A Paradigmatic Example (from William Baker (2015)

Consider the square root domain question in the classroom of a teacher-researcher, demonstrating the interaction between student and instructor, in which the latter is able to get the student engaged in the thinking process and, hence, to facilitate student creativity. The domain of the function $\sqrt{x+3}$ is at the centre of the dialog.
Note that it is the spontaneous responses of the student from which the teacher-researcher creates/determines the next set of questions, thus, balancing two frames of reference, his/her own mathematical knowledge and the direction taken by the student. Similarly the student has his or her own train of thought and prompted by the teacher-researcher's questions, he or she must now balance two frames of reference to determine his or her next response.

The problem starts with the function $f(x) = \sqrt{x+3}$. The teacher asked the students during the review: "Can all real values of *x* be used for the domain of the function $\sqrt{x+3}$?"
Student (**S**): (**1**) "No, negative *x*'s cannot be used." (The student habitually confuses the general rule which states that for the function $\sqrt{x}$ only non-negative values can be used as the domain of definition, with the particular application of this rule to $\sqrt{x+3}$.)
Teacher (**T**): (**2**) "How about $x = -5$?"
S: (**3**) "No good."
T: (**4**) "How about $x = -4$?"
S: (**5**) "No good either."
T: (**6**) "How about $x = -3$?"
Student, after a minute of thought: (**7**) "It works here."
T: (**8**) "How about $x = -2$?"
S: (**9**) "It works here too."
A moment later the **student** adds: (**10**) "Those *x*'s which are smaller than -3 can't be used here." (***Elimination of the habit through original creative generalization.***)
T: (**11**) "How about $g(x) = \sqrt{x-1}$?"
Student, after a minute of thought: (**12**) "Smaller than 1 can't be used."
T: (**13**) "In that case, how about $h(x) = \sqrt{x-a}$?"
S: (**14**) "Smaller than *a* cannot be used."(***Second creative generalization***)

As was discussed earlier, Koestler defines a matrix as, "any pattern of behaviour governed by a code of fixed rules," (p. 38) and, in statement (**1**) above, the limitations of the students' internal matrix, or problem representation, are demonstrated. The teacher, adjusting to the students' limited matrix provides two examples (lines (**6**) and (**8**)) that provide a *perturbation,* or a *catalyst*, for cognitive conflict and change. Recall that, as Von Glasersfeld (1989) writes, "...perturbation is one of the conditions that set the stage for cognitive change" (p. 127).

In lines (**6**) – (**9**) the student reflects upon the results of the solution activity. Through the comparison of the results (records) they abstract a pattern, — "the learners' mental comparisons of the records allows for recognition of patterns" (Simon et al., 2004). Thus, in this example the synthesis of the student's matrix for substitution and evaluation of algebraic expressions with their limited matrix of what constitutes an appropriate domain for radical functions (bisociation) resulted in the cognitive growth demonstrated in line (**10**).

In lines (**11**) and (**12**), the perturbation brought about by the teacher's questions, leads the student to enter the second stage of the Piaget & Garcia's Triad. The student understood that the previously learned matrix or domain concept of radical functions, with proper modifications, extended to this example. They student was then able to reflect upon this pattern and abstract a general structural relationship in line (**14**), characteristic of the third stage of the Triad. (Piaget & Garcia, 1983)

The creativity of the teacher manifests itself in the scaffolding which led the student to the cognitive conflict between the two frames of reference. In the first case, the data driven results obtained through the matrix-process of substitution was synthesized with their limited matrix of the possible domain of a radical function. This bisociation, and the resulting abstraction, led to a more complete understanding of the possible domain for specific functions. This represents a transition from the first to the second stage of the Triad. Continuation of this questioning process led to further creative moments of understanding, in which the student was able to synthesize their understanding of the domain for two separate special cases of radical functions. This bisociation, and the resulting abstraction into structural understanding (line (**14**)), suggests that the student had crossed the ZPD from the second to the third stage of the Triad.