CREATIVITY ≠ CREATIVITY

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Aha!Moments might have a big effect on the motivation of students. A cause might be their perception of creativity which often is demanded in mathematics lessons. As appeared with the study of students of grade 12 and 13 the understanding of creativity might be divided into two different classes in connection with extension of knowledge and the application already familiar contains concerning the topic. During student's projects differences appeared in whether foreknowledge of students was used to attain solutions and applied to the projects or they became acquainted with some new topics to apply it to their project. Connections appeared between different student's perception of creativity and the elaboration to their projects and lead to Aha!Moments.

INTRODUCTION

Everybody got a certain imagination of creativity. In literature might be found a lot of different perceptions of creativity exist. Wallace (1926) defined creativity based on the Gestalt theories by (i) *preparation*, (ii) *incubation*, (iii) *intimation*, (iv) *illumination* and (v) *verification*. On the other hand Wallas & Kogan (1965) gave another definition of creativity by (i) *originality*, (ii) *fluency*, (iii) *flexibility* and (iv) *elaboration*. In Torrance's test of creative thinking with words creativity (Torrance, 1974) is given by *fluency*, *flexibility*, *originality* and *elaboration*.

In accordance to Torrance (1974) and Silver (1997) in Leikin (2009) the measurement of creativity in solutions of mathematics exercises is divided in *fluency*, *flexibility*, *total creativity* and *final creativity*, which got a scaling for specific exercises.

As is displayes, no single, authoritative perspective of definition of creativity exists (Mann, 2006; Sriraman, 2005; Kattou et al., 2011; Nadjafikhah, Yaftian & Bakhshalizadeh, 2012).

In literature sometimes is distinguished between *creativity* and *mathematical* creativity. In Liljedahl & Sriraman (2006) a definition of professional level mathematical creativity is given by (p. 18)

(i) the ability to produce original work that significantly extends the body of knowledge (which could also include significant syntheses and extensions of known ideas)

(ii) opens up avenues of new questions for other mathematicians.

Creativity at the school levels are (p. 19)

(iii) the process that results in unusual (novel) and/or insightful solution(s) to a given problem or analogous problems, and/or

(iv) the formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle.

Connections between Aha!Moments and creativity exist (Liljedahl, 2004; Liljedahl, 2005). Thus Aha!Moments might be influenced by conjunctions between creativity and phases of projects. Therefore different Aha!Moments appear by different opinions of students about creativity in relationship with the procedure of their projects. Aha!Moments of distinct types might lead to increased motivation anyway.

Previous studies concerning creativity did not regard the opinion of students about creativity. Furthermore up to now no investigation of relationships between students' opinion about creativity in relationship to sequences of lessens or projects exist. The coherence between the two is focus of this article. Therefore results of students' projects and their description of creativity contained in videos are considered.

STRUCTURE OF COURSES

According to the Conference of the Ministers of Education of the Länder in the Federal Republic of Germany (Kultusministerkonferenz, 2013), students can improve their ability to study with help of a "special learning performance" (*besondere Lernleistung*). As response some of the federal states designed special courses in the last three years before graduation. In North Rhine-Westphalia (NRW) those project courses started in the school year 2011/2012 and take place for a year during the last two years before graduation (*Abitur*, Ministry of Education in NRW, 2010). By the Ministry of Education in NRW, students should have the possibility to study autonomously and cooperatively in connection with projects and applications as well as in interdisciplinary contexts.

Because of shortage in mathematics teachers[1], a lot of schools were unable to establish project courses. Inspired hereby the Institute of Mathematics and Computer Science at the University of Münster in connection with the Institute of Education of Mathematics and Computer Science established project courses of different topics in connection with the study described above.

The author taught two courses of the topic *coding and cryptography* in each of the school years 2013/2014 and 2014/2015. In the school year 2013/2014 the courses started with an introduction to mathematical foundations as numbers, groups, rings and fields, matrices, basics of topology, plane algebraic curves, probability and different algorithms. Over this phase foundations were set inside the lessons, and the students had to study autonomously mathematical and application-oriented backgrounds in between the lessons, see the left side of figure 1. In the courses over the school year

[1] These shortages appear besides mathematics in all sciences and computer science.

2014/2015 instead of the introduction to mathematical foundation the students started with short projects over coding including mathematical contains. Results were presented each other at the end of every project.

After the phases of introduction to mathematical foundations or short projects, the students were encouraged to look for topics for projects on coding. In case they were unable to find topics themselves, the teacher suggested topics. With one exception, all students were unable to find topics themselves and accepted suggestions by the teacher. After choosing topics the students started to elaborate their projects for two months and to prepare presentations. Except one group of two members, all of the students worked alone. Later they collected their results alone or two by two.

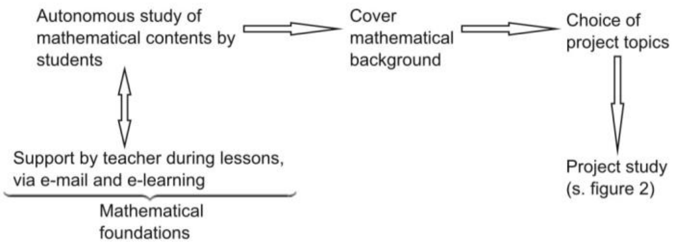

FIG. 1. PHASES OF THE COURSE IN THE FIRST QUARTER OF THE SCHOOL YEAR.

The students began to process their projects in a way which might be described by cycles. At the beginning they studied the topics autonomously. During the time of four up to six weeks they had to study their topics. All this time the students were offered support by the teacher via e-mail or by e-learning and were given hints of literature as well as copied material. In the end they were expected to prepare a presentation. Afterwards they presented the results of their work up to then to the class, followed by a short discussion including improvement suggestions by the other students and the teacher. The students extended their projects afterwards, see figure 2.

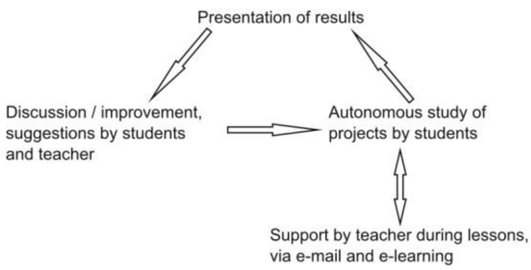

FIG. 2. CYCLE OF PROJECT STUDY.

Five months after the beginning of the school year the students presented the final results of their projects for about 15 minutes each. Because the mathematical foundations in the beginning of the school year included the principles of all topics, the students were able to understand the mathematical contents of the presentations.

The next phase of lessons began with an introduction to cryptography by lectures of the teacher. Afterwards the students worked on another two projects phased analogous to project 1 and shown in figures 1 and 2. The topics of projects might be connected, e.g. by project 2 *cryptography with elliptic curves* and project 3 *public key in relationship with elliptic curves*. The execution of each project took two months. The school year finished immediately after the presentation of the last projects.

METHODS

The assessment includes qualitative and quantitative elements which are aligned by isomorphic transformation following Mixed Methods. Quantitative data were taken at five different moments all over the school year with identical questionnaires. Furthermore in the middle and at the end of the school year semi-structured interviews were taken. The moments of the data collections were taken in connection with changes of topics of the courses over the whole year, see figure 3.

The author considered the questionnaires and created the semi-structured interviews based on observation at the first questionnaires. Because the courses were taught by the author, different people conducted the interviews. The author assured his students that he would not listen to the interviews until the end of the school year.

The students kept research notebooks and learning diaries in one folder. The parallel structure of these parts of the folder allowed for interconnections between research notes and diary texts. At the end of the school year the author made copies of them.

As mentioned above the students presented their preliminary and final results of their projects. The final presentations of each project included a summary of contents for all the other students. The author took copies from every presentation and all summaries.

The analysis of data is based on interviews, questionnaires, notebooks, learning diaries, presentation and summaries of students. For triangulation of questionnaires and interviews several parts of the interviews were transformed from qualitative to quantitative form after transcription.

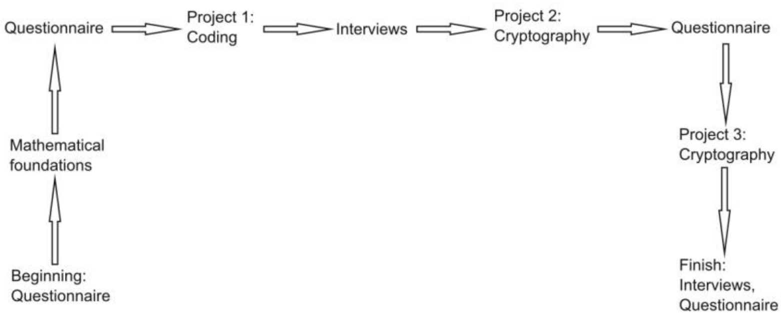

Fig. 3. Data collection during the study.

DATA COLLECTION AND PROGRESSION OF THE COURSES

Over the school year all students filled out four questionnaires and gave two interviews. The questionnaires are created based on Schommer-Aikins et al. (2000) and Urhahne and Hopf (2004) concerning epistemological beliefs, additional aspects of beliefs related to mathematics (from now on denoted as *mathematical beliefs*) based on Kloosterman and Stage (1992). All the questionnaires used over the year were the same.

The interviews covered *knowledge and understanding of educational contents*, *perception of mathematical contents, association of mathematics and its implementation* and attitudes about statements concerning mathematics: *logical nature, empirical nature, creativity and imagination, discovered vs. invented, socio-cultural aspects, scientific aspects* (Liu & Liu, 2011). The answers are categorized by *mathematical understanding, opinion about mathematical background, definition of mathematics, application of mathematics in society* and *meaning of mathematics for nature.*

Moreover, the first interviews started with a narrative part concerning the contents of the courses, where students were asked to give a description of the mathematical foundations of the course and their own projects as well as projects by other students. The second interviews differed from the first ones in the narrative part about the contents of the course. Students were required to describe contents of their own projects and projects of the other students from all over the school year, not only about the second half.

The interviews were taken by colleagues of the author, the questionnaires were distributed and collected by himself. The first interviews were recorded with 35 students. The second interviews were conducted with only 27, as some of the students did not appear. With 39 students at least one interview was recorded.

At the beginning of the first lesson the students filled in questionnaires. Then the course started with an introduction to coding and cryptography including lectures and exercises over four lessons followed by the treatment of mathematical foundations useful for the prospective contains of the course for more than three months. The second questionnaires were filled in at the end of the phase of mathematical foundations for students' projects. Immediately afterwards the students started their first projects about coding and processed them for about two months.

After the presentation of final results of students' first projects semi-structured interviews were conducted. Subsequent to the interviews an introduction in cryptography started followed by students' first projects about cryptography. At the end of the next quarter of the school year the students presented the results of their projects. Immediately after the presentations the students filled in the third questionnaires.

Afterwards the students started the last projects of the course and presented results about two months later. Immediately after the presentation of results students gave the second interviews and then completed the fourth questionnaires in the last lesson of the school year.

RESULTS

The interviews of all the 39 students are used for this investigation. If the second interview exists it was taken for the study. If only the first interview exists this one was taken for the analysis.

The progressions of projects will be divided into two types. In *Progress 1* the projects only include application of topics of the course or regular mathematics lessons processed before the beginning of projects. If students came to grips with new mathematical contains while editing their projects, the progression will be denoted by *Progress 2*.

In the following three project topics will be described and analyzed. Every time projects of type *Progress 1* and *Progress 2* approaching the same topic will be analyzed and compared. On the one hand contents and progression of each project will be described. Afterwards both projects will be analyzed concerning creativity. As progression creativity will be divided in two cases. In the case *Creativity 1* students understand creativity as an extension of knowledge to reach more comprehension of mathematics with a look on their projects. The case that students only consider creativity in connection with topics treated before in lessons of mathematics and the course will be denoted by *Creativity 2*. Here relationships between creativity and the progression of projects will be considered.

An example of Creativity 1 of student [S1] is given by a project of coding with Compact Disk (CD). In the beginning he attends to coding, decoding and implied the topic binary code treated before. Interest in details of coding and the transfer of waves appeared. Furthermore he asked himself what might happen with damages or pollutions of a CD. Here [S1] included the Reed-Solomon error correction treated before.

Afterwards [S1] examined the functionality of the phonograph record. He explored the difference between the data transfer of phonograph record and CD. Moreover he analyzed the working of MP3-format. During this sequence he came across to Fourier-transformation and had a look at complex numbers and the exponential function.

In contrast to [S1] the student [S2] processed the topic CD with Creativity 2. He included trigonometry, binary code, Hamming distance, and Reed-Solomon code in his project. All the topics were included in lessons before. [S2] did not include more topics into his project.

Students [S3] und [S4] chose the topic entropy in connection to coding theory. Based on a short introduction by the teacher and foundations of school [S3] considered the entropy from coding for some time. He came upon entropy from thermodynamics. Afterwards [S3] discovered several aspects of entropy in mathematics and physic and even connection between them. In addition he had a look at mathematical foundations of thermodynamics. While editing the topic [S3] wanted to calculate the entropy of several systems. To avoid calculation by hand he learned foundations of Python. Furthermore while starting the documentation of his project [S3] came across a lot of formulae. To avoid writing down a lot of formulae with Word he learned LaTeX.

[S4] again processed the contents of the document of the teacher and added aspects of logarithm function and probability theory. Everything has been included in mathematics at school before.

[S5] and [S6] chose for their projects the topic about elliptic curves in coding. After a short introduction into elliptic curves by the teacher [S5] and [S6] started their projects. First they had a look at more foundations of elliptic curves. Afterwards [S6] continued coding with elliptic curves with already known algebra and in the beginning of the course treated finite fields $\mathbb{Z}_p$ with p prime as well as familiar analysis. By contrast [S5] continued with elliptic curves and a look at coding but added partial derivation, Public Key as well as proofs of mathematical contains.

An exception is given by another student [S7]. After sometime of his project concerning elliptic curves he attended to a lot of topics:

- Euclidean algorithm for computing the GCD
- Public Key and RSA (Rivest, Shamir & Adleman)
- Riemanns ζ-function
- Multidimensional calculus

[S7] left the topic with elliptc curves this way.

After induction [S1], [S3], and [S5] conducted their projects with consolidation of functional backgrounds. One can find the connection to Creativity 1, e.g. by [S1]:

> [S1]: […] if it’s a more complicated exercise, one needs creativity to apply the right ones (algorithm a.s.o.)
>
> ([…] wenn das ne größere, komplexere Aufgabe ist, […] muss man schon eine gewisse Kreativität beweisen, um die Richtigen immer anwenden zu können)

[S3] described creativity in a similar way:

> [S3]: […] one needs creativity anyway, because proofs or something else are put together with different parts of mathematics
>
> ([…] trotzdem brauch man noch Kreativität, denn oft genug sind ja noch Beweise oder so, aus verschiedenen Bereichen der Mathematik zusammengesetzt)

Alike described by [S5]:

> [S5]: […] that somebody, who is able to understand all of it and knows, why it works, […] finds a solution with his creativity
>
> ([…] dass jemand, der wirklich das gesamte mathematische Verständnis hat und genau weiß wieso die Sachen so funktionieren wie sie funktionieren, […] eben mit Hilfe seiner Kreativität sich einen eigenen Lösungsweg herausfindet)

On the other hand Creativity 2 appears with [S2]:

> [S2]: […] one can choose [creative], which procedures to take, but one always needs to choose the same procedure
>
> ([…] man kann zwar [kreativ] auswählen, welche Prozeduren, aber man muss […] schließlich […] immer die gleichen Prozeduren wählen)

Alike with [S4]:

> [S4]: Creativity allways plais a role, because if one […] applies given formulae one will find the right solution.
>
> (Kreativität spielt […] immer eine Rolle, denn wenn man […] den vorgegebenen Formeln […] folgt kann man zu den richtigen Lösungen kommen.)

[S6] shares this opinion.

Connections between the processes of projects and the opinion of creativity are visible for all students [S1] to [S6]. If a student described creativity by Creativity 1, his project’s process was of type Progress 1. In case of Creativity 2 the process was of type Progress 2. These relations appeared for 35 of 39 students of the four project courses.

REFLECTION

Projects and their processing as well as students' opinion might be divided into two types. Furthermore one can find relationships between student's project type and their perception of creativity. The results offer a relationship to school levels of Liljedahl & Sriraman (2006). The school level (iii) is connected with Creativity 2, where the process result insightful solution under usage of own technical foundations. In Creativity 1 besides school level (iii) one might find (iv), the formulation of new questions. Afterwards old problems/projects might be regarded from a new point of view and lead to new parts of the projects, e.g. by [S1], [S3] and [S5] or even encourage for new topics, for example [S7].

The perceptions of creativity by students might disagree with the perception of creativity of outsiders. Students often are postulated creativity by solving exercises a.s.o. In Leikin et al. (2013) teachers "consider students to be creative if they have investigative abilities, are mathematically flexible, and succeed in problem solving". In contrast the study shows that students' perception of creativity distinct from each other. Aha!Moments might appear in different ways and maybe not appear to teachers as Aha!Moments from their point of view. But with another look at the Progress of results one might realise Aha!Moments.

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