group. We chose as our control group a past class of the same ESL instructor, which wrote an essay on the topic Our Family Conflicts. This topic was judged to be the closest in meaning to the topic In Between the Two Cultures, assigned to the experimental group.

The judgment of the ESL instructor after reading all the essays of the experimental group was that they were more cohesive. As cohesiveness is closely related to the use of connectors - words such as "because", "yet", "although", etc.- all the connectors used by all students in their essay were categorized, counted, averaged per 22-line page, and the results compared with the corresponding numbers from the control group. _Our conclusion was that there was an average 15% increase in the number of connectors and subordinating clauses in the essays of the experimental group. This confirmed the ESL instructor's assessment that the long-term essays of the experimental group were more cohesive than the essays of the students who did not participate in the instructional link under discussion.

Comparison of the use of connectors in student writing:

	Experiment	**Control**	**% increase**
Time	228/**5.16**	217/**4.8**	**8%**
subordination (when)	114/**2.58**	106/**2.35**	**10%**
others	38/**.86**	53/**1.17**	**-36%**
connectors	66/**1.49**	56/**1.24**	**20%**
Cause	181/**4.09**	152/**3.37**	**21%**
subordination (because)	140/**3.17**	118/**2.62**	**21%**
others	20/**.45**	20/**.44**	**2%**
connectors	21/**.48**	14/**.31**	**55%**
Purpose	29/**.66**	20/**.44**	**50%**
subordination	27/**.61**	20/**.44**	**37%**
connectors	2/		
Condition	21/**.44**	28/**.6**	**22%**
Contrast	29/**.66**	22/**.4**	**39%**
subordination	14/**.32**	18/**.40**	**20%**
connectors	13/**.29**	4/**.09**	**222%**
Place	19/**.43**	3/**.07**	

Addition	46/**1.04**	38/**.84**	**24%**
TOTAL	553/**12.52**	489/**10.86**	**15%**

Subordinating Conjunctions:

Time: When (the most common one)/ As soon as/ Before/ After/ etc.

Cause: Because (the most common one)/ As/ etc.

Purpose: In order that/ So that / etc.

Condition: If / Unless/ etc.

Contrast: Although/ Even though/etc.

Place: wherever/ etc.

Transitional Words or Connectors:

Time: First/ Second/ Finally/etc.

Cause: Therefore/ As a result/ In consequence/ Because of this/etc.

Contrast: However/ Nevertheless/ On the contrary/etc.

Addition: In addition/ Moreover/ Also etc.

Example: For example/ In fact/ etc.

These measurements provided an independent confirmation of the holistic assessment that the essays written under the influence of algebraic thinking were more cohesive, more thoughtfully written. Despite the novelty of this observation one should not be surprised by it. Algebra, as an abstract area, depends a lot on the relationship between different concepts, ideas and mental actions. Connectors and subordinating clauses are those particles of language which are used to express the relationship between ideas, events or facts; these are words such as "because", "in order to", "finally", "if….then." They are used to express cause and effect relationships, conditions, reasons, and contrast; thus, they seem to be closely related to what is called a critical (or analytical) mode of thinking. ". A correct use of connectors determines the organization of ideas within an essay. The increase in the (correct) use of these linguistic tools meant that there was an increase in the number of relationships between ideas, their better organization expressed by our students in their writing, making it more cohesive. The correlation of the ESL syllabus with the Algebra course - whose dense mathematical relationships, when translated into natural language with the help of connectors, were able to penetrate the simpler language of descriptive writing - induced an increase in the level of thinking effected by the ZPD.

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