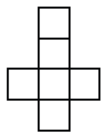
FIR TREE Aha! Moment

Consider the following function that generates the geometric pattern of a reverse growing fir tree.

Stage 1

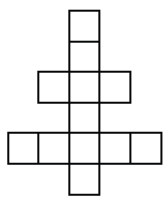
2 unit squares

Stage 2



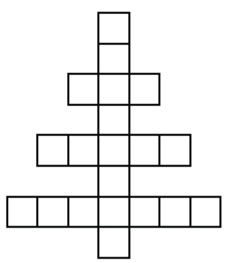
6 unit squares

Stage 3

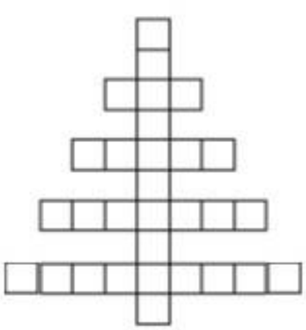


12 unit squares

Stage 4



1. Draw and describe **Stage 5** of the pattern in terms of its shape and number of unit squares needed to construct the fir tree.



1. Stage 5's shape is bigger than the previous one; its shape grows horizontally and follows the pattern of stage 4. It has 30 unit squares meaning that it increased by 10 units.

2. Describe how the pattern is growing?

   2. *The pattern is growing by increasing 1 unit square to the left, 1 unit square to the left, one additional unit in the center and 1 unit down compared to stage 4.*

3. How many unit squares are needed to build a **Stage 10** Aussie Fir Tree? Show your work.

It would need 110 unit squares.

I developed this answer in the next page.

4. Given any stage number **n**, determine a closed form equation to determine the amount of unit squares needed to build the tree.

The formula is: n(n+1).
Any stage number multiply by its following number equals the number of unit square, for instance: stage number: 3 multiplied by its following number meaning 4 equals 12 (unit squares.)

5. Your mate tells you that exactly 274 unit squares will make an Aussie Fir Tree. He is wrong. Explain to him why his statement is false.

Stage 1	1(1+1)= 2

Answer #3

Stage 2	2(2+1)=6
Stage 3	3(3+1)=12
Stage 4	4(4+1)=20
Stage 5	5(5+1)=30
Stage 6	6(6+1)=42
Stage 7	7(7+1)=56
Stage 8	8(8+1)=72

Stage 9	9(9+1)= 90
Stage 10	10(10+1)= 110

Answer #5
It is really wrong because:
Stage 16= 16(16+1)= 272
Stage 17= 17(17+1)= 306
The fir tree would never have 274 unit squares.

had a tremendous Aha! Moment. I just realized that the formula I got from the patterns was a factorized expression and if I multiply it "n(n+1)= units square" I would have something like an algebraic expression exactly a trinomial expression that can be factorized as well, and it equals real numbers for example: n(n+1)= 12 n^2+n= 12 n^2+n-12=0 n^2+n-20=0 n^2+n-56=0 n^2+n-90 (n+4)(n-3)=0 (n+5)(n-4)=0 (n+8)(n-7)=0 (n+10)(n-9) And when it comes to n^2+n-274 it cannot be factorized.

I just realised that if I solve the formula "n(n+1)= units square" I have something like an algebraic expression that can be factorize and it equals real numbers for example:
n(n+1)= 12
n^2+n= 12
n^2+n-12=0 n^2+n-20=0 n^2+n-56=0 n^2+n-90
(n+4)(n-3)=0 (n+5)(n-4)=0 (n+8)(n-7)=0 (n+10)(n-9)
And when it comes to n^2+n-274 it cannot be factorize.

Domain of the Square Root function.

The problem starts with the function $f(x) = \sqrt{x+3}$. The teacher asked the students during the review: "Can all real values of *x* be used for the domain of the function $\sqrt{x+3}$?"

Student (**S**): (**1**) "No, negative *x*'s cannot be used." (The student habitually confuses the general rule, which states that for the function $\sqrt{x}$ only non-negative values can be used as the domain of definition, with the particular application of this rule to $\sqrt{x+3}$.)

Teacher (**T**): (**2**) "How about $x = -5$?"
S: (**3**) "No good."
T: (**4**) "How about $x = -4$?"
S: (**5**) "No good either."
T: (**6**) "How about $x = -3$?"
Student, after a minute of thought: (**7**) "It works here."
T: (**8**) "How about $x = -2$?"
S: (**9**) "It works here too."
A moment later the **student** adds: (**10**) "Those x's which are smaller than -3 can't be used here." (***Elimination of the habit through original creative generalization.***)
T: (**11**) "How about $g(x) = \sqrt{x-1}$?"
Student, after a minute of thought: (**12**) "Smaller than 1 can't be used."
T: (**13**) "In that case, how about $h(x) = \sqrt{x-a}$?"
S: (**14**) "Smaller than *a* cannot be used."(***Second creative generalization***)

Aha!Moment in Physics

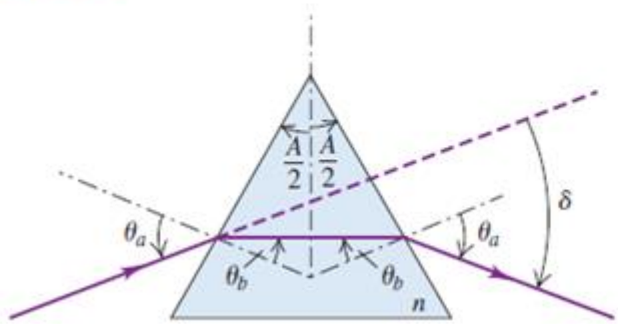
I experienced an Aha!Moment when I felt a sense of contentment. Prior to this emotion I had spent about an half an hour trying to figure out the following problem:

33.59 •• Angle of Deviation. The incident angle θ_a shown in Fig. P33.59 is chosen so that the light passes symmetrically through the prism, which has refractive index n and apex angle A. (a) Show that the angle of deviation δ (the angle between the initial and final directions of the ray) is given by

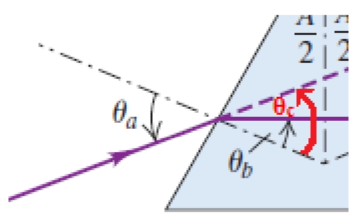
$$\sin\frac{A+\delta}{2} = n\sin\frac{A}{2}$$

(When the light passes through symmetrically, as shown, the angle of deviation is a minimum.)

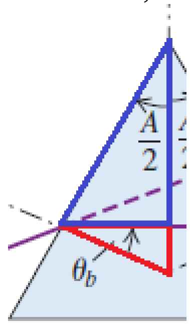
Figure **P33.59**



After a significant amount of trial and error, I realized that to be able to solve this problem I would have to use the basic geometry that I was taught as a child. I realized that awas equal to b+cas shown in the below image.



Once I remembered the rule to angles called the X angle property, I was able to apply it to the problem. Now that I had remembered the X angle property I had to figure out what either bor cwere to be able to continue to solve the problem. I could not bring myself to figure this out so I began to play with the triangles within the prism to see if anything could jog my memory. After a lot of sketching and redrawing I tried the two right angle triangles. I thought that my thought process was wrong since the two right angles that I could see were both different sizes. Using the right triangle rules and opening the right triangles on a piece of paper I was able to see the problem differently. When I opened the right triangles I noticed that they were the same and since we were dealing with variables and not actual numbers I could apply the given variables to continue solving the problem. I determined that the blue right triangle and the red right triangle were equal when dealing with variables, as shown below.



This meant that b was also equal to A2. Now I could proceed with the problem, I started with Snell's Law of Refraction, $n_a \sin\theta_a = n_b \sin\vartheta_b$. I had already determined that a=A2+cand because we're only dealing with a prism and air I remembered that the index of refraction for air is $n_{air}=1.00$. I substituted the known variables into Snell's Law of Refraction equation to show that my final equation was equal to the equation given.

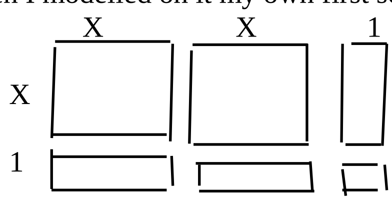
AHA!MOMENT: $2x^2 + 3x + \square$

I did experience gentle Aha! Moment, when suddenly I felt empowered to go that is to generalize from the separate concrete examples into the general case, which helped me to solve the problem.
I found one case without the hint but did not know what does it mean here looking for maximum and minimum, and how to look for Them. The case I found was $2x^2 + 3x + 1$ and I found following the method I teach that is multiply a by c and then find its two factors whose sum is b. Then I looked at the Hint, which suggested the visual model of the tiles asking for its possible transformations.

$2x^2 + 3x + \square$ Fill in the blanks by finding the largest and smallest integers that will make the quadratic expression factorable.

Hint
How would you represent this problem using visually (such as by using algebra tiles)?

Then I recalled the method of the tiles on the example $x^2 + 3x + 2 = (x+1)(x+2)$.
Then I modelled on it my own first solution, which was $2x^2 + 3x + 1$,



and then I started playing with trying different possibilities, of which I could not find any by changing for different positive numbers, even daring to employ fractions. Only when I tried (2x-1)(x+2) and saw, it works, I had the gentle, but definitely noticeable on both cognitive as generalization and affective as empowerment, Aha! Moment, which sent me to formulate the general condition of (2x + a)(x + b) as the hidden analogy between examples I did with the condition that a+2b = 3. Once I had this condition I could start getting the polynomials that work by assuming b and getting a, showing that only for b=1 I get positive a, for any other positive b I get negative a. For negative b I get positive a. However since, the coefficient $c = ab < 0$ for $b \neq 1, 0$, and that means that 1 is maximum, while there is no minimum.

An elephant Aha!Moment

Linear equations with one unknown can be solved already by students in the elementary school. Those are simple equations and students often formulate them by themselves while solving word problems. Sometimes the problems lead to equations a bit more complex than the elementary additive equations of the type $x + a = b$.

I have had an opportunity to listen to the discussion of two enthusiastic students solving a standard word problem: *The sum of two numbers is 76. One of the numbers is 12 more than the other. Find both numbers.* It was a problem from Semadeni's set of problems for the 3rd grade and one had to solve it using equations and that's where the difficulty appeared:

Przemek (read Pshemik) wrote the equation: $x + (x+12) = 76$. To solve it was a bit of a problem for him, but still he dealt with it. He drew an interval and then a following dialog had taken place [between him and his friend Bart]:
P: *That is that number*: he extended this interval by almost the same length, and the another one like that.
And this is that number plus 12
B*: and this all together is equal to 76…*
P*: No, this is an equation, d'you understand…*
B could not accept it…
B: *Why did you draw this interval? You don't know yet what it's supposed to be?*
P: *That's not important.*
B: *Why 76?*
P: '*cause that's what is in the problem*
B*: that iks, that iks add 12 and that's supposed to be 76..?*
P: *Look instead of iks there is a little square in the book* – P showed the little square in the book.
B: *Aha, but here, here is written something else*
P: *But it could be as here. And now I am inputting a number into this square.*
B: *A number?! Why into the square?*
P: *No, it's into the window. Into this window I input the number which comes out here.*
B: *But here is a square* – B insisted.
P: *It's not a square but a window, and one inputs the numbers into that window.*
B: *How so*?:
P: *Two windows are equal 64, one window is equal 32. Well, now, you subtract 12 from both sides*, and *you see that the two windows are equal to 64.*
B: *But are there numbers in the windows*?
P: *Two windows are 64, so one window is 32*
B: *Window!?*
P: *That's right, a window. Look here:* ***an elephant*** *and* ***an elephant*** *is equal* ***64****. Therefore what is* ***one elephant*** *equal to?* ***Two elephants*** *are equal* ***64****. So,* ***one elephant*** *is* ***equal*** *to what*?
B: ***An elephant****? Hmm, I see.* ***One elephant equals 32****. I understand now… so now the equation…*
P: *If* ***two elephants*** *are equal* ***60****, then* ***one elephant*** *is* ***equal*** *what?*
B: An elephant?, ok, one elephant equals 30. I see it now…..Now equation…………..**aaaaaaa**

Calculus Aha! Moment

During my Calculus 1, the teacher gave us an example to solve: $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

1- I verify if the limit is defined when X approaching to 0. It is not.

I asked myself "how can I do and find a way for this limit can be define?"
I remember in my previews class math 150 when the teacher gave us a rational fraction to solve, he said that we must eliminate the radical in the denominator by multiplicated by the conjugate. But for this equation we don't have radical in the denominator but in the numerator.
I'm a little bit struggling. What can I do?
I was looking at the limit, and said to myself why not apply the same rule for the fraction when we have the radical in the denominator.
Looking at the limit in my mind I try to solve the problem and ….. Aha!!!!! Multiplying by the conjugate in the numerator and the denominator will solve the problem. The feeling of satisfaction and to be powerful invaded me.

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x} = \lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x})(\sqrt{1+x}+\sqrt{1-x})}{x(\sqrt{1+x}+\sqrt{1-x})}$$

$$= \lim_{x\to 0}\frac{(\sqrt{1+x})^2-(\sqrt{1-x})^2}{x(\sqrt{1+x}+\sqrt{1-x})}$$

$$= \lim_{x\to 0}\frac{1+x-1+x}{x(\sqrt{1+x}+\sqrt{1-x})}$$

$$= \lim_{x\to 0}\frac{2x}{x(\sqrt{1+x}+\sqrt{1-x})}$$

$$= \lim_{x\to 0}\frac{2}{(\sqrt{1+x}+\sqrt{1-x})}$$

Now the limit is defined. I can solve it and finish.

Composition of functions Aha!Moment

I was given the problem gof(x) where f(x) = 2x+1 and g(x)= X^2. I went ahead and tried to solve the problem.

1. I rewrote the equation to make it clear, giving me g(f(x)).
2. Then, g(2x+1)
3. Lastly, I got the answer $2x^2+1$

But the answer was wrong. Then I tried to figure out why the answer was wrong. I assume that g(x) = x^2 meant, wherever x is, it must be squared, that's not the case. I realized that x actually served as an input. So wherever x was, you just have to input the corresponding equation. For example, g(x) = x^2 would be written as g(input)=$(\text{input})^2$. As a result of restructuring the problem, g$(\text{input})^2$, I got the answer $(2x+1)^2$.